PHASE TRANSITIONS IN THE EXTENDED PARTICLE SYSTEMS, HAGEDORN TEMPERATURE AND CRITICAL DENSITY AND THE FRACTAL DIMENSION OF SPACE, AS A CONFINEMENT PHASE TRANSITION ORDER PARAMETER

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Talk at the XXIVth International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"
17-22 September 2018 in Dubna, Russia

One must be prepared to follow up the consequence of theory,
and feel that one just has to accept the consequences
no matter where they lead.
Paul Dirac

"... all things physical are information-
theoretic in origin ..."
John A. Wheeler

We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value.

In 1900, the British physicist Lord Kelvin is said to have pronounced: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Within three decades, quantum mechanics and Einstein's theory of relativity had revolutionized the field. Today, no physicist would dare assert that our physical knowledge of the universe is near completion. To the contrary, each new discovery seems to unlock a Pandora's box of even bigger, even deeper physics questions.

I always knew that sooner or later p - adic numbers will appear in Physics - André Weil.

In the Universe, matter has manly two geometric structures, homogeneous, [Weinberg, 1972] and hierarchical, [Okun, 1991] . The homogeneous structures are naturally described by real numbers with an infinite number of digits in the fractional part and usual archimedean metrics. The hierarchical structures are described with p-adic numbers with an infinite number of digits in the integer part and non-archimedean metrics, [Koblitz, 1977].

A discrete, finite, regularized, version of the homogenous structures are homogeneous lattices with constant steps and distance rising as arithmetic progression. The discrete version of the hierarchical structures is hierarchical lattice-tree with scale rising in geometric progression.

There is an opinion that present day theoretical physics needs (almost) all mathematics, and the progress of modern mathematics is stimulated by fundamental problems of theoretical physics.

Quarks and gluons can break free from their confinement inside protons and neutrons at a temperature of around $200 MeV$ - the temperature of the universe a fraction of a second after the Big Bang. We arrived at this figure by combining the results of supercomputer calculations and heavy-ion collision experiments. It puts our knowledge of quark matter on a firmer footing. According to the Big Bang model, the very early universe was filled with quark-gluon plasma, in which quarks and gluons (the carriers of the strong nuclear force) existed as individual entities. The strong force between quarks increases rapidly with distance, which means that the quarks need large amounts of energy to remain free - and therefore the plasma can only exist at extremely high temperatures. When the cosmos was only about a millionth of a second old, it had cooled to the point where quarks and gluons combined to form composite particles such as protons and neutrons.

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in the description of a number of various physical systems. Two important ones are neutron stars, which probe the low temperature and intermediate baryon chemical potential domain, and heavy ion collision experiments, which explore the region of the high temperature and low baryon chemical potential. There exist low-dimensional theories, such as (1+1)-dimensional chiral Gross-Neveu (GN) type models, that possess a lot of common features with QCD (renormalizability, asymptotic freedom, dimensional transmutation, the spontaneous breaking of chiral symmetry) and can be used as a laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies. Besides temperature and baryon density, there are additional parameters, for instance, an isotopic chemical potential μ_I . It allows to consider systems with isospin imbalance (different numbers of u and d quarks). It is realized, e.g., in neutron stars, heavy-ion experiments, etc.

The thermodynamics of QCD is most conveniently described by the grand canonical partition function [Le Bellac 1996]

$$\begin{aligned} Z(\alpha,\beta) &= tre^{-\alpha Q-\beta H} = \int dA d\bar{q} dq \exp(-S_A - S_q), \\ S_A &= \int_0^\beta dx_0 \int_V d^3x tr(F_{\mu\nu}^2/2), \\ S_q &= \int_0^\beta dx_0 \int_V d^3x \prod_{f=1}^{N_f} \bar{q}_f(\gamma_\mu D_\mu - m_f - \alpha/\beta\gamma_0)q_f \end{aligned} \tag{1}$$

Another form of functional representation is

$$Z(\alpha,\beta) = tre^{-\alpha Q-\beta H} = \int dA d\bar{q} dq \exp(-S_A - S_q),$$
$$S_A = \beta/\alpha \int_0^\alpha dx_0 \int_V d^3x tr(F_{\mu\nu}^2/2),$$
$$S_q = \int_0^\alpha dx_0 \int_V d^3x \prod_{f=1}^{N_f} \bar{q}_f(\gamma_\mu D_\mu - m_f - \gamma_0) q_f \quad (2)$$

The charge density we may interpret as a second hamiltonian: $Q = H_2,\ H_1 = H$ and consider corresponding classical Nambu's dynamics

$$\dot{A} = \{A, H_1, H_2\} \quad (3)$$

As a quantum Nambu's dynamics we consider above functional representations.

The partition function depends on the external macroscopic parameters V, T, μ , as well as on the microscopic parameters like masses and the coupling constant. Once the partition function is known, thermodynamic properties such as free energy, pressure, average particle numbers or the thermal expectation value of an operator A readily follow,

$$F = -T \ln Z \ , \ P = -\frac{\partial F}{\partial V}, \ < Q >= -\frac{\partial \ln Z}{\partial \mu} \tag{4}$$

Note that the functional form is a trace from evolution operator in imaginary time. For real time the definition is formal. Similarly, it is defined for imaginary chemical potential $i\mu$ and for real μ the definition is formal. So it is natural to consider Wick rotation for both parameters.

The rich structure of separated energy scales makes quarkonium an ideal probe of confinement and deconfinement. The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state. Different quarkonia will dissociate in a medium at different temperatures, providing a thermometer for the plasma.

Let us construct a model of a physical system with maximal (Hagedorn) temperature $T_H = \beta_H^{-1},\ T < T_H$. The system consists from N identical (noninteracting) subsystems s_n each of which can be in $p > 1$ states with same energy ε. So the number of states of the system with given energy $E = N\varepsilon$ is $M = p^N$ and corresponding entropy is $S = N \ln p$. To different energies E corresponds different $N = E/\varepsilon$. The statistical sum of the system is

$$Z(\beta) = \sum_E \rho(E) e^{-\beta E} = \sum_N e^{-(\beta - \beta_H)\varepsilon N},\ \beta > \beta_H = \varepsilon \ln p. \quad (5)$$

Note that, for QCD $T_H \simeq 150$ MeV. Quantum computers (quanputers) will become commercial interesting for systems with $N > 100$.

Let us consider a potential V and corresponding force F of an elementary charge e in a scalar field φ generated in $D-$dimensional euclidian space by corresponding point charge-source g defined as a solution of the following equation

$$\Delta_D\varphi = g\delta^D(x),$$
$$\int dx^D \Delta_D\varphi = \int dS_{D-1}\nabla\varphi = \Omega_D r^{D-1}\varphi'(r) = g \Downarrow$$
$$\varphi(r) = -\frac{1}{(D-2)\Omega_D}\frac{g}{r^{D-2}},\ D \neq 2$$
$$D = 2 : 2\pi r\varphi'(r) = g \Rightarrow \varphi(r) = \frac{g}{2\pi}\ln\frac{r}{r_0},$$
$$V_D = -\frac{1}{(D-2)\Omega_D}\frac{eg}{r^{D-2}},\ F_D = -\nabla V_D = -\frac{eg}{\Omega_D r^{D-1}},\ \Omega_3 = 4\pi$$
$$V_2 = \frac{eg}{2\pi}\ln\frac{r}{r_0},\ F_2 = -\frac{eg}{2\pi r} \quad (6)$$

For Newton potentials charges e, g play pole of masses, they are, by definition and in correspondence with observations, positive and for $D \geq 2$ we have attraction of masses. For $D < 2$ we have attraction and confinement. From Newton potential we obtain the Coulomb one if we take imaginary masses $m = ie_{\pm}$.

It is well known the Dirac-Pauli argument of the absence of a good time observable for the usual quantum dynamical systems. The Dirac's part of the argument consists in the statement that there is not a good momentum operator $\hat{p}$ if corresponding coordinate $\hat{x}$ has restricted spectrum. Any good momentum observable permits to reach from the state with coordinate x_0 any stay with coordinate x,

$$U(x-x_0)\psi(x_0)=\psi(x),\ \hat{x}\psi(x)=x\psi(x),\ U(x)=\exp(ix\hat{p}),\ [\bar{x},\hat{p}]=i, \quad (7)$$

In the coordinate representation: $\hat{x}=x,\ \hat{p}=-i\partial_x$. In momentum representation: $\hat{p}=p,\ \hat{x}=i\partial_p$. If we suppose that there is a good time observable $\hat{t},\ [\hat{H},\hat{t}]=i$, than it permits from a given state corresponding to a value of energy E_0 to reach any state with corresponding energy value E. But for usual systems the spectrum is restricted from below by the ground state energy, so there can not be a good time observable. This is the Pauli's part of the argument.

Let us consider rather general example of a dynamical system with good time observable. Let us consider nonrelativistic particle of mass m in the earth gravitational field $V = -mgz$,

$$H = \frac{p^2}{2m} + V(z), \tag{8}$$

with corresponding motion equations

$$\begin{aligned}
&\dot{z} = \frac{\partial H}{\partial p_z} = \frac{p_z}{m},\\
&\dot{p_z} = -\frac{\partial H}{\partial z} = mg \Rightarrow p_z(t) = mgt + p_{0z} \Downarrow\\
&t = \frac{p_z(t) - p_{0z}}{mg} \Rightarrow \hat{t} = \frac{\hat{p}_z(t) - \hat{p}_{0z}}{mg},\\
&[\hat{H}, \hat{t}] = [V, p_z(t)]/mg = i.
\end{aligned} \tag{9}$$

So, dynamical systems with unbounded energy spectrum may play role of the quantum clock with good time observable.

Let me draw the following scenario of confinement and particle production in QCD. In classical gluodynamics (in the simplest case, $A_0 = 0$ and finite energy assumption) particle-like solutions, monopoles, can not be due to scale (conformal) invariance. For nontrivial asymptotic A_0, we may have monopole states. In quantum gluodynamics we have not conformal invariance beyond the renormdynamic fixed points and monopoles can exist. In (one charge) quantum gluodynamics we have the trivial ultraviolet fixed point at $g = 0$ and nontrivial infrared fixed point at some $g = g_c$. The Higher energy multiparticle production processes follow the following scenario: higher energy quarks and gluons (perturbatively) produce lower energy gluons and quarks until the intermediate energy-scale where running coupling constant reach the selfdual (fixed) beyond of which monopoles start to produce. Later at the valence quark energies-scales, (at which $\alpha_s = 2$,) monopoles become unstable and decay into hadrons.

Acoustic cavitation-the formation and implosive collapse of bubbles-occurs when a liquid is exposed to intense sound. Cavitation can produce the emission of light, or sonoluminescence. The concentration of energy during the collapse is enormous: the energy of an emitted photon can exceed the energy density of the sound field by about twelve orders of magnitude, and it has long been predicted that the interior bubble temperature reaches thousands of degrees Kelvin during collapse.
Sonoluminescence refers to that remarkable phenomenon in which a small bubble of air injected into a container of water and suspended in a node of a strong acoustic standing wave emits light. More precisely, if it is driven with a standing wave of about 20,000 Hz at an overpressure of about 1 atm, the bubble expands and contracts in concert with the wave, from a maximum radius $R \sim 10^{-3}$ cm to a minimum radius of $r \sim 10^{-4}$ cm. Note that $R/r \sim 10$ one order of magnitude as in QCD with size for hadrons $R \sim 10^{-13}$ cm and perturbative size $r \sim 10^{-4}$ cm. Exactly at minimum radius roughly 1 million optical photons are emitted, for a total energy liberated of 10 MeV.

Let as consider the following formula

$$\frac{1}{1-x} = (1+x)(1+x^2)(1+x^4)..., \ |x| < 1. \tag{10}$$

which can be proved as

$$p_k \equiv (1+x)(1+x^2)(1+x^4)...(1+x^{2^k}) = \frac{1-x^{2^{(k+1)}}}{1-x},$$
$$|p_k| < c(1+|x|^{2^{(k+1)}}), \ \lim_{k\to\infty} p_k = c = 1/(1-x). \tag{11}$$

The formula (10) reminds us the boson and fermion statsums

$$Z_b = \frac{\sqrt{x}}{1-x}, \ Z_f = \frac{1+x}{\sqrt{x}}, \ x = \exp\left(-\beta\hbar\omega\right) \tag{12}$$

and can be transformed in the following relation

$$Z_b(\omega) = Z_f(\omega)Z_f(2\omega)Z_f(4\omega)... \tag{13}$$

Indeed, [Makhaldiani 2018]

$$Z_b(\omega) = \frac{\sqrt{x}}{1-x} = x^a Z_f(\omega) Z_f(2\omega) Z_f(4\omega)...,$$
$$a = 1 + (1 + 2 + 2^2 + ...) = 1 + \frac{1}{1-2} = 0, \ |2|_2 = 1/2 < 1. \quad (14)$$

By the way we have an extra bonus! We see that the fermi content of the boson wears the p-adic sense. The prime $p = 2$, in this case. Also, the vacuum energy of the oscillators wear p-adic sense.
What about other primes p? For the finite fields,

$$z_n(p) = \exp(2\pi i n/p), \ n = 0, 1, ..., p-1, \ \sum_n z_n = 0,$$
$$Z_p(\beta) = \sum_{n=1}^{p-1} \exp\left(-\beta E_n/\hbar\right), \ E_n = 2\pi\hbar(n+a),$$
$$Z_p(-i/p) = 0, \ p = 2, 3, 5, ...13...29...137... \quad (15)$$

In polynomial approximation of a function
$f(x) \simeq P_N(x) = a_0 + a_1 x + ... + a_N x^N$,

$$\begin{aligned} & a_0 + a_1 x_0 + a_2 x_0^2 + ... + a_N x_0^N = f(x_0) = f_0, \\ & a_0 + a_1 x_1 + a_2 x_1^2 + ... + a_N x_1^N = f(x_1) = f_1, \\ & ..., \\ & a_0 + a_1 x_1 + a_2 x_N^2 + ... + a_N x_N^N = f(x_N) = f_N, \end{aligned} \tag{16}$$

the coefficients $a_n, \ n = 0, 1, ..., N$ are defined as a solutions of the linear system of equations

$$VA = F, \ A^T = (a_0, a_1, ..., a_N), \ F^T = (f_0, f_1, ..., f_N),$$
$$V = \begin{pmatrix} 1 & x_0 & x_0^2 & ... & x_0^N \\ 1 & x_1 & x_1^2 & ... & x_1^N \\ 1 & x_2 & x_2^2 & ... & x_2^N \\ . & . & . & . & . \\ 1 & x_N & x_N^2 & ... & x_N^N \end{pmatrix} \tag{17}$$

Determinant of the Vandermonde matrix
$\Delta_N = \prod_{N \geq m > n \geq 0} (x_m - x_n)$, ($\Delta_0 = 1$, by definition). Indeed,

$$\Delta_1 = x_1 - x_0, \tag{18}$$

$$\begin{aligned}
\Delta_2 &= det \begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{pmatrix} = det \begin{pmatrix} 1 & x_0 & x_0^2 \\ 0 & x_1 - x_0 & x_1^2 - x_0^2 \\ 0 & x_2 - x_0 & x_2^2 - x_0^2 \end{pmatrix} \\
&= (x_1 - x_0)(x_2 - x_0) det \begin{pmatrix} 1 & x_1 + x_0 \\ 1 & x_2 + x_0 \end{pmatrix} \\
&= (x_2 - x_1)(x_2 - x_0)(x_1 - x_0), \\
\Delta_N &= (x_N - x_{N-1})...(x_N - x_0)\Delta_{N-1} = \prod_{1 \leq n \leq N} Z_n, \\
Z_n &= (x_n - x_{n-1})...(x_n - x_0)
\end{aligned} \tag{19}$$

There are two exceptional (simplest) case for discrete values of x: when $x_n = p^n, \ n = 0, 1, 2, ..., N$, and $x_n = x_0 + nh, \ n = 0, 1, 2, ..., N$.

In the first, geometric progression, case

$$
\begin{aligned}
&Z_n = (p^n - p^{n-1})(p^{n-1} - p^{n-2})...(p^n - 1) \\
&= p^{(1+2+...+n-1)}(p-1)^n \frac{p^n-1}{p-1}\frac{p^{n-1}-1}{p-1}...\frac{p-1}{p-1} \\
&= p^{n(n-1)/2}(p-1)^n[n]_p!,\ [n]_p = \frac{p^n-1}{p-1},\ \Delta_1 = Z_1 = (p-1), \\
&\Delta_2 = (p^2-p)(p^2-1)(p-1) = p(p-1)^3(p+1) \\
&= Z_2 Z_1 = p(p-1)^2(p+1)(p-1), \\
&\Delta_N = \prod_{1\le n\le N} Z_n = p^a(p-1)^b \prod_{1\le n\le N}[n]_p!,
\end{aligned}
\tag{20}
$$

$$a = \frac{1}{2}\sum_0^N n(n-1) = \frac{1}{2}(\sum_0^N x^n)^{(2)}|_{x=1} = \frac{1}{2}\frac{(1+\varepsilon)^{N+1}-1}{\epsilon}|_{\varepsilon=0}$$
$$= \frac{1}{2}(N+1+\frac{(N+1)N}{2}\varepsilon + \frac{(N+1)N(N-1)}{3!}\varepsilon^2 + ...)^{(2)}|_{\varepsilon=0}$$
$$= \frac{(N+1)N(N-1)}{6},$$
$$b = \sum_0^N n = N(N+1)/2,$$
$$\Delta_2 = p(p-1)^3(p+1),\ a=1,\ b=3. \tag{21}$$

For $p \gg 1$,

$$[n]_p \simeq p^{n-1},\ [n]_p! \simeq p^{n(n-1)/2},$$
$$\Delta_N \simeq p^{2a+b} = p^c,\ c = \frac{N(N+1/2)(N+1)}{3} = \sum_1^N n^2,$$
$$\Delta_1 \simeq p,\ \Delta_2 \simeq p^5. \tag{22}$$

For $p \ll 1, \Delta_N \simeq (-1)^b p^a,\ a = N(N^2-1)/6,\ b = N(N+1)/2,\ [n]_p \simeq 1,\ \Delta_2 \simeq -p$. Having expression for Δ_N in p, it is ease to obtain corresponding expression in arithmetic progression case by putting $p = 1+h:\ \Delta_N(h) = h^b \prod_1^N n!,\ b = N(N+1)/2, \Delta_2 = 2h^3$. We obtain the same result by direct calculation:
$Z_n = h \times 2h \times ... \times nh = h^n n!,\ \Delta_N(h) = \prod Z_n.$

Let us consider the integer derivatives of the monomials

$$\frac{d^n}{dx^n}x^m = m(m-1)...(m-(n-1))x^{m-n}, \quad n \le m, \\ = \frac{\Gamma(m+1)}{\Gamma(m+1-n)}x^{m-n}. \tag{23}$$

L.Euler (1707 - 1783) invented the following definition of the fractal derivatives,

$$\frac{d^\alpha}{dx^\alpha}x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha}. \tag{24}$$

J.Liouville (1809-1882) takes exponents as a base functions,

$$\frac{d^\alpha}{dx^\alpha}e^{ax} = a^\alpha e^{ax}. \tag{25}$$

The following Cauchy formula

$$I_{0,x}^{n} f = \int_0^x dx_n \int_0^{x_{n-1}} dx_{n-2} ... \int_0^{x_2} dx_1 f(x_1) = \frac{1}{\Gamma(n)} \int_0^x dy (x-y)^{n-1} f(y)($$

permits analytic extension from integer n to complex α,

$$I_{0,x}^{\alpha} f = \frac{1}{\Gamma(\alpha)} \int_0^x dy (x-y)^{\alpha-1} f(y) \quad (27)$$

J.H. Holmgren invented (in 1863) the following integral transformation,

$$D_{c,x}^{-\alpha} f = \frac{1}{\Gamma(\alpha)} \int\limits_c^x |x-t|^{\alpha-1} f(t) dt. \quad (28)$$

It is easy to show that

$$\begin{aligned} D_{c,x}^{-\alpha} x^m &= \frac{\Gamma(m+1)}{\Gamma(m+1+\alpha)} (x^{m+\alpha} - c^{m+\alpha}), \\ D_{c,x}^{-\alpha} e^{ax} &= a^{-\alpha} (e^{ax} - e^{ac}), \end{aligned} \quad (29)$$

so, $c = 0$, when $m + \alpha \geq 0$, in Holmgren's definition of the fractal calculus, corresponds to the Euler's definition, and $c = -\infty$, when $a > 0$, corresponds to the Liouville's definition.
Holmgren's definition of the fractal calculus reduce to the Euler's definition for finite c, and to the Liouvill's definition for $c = \infty$,

$$\begin{aligned} &D_{c,x}^{-\alpha} f = D_{0,x}^{-\alpha} f - D_{0,c}^{-\alpha} f, \\ &D_{\infty,x}^{-\alpha} f = D_{-\infty,x}^{-\alpha} f - D_{-\infty,\infty}^{-\alpha} f, \\ &D_{-\infty,x}^{-\alpha} f = D_{0,x}^{\alpha} f - D_{0,-\infty}^{-\alpha} f. \end{aligned} \quad (30)$$

We considered the following modification of the $c = 0$ case [Makhaldiani, 2003],

$$D_{0,x}^{-\alpha} f = \frac{|x|^\alpha}{\Gamma(\alpha)} \int_0^1 |1-t|^{\alpha-1} f(xt)dt, = \frac{|x|^\alpha}{\Gamma(\alpha)} B(\alpha,\ \partial x) f(x)$$
$$= |x|^\alpha \frac{\Gamma(\partial x)}{\Gamma(\alpha+\partial x)} f(x),\ f(xt) = t^{x\frac{d}{dx}} f(x). \quad (31)$$

As an example, consider Euler B-function,

$$B(\alpha,\beta) = \int_0^1 dx|1-x|^{\alpha-1}|x|^{\beta-1} = \Gamma(\alpha)\Gamma(\beta) D_{01}^{-\alpha} D_{0x}^{1-\beta} 1 = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad (32)$$

In QFT existence of a given theory means, that we can control its behavior at some scales (short or large distances) by renormalization theory [Collins, 1984].
If the theory exists, than we want to solve it, which means to determine what happens on other (large or short) scales. This is the problem (and content) of Renormdynamics.
The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one).
We can invent scale variable λ and consider QFT on $D+1+1$ dimensional space-time-scale. For the scale variable $\lambda \in (0,1]$ it is natural to consider q-discretization, $0<q<1,\ \lambda_n = q^n,\ n=0,1,2,\ ...$ and p - adic, nonarchimedian metric, with $q^{-1}=p$ - prime integer number.
The field variable $\varphi(x,t,\lambda)$ is complex function of the real, x, t, and p - adic, λ, variables. The solution of the UV renormdynamic problem means, to find evolution from finite to small scales with respect to the scale time $\tau = \ln \lambda/\lambda_0 \in (0,-\infty)$. Solution of the IR renormdynamic problem means to find evolution from finite to the large scales, $\tau = \ln \lambda/\lambda_0 \in (0,\infty)$.

We will call RDF functions $g_n = f_n(t)$ which are solutions of the RD motion equations

$$\dot{g}_n = \beta_n(g), 1 \leq n \leq N. \tag{33}$$

In the simplest case of one coupling constant the function $g = f(t)$ is constant, $g = g_c$ when $\beta(g_c) = 0$, or is invertible (monotone). Indeed,

$$\dot{g} = f'(t) = f'(f^{-1}(g)) = \beta(g). \tag{34}$$

Each monotone interval ends by UV and IR fixed points and describes corresponding phase of the system.
Note that the simplest case of the classical dynamics, the Hamiltonian system with one degree of freedom, is already two dimensional, so we have no analog of one charge renormdynamics.

The regular Hamiltonian systems of the classical mechanics are defined on the even dimensional phase space, so there is no analog of the three dimensional renormdynamics for the coupling constants of the SM. The fixed points of renormdynamics belong to a set of solutions of the polynomial system of equations $\beta_n(g) = 0, 1 \leq n \leq N$, in the perturbative renormdynamics. Describing the solutions is the task of contemporary algebraic and computational geometry.

The quantitative values and qualitative content of the given field theory depend on the scale (parameter, e.g. $\mu-$renormalization point, $g = g(\mu),\ A = A(\mu)$). In QCD e.g. the effective action has the following form:

$$S(\mu) = \frac{1}{g^2(\mu)} \int d^D x \mathcal{L}(A(\mu)), \tag{35}$$

variation with respect to the change of scale gives

$$\delta S = -2\frac{\beta(g)}{g\mu}\delta\mu S + \frac{1}{g^2}\int d^D x \frac{\delta \mathcal{L}}{\delta A}\delta A \tag{36}$$

and the following two statements are equivalent:

$$\delta S = 0,\ \beta(g) = 0 \Leftrightarrow \delta S = 0,\ \frac{\delta \mathcal{L}}{\delta A} = 0. \tag{37}$$

So, from renorminvariance of the effective action follows that at the conformal symmetric points, the motion equations for fields are satisfied. Generalization for the several coupling constants and other models is obvious.

In string theory, the connection between conformal invariance of the effective theory on the parametric world sheet and the motion equations of the fields on the embedding space is well known [Ketov, 2000]. A more recent topic in this direction is AdS/CFT Duality [Maldacena, 1988]. In this approach for QCD coupling constant the following expression was obtained [Brodsky, de Tèramond, Deur, 2010]

$$\alpha_{AdS}(Q^2) = \alpha(0)e^{-Q^2/4k^2}. \tag{38}$$

A corresponding β-function is

$$\beta(\alpha_{AdS}) = \frac{d\alpha_{AdS}}{\ln Q^2} = -\frac{Q^2}{4k^2}\alpha_{AdS}(Q^2) = \alpha_{AdS}(Q^2)\ln\frac{\alpha_{AdS}(Q^2)}{\alpha(0)} \leq 0 \tag{39}$$

So, this renormdynamics of QCD interpolates between the IR fixed point $\alpha(0)$, which we take as $\alpha(0) = 2$, and the UV fixed point $\alpha(\infty) = 0$.

For the QCD running coupling considered in [Diakonov, 2003]

$$\alpha(q^2) = \frac{4\pi}{9\ln(\frac{q^2+m_g^2}{\Lambda^2})}, \quad (40)$$

where $m_g = 0.88GeV,\ \Lambda = 0.28GeV$, the $\beta-$function of renormdynamics is

$$\beta(\alpha) = -\frac{\alpha^2}{k}(1 - c\exp(-\frac{k}{\alpha})),$$
$$k = \frac{4\pi}{9} = 1.40,\ c = \frac{m_g^2}{\Lambda^2} = e^{k/\alpha} = (3.143)^2 = 9.88, \quad (41)$$

for a nontrivial (IR) fixed point we have

$$\alpha_{IR} = k/\ln c = 0.61 \quad (42)$$

For $\alpha(m) = 2$, at valence quark scale m we predict the gluon (or valence quark) mass as

$$m_g = \Lambda e^{\frac{k}{2\alpha(m)}} = 1.42\Lambda = m_N/3,\ \Lambda = 220MeV. \quad (43)$$

Equality of the gluon and quark masses indicates on effective IR supersymmetry in QCD.

It is nice to have a nonperturbative $\beta-$function like (41), but it is more important to see which kind of nonperturbative corrections we need to have a phenomenological coupling constant dynamics.
It was noted [Voloshin, Ter-Martyrosian, 1984] that in valence quark parametrization $\alpha_s(m) = 2$, at a valence quark scale m.

In the 1870's G.J. Stoney [Stoney, 1881], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M :

$$l_S = \frac{e}{c^2}\sqrt{G},\ t_S = \frac{e}{c^3}\sqrt{G},\ m_S = \frac{e}{\sqrt{G}} \tag{44}$$

M. Planck introduced [Planck, 1899] as universal units of Nature for L, T, M:

$$m_P = \sqrt{\frac{hc}{G}} = \frac{m_S}{\sqrt{\alpha}},\ l_P = \frac{h}{cm_P} = \frac{l_S}{\sqrt{\alpha}} = 11.7 l_S,\ t_P = \frac{l_P}{c} = \frac{t_S}{\sqrt{\alpha}} \tag{45}$$

Stoney's fundamental constants are more fundamental just because they are less than Planck's constants :) Due to the value of $\alpha^{-1} = 137$, we can consider relativity theory and quantum mechanics as deformations of the classical mechanics when deformation parameter $c = 137$ (in units $e = 1, \hbar = 1$) and $\hbar = 137$ (in units $e = 1, c = 1$), correspondingly. These deformations have an analytic sense of p-adic convergent series. The number 137 has a very interesting geometric sense,

$$137 = 11^2 + 4^2, \qquad (46)$$

so, $\sqrt{137}$ is the hypotenuse length of a triangle with other sides of lengths 11 and 4.

The Babylonians used a base 60 number system which is still used for measuring time - 60 seconds in a minute, 60 minutes in an hour - and for measuring angle - 360 degrees in a full turn. The base 60 number system has its origin in the ration of the Sumerian mina (m) and Akkadian shekel (s), $m/s \simeq 60 = 3 \cdot 4 \cdot 5.$
We also can consider base 137 system for fundamental theories.

For the nuclear physics strong coupling phenomena description we may take as a base $p = 13$.
For the hadronic physics, valence scale QCD, and graphen strong coupling phenomena description we may take as a base $p = 2$.
For the weak coupling physics SM m_Z scale and MSSM unification scale phenomena description we may take as a base $p = 29$.

There are different opinions about the number of fundamental constants [Duff, Okun, Veneziano, 2001].
According to Okun, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$; the velocity of light, c; and Newton's constant, G.
According to Veneziano, there are only two: the string length L_s and c.
According to Duff, there are not fundamental constants at all.
Usually $L_s = l_p$, so, the fundamental area is $L_s^2 = 137 l_s^2$.
The value $s_s = l_s^2 -$ Stoney area, is more like on a fundamental area :)

In mathematics we have two kind of structures, discrete and continuous one. If a physical quantity has discrete values, it might have no dimension. If the values are continuous - the quantity might have a dimension, a unit of measure. These structures may depend on scale, e.g. on macroscopic scale condensed state of matter (and time) is well described as continuous medium, so we use dimensional units of length (and time). On the scale of atoms, the matter has a discrete structure, so we may count lattice sites and may not use a unit of length. If at small (e.g. at Plank) scale space (and/or time) is discrete, then we do not need a unit of length (time) for measuring, there is a fundamental length and we can just count.

Negative binomial distribution (NBD) is defined as

$$P(n) = \frac{\Gamma(n+r)}{n!\Gamma(r)} p^n (1-p)^r, \ \sum_{n \geq 0} P(n) = 1, \tag{47}$$

The Bose-Einstein distribution is a special case of NBD with $r = 1$.
NBD provides a very good parametrization for multiplicity distributions in e^+e^- annihilation; in deep inelastic lepton scattering; in proton-proton collisions; in proton-nucleus scattering. Hadronic collisions at high energies (LHC) lead to charged multiplicity distributions whose shapes are well fitted by a single NBD in fixed intervals of central (pseudo)rapidity η.

A Bose-Einstein, or geometrical, distribution is a thermal distribution for single state systems. An useful property of the negative binomial distribution with parameters

$$< n >,\ k$$

is that it is (also) the distribution of a sum of k independent random variables drawn from a Bose-Einstein distribution with mean $< n > /k$,

$$\begin{aligned} & P_n = \frac{1}{< n > +1}(\frac{< n >}{< n > +1})^n \\ & = (e^{\beta\hbar\omega/2} - e^{-\beta\hbar\omega/2})e^{-\beta\hbar\omega(n+1/2)},\ T = \frac{\hbar\omega}{\ln\frac{<n>+1}{<n>}} \\ & \sum_{n\geq 0} P_n = 1,\ \sum nP_n =< n >= \frac{1}{e^{\beta\hbar\omega - 1}},\ T \simeq \hbar\omega < n >,\ < n >\gg 1, \\ & P(x) = \sum_n x^n P_n = (1+ < n > (1-x))^{-1}. \end{aligned} \tag{48}$$

Indeed, for

$$n = n_1 + n_2 + ... + n_k, \tag{49}$$

with n_i independent of each other, the probability distribution of n is

$$P_n = \sum_{n_1,...,n_k} \delta(n - \sum n_i) p_{n_1} ... p_{n_k},$$
$$P(x) = \sum_n x^n P_n = p(x)^k \tag{50}$$

This has a consequence that an incoherent superposition of N emitters that have a negative binomial distribution with parameters $k, < n >$ produces a negative binomial distribution with parameters $Nk, N < n >$.
So, for the GF of NBD we have (N=2)

$$F(k, < n >)F(k, < n >) = F(2k, 2 < n >) \tag{51}$$

And more general formula (N=m) is

$$F(k, < n >)^m = F(mk, m < n >) \tag{52}$$

We can put this equation in the closed nonlocal form

$$Q_q F = F^q, \tag{53}$$

where

$$Q_q = q^D, \quad D = \frac{kd}{dk} + \frac{< n > d}{d < n >} = \frac{x_1 d}{dx_1} + \frac{x_2 d}{dx_2} \tag{54}$$

Note that temperature defined in (48) gives an estimation of the Glukvar temperature when it radiates hadrons. If we take $\hbar\omega = 100MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 1.5$ If we take $\hbar\omega = 10MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 20$. A singular behavior of $< n >$ may indicate corresponding phase transition and temperature. At that point we estimate characteristic quantum $\hbar\omega$.
We see that universality of NBD in hadron-production is similar to the universality of black body radiation.

The Riemann zeta function $\zeta(s)$ is defined for complex $s = \sigma + it$ and $\sigma > 1$ by the expansion

$$\zeta(s) = \sum_{n \geq 1} n^{-s}, \; Re \; s > 1,$$
$$= \delta_x^{-s} \frac{x}{1-x}|_{x \to 1} = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{-\delta_x t} \frac{x}{1-x}|_{x \to 1}$$
$$= \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{t \partial_\tau} \frac{1}{e^\tau - 1}|_{\tau \to 0} = \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1} dt}{e^t - 1}, \; x = e^{-\tau} \quad (55)$$

All complex zeros, $s = \alpha + i\beta$, of $\zeta(\sigma + it)$ function lie in the critical stripe $0 < \sigma < 1$, symmetrically with respect to the real axe and critical line $\sigma = 1/2$. So it is enough to investigate zeros with $\alpha \leq 1/2$ and $\beta > 0$. These zeros are of three type, with small, intermediate and big ordinates.

The Riemann hypothesis states that the (non-trivial) complex zeros of $\zeta(s)$ lie on the critical line $\sigma = 1/2$.
At the beginning of the XX century Polya and Hilbert made a conjecture that the imaginary part of the Riemann zeros could be the oscillation frequencies of a physical system (ζ - (mem)brane).
After the advent of Quantum Mechanics, the Polya-Hilbert conjecture was formulated as the existence of a self-adjoint operator whose spectrum contains the imaginary part of the Riemann zeros.
The Riemann hypothesis (RH) is a central problem in Pure Mathematics due to its connection with Number theory and other branches of Mathematics and Physics.

Functional Equation for Zeta Function

The functional equation is

$$\zeta(1-s) = \frac{2\Gamma(s)}{(2\pi)^s}\cos(\frac{\pi s}{2})\zeta(s) \quad (56)$$

From this equation we see the real (trivial) zeros of zeta function:

$$\zeta(-2n) = 0,\ n = 1, 2, ... \quad (57)$$

Also, at s=1, zeta has pole with reside 1.
From Field theory and statistical physics point of view, the functional equation (56) is duality relation, with self dual (or critical) line in the complex plane, at $s = 1/2 + i\beta$,

$$\zeta(\frac{1}{2} - i\beta) = \frac{2\Gamma(s)}{(2\pi)^s}\cos(\frac{\pi s}{2})\zeta(\frac{1}{2} + i\beta), \quad (58)$$

we see that complex zeros lie symmetrically with respect to the real axe.
On the critical line, (nontrivial) zeros of zeta corresponds to the infinite value of the free energy,

$$F = -T\ln\zeta. \quad (59)$$

At the point with $\beta = 14.134725...$ is located the first zero. In the interval $10 < \beta < 100$, zeta has 29 zeros. The first few million zeros have been computed and all lie on the critical line. It has been proved that uncountably many zeros lie on critical line.
The first relation of zeta function with prime numbers is given by the following formula,

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}, \ Re\ s > 1. \tag{60}$$

Another formula, which can be used on critical line, is

$$\begin{aligned}\zeta(s) &= (1 - 2^{1-s})^{-1} \sum_{n \geq 1} (-1)^{n+1} n^{-s} \\ &= \frac{1}{1 - 2^{1-s}} \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1} dt}{e^t + 1}, \ Re\ s > 0\end{aligned} \tag{61}$$

Let us consider the values $q = n, n = 1, 2, 3, ...$ and take sum of the corresponding equations (53), we find

$$\zeta(-D)F = \frac{F}{1-F} \tag{62}$$

In the case of the NBD we know the solutions of this equation.
Now we invent a Hamiltonian H with spectrum corresponding to the set of nontrivial zeros of the zeta function, in correspondence with Riemann hypothesis,

$$-D_n = \frac{n}{2} + iH_n,\ H_n = i(\frac{n}{2} + D_n),$$
$$D_n = x_1\partial_1 + x_2\partial_2 + ... + x_n\partial_n,\ H_n^+ = H_n = \sum_{m=1}^{n} H_1(x_m),$$
$$H_1 = i(\frac{1}{2} + x\partial_x) = -\frac{1}{2}(x\hat{p} + \hat{p}x),\ \hat{p} = -i\partial_x \tag{63}$$

The Hamiltonian $H = H_n$ is hermitian, its spectrum is real. The case $n = 1$ corresponds to the Riemann hypothesis.

From Qlike to Zeta Equations

The case $n = 2$, corresponds to NBD,

$$\zeta(1+iH_2)F = \frac{F}{1-F},\ \zeta(1+iH_2)|_F = \frac{1}{1-F},$$
$$F(x_1,x_2;h) = (1+\frac{x_1}{x_2}(1-h))^{-x_2} \qquad (64)$$

Let us scale $x_2 \to \lambda x_2$ and take $\lambda \to \infty$ in (64), we obtain

$$\zeta(\frac{1}{2}+iH_1(x))e^{-(1-h)x} = \frac{1}{e^{(1-h)x}-1},$$
$$\frac{1}{\zeta(\frac{1}{2}+iH(x))}\frac{1}{e^{\varepsilon x}-1} = e^{-\varepsilon x},$$
$$H(x) = i(\frac{1}{2}+x\partial_x) = -\frac{1}{2}(x\hat{p}+\hat{p}x),\ H^+ = H, \varepsilon = 1-h. \qquad (65)$$

Let us take an eigenvector $|n>$ with eigenvalue E_n of H, than

$$<n|\zeta(\frac{1}{2}+iH(x))e^{-(1-h)x}> = \zeta(\frac{1}{2}+iE_n(x)) <n|e^{-(1-h)x}>$$
$$= <n|\frac{1}{e^{(1-h)x}-1}> \qquad (66)$$

For zeros of Zeta function, E_n, the eigenfunctions fulfils the following conditions

$$<n|\frac{1}{e^{(1-h)x}-1}> = 0,\ <n|e^{-(1-h)x}> \neq 0. \qquad (67)$$

For eigenvalues of H, we have

$$
\begin{aligned}
&H|n>=E_n|n>,\ H=i(\frac{1}{2}+x\partial_x),\ |n>\sim x^{s_n},\ s_n=-\frac{1}{2}-iE_n,\\
&<n|\frac{1}{e^{(1-h)x}-1}>\sim\zeta(\frac{1}{2}+iE_n),\\
&<n|e^{-(1-h)x}>\sim\Gamma(\frac{1}{2}+iE_n).
\end{aligned}
\tag{68}
$$

Let us consider the following finite approximation of the Riemann zeta function

$$\zeta_N(s) = \sum_{n=1}^{N} n^{-s} = \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} \frac{e^{-t} - e^{-(N+1)t}}{1 - e^{-t}}$$
$$= \zeta(s) - \Delta_N(s), \;\; Re\ s > 1$$
$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1}}{e^t - 1}, \;\; \Delta_N(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1} e^{-Nt}}{e^t - 1} \quad (69)$$

Another formula, which can be used on critical line, is

$$\zeta(s) = (1 - 2^{1-s})^{-1} \sum_{n \geq 1} (-1)^{n+1} n^{-s}$$
$$= \frac{1}{1 - 2^{1-s}} \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1} dt}{e^t + 1}, \;\; Re\ s > 0 \quad (70)$$

Corresponding finite approximation of the Riemann zeta function is

$$\zeta_N(s) = (1-2^{1-s})^{-1}\sum_{n=1}^{N}(-1)^{n-1}n^{-s}$$
$$= \frac{1}{1-2^{1-s}}\frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}(1-(-e^{-t})^N)dt}{e^t+1} = \zeta(s)-\Delta_N(s),$$
$$\Delta_N(s) = \frac{1}{\Gamma(s)}\int_0^\infty dt\frac{t^{s-1}(-e^{-t})^N)}{e^t+1} \sim \pm N^{-s} \quad (71)$$

at a (nontrivial) zero of the zeta function, s_0, $\zeta_N(s_0) = -\Delta_N(s_0)$. In the integral form, dependence on N is analytic and we can consider any complex valued N.

It is interesting to see dependence (evolution) of zeros with N. For the simplest nontrivial integer $N=2$,

$$\zeta_2(s) = (1-2^{1-s})^{-1}(1-2^{-s}) \\ = \frac{1-2^{-s}}{1-2^{1-s}} = \frac{2^s-1}{2^s-2} = \frac{2^{s-1/2}-1/\sqrt{2}}{2^{s-1/2}-\sqrt{2}} \quad (72)$$

we have zeros at $s_n = 2\pi i n/\ln 2,\ n = 0, \pm 1, \pm 2, ...$
$2\pi/\ln 2 = 9.06$, so, in the interval $Ims_n \in (0,100)$ we have 10 nontrivial zeros. The first nontrivial zero of the zeta function, by Mathematica, is: $s_1 = 1/2 + i14.1347$. The last zero in the interval $Ims_n \in (0,100)$ is: $s_{29} = 1/2 + i98.8312$.
Another finite approximation of the zeta function is

$$\zeta_{p^N}(s) = \prod_{n=1}^{N}(1-p_n^{-s})^{-1} = \prod_{n=1}^{N}\zeta_{pn}(s),\ \zeta_{pn}(s) = (1-p_n^{-s})^{-1} \quad (73)$$

where p_N is N-th prime number.

We consider a novel method to generate a polynomial expression for each of the Euler sums,

$$E_k = \sum_{n=1}^{N} n^k, \; k \in Z^+(k = 0, 1, 2, ...) \qquad (74)$$

One of the way of calculation of the sum

$$E_k(N) = \frac{N^{k+1}}{k+1} + P_k(N), \; P_k = x_k N^k + x_{k-1} N^{k-1} + ... + x_0 \qquad (75)$$

we show by explicit calculation of E_2.
For particular values $N = 1, \; 2$ and 3, we have

$$\begin{aligned} x_2 + x_1 + x_0 = 1 - 1/3 = 2/3, \\ 4x_2 + 2x_1 + x_0 = 5 - 8/3 = 7/3, \\ 9x_2 + 3x_1 + x_0 = 14 - 27/3 = 5 \end{aligned} \qquad (76)$$

Subtracting from the second equation the first and from third the second, we obtain

$$3x_2 + x_1 = 5/3,$$
$$5x_2 + x_1 = 8/3 \tag{77}$$

than we have

$$2x_2 = 1 \Rightarrow x_2 = 1/2 \Downarrow$$
$$x_1 = 5/3 - 3x_2 = 1/6 \Rightarrow x_0 = 2/3 - x_1 - x_2 = 2/3 - 1/6 - 1/2 = 0, \Downarrow$$
$$E_2(N) = N^3/3 + N^2/2 + N/6$$
$$= N(N+1)(2N+1)/6 = N(N+1/2)(N+1)/3 \tag{78}$$

Note that, the right hand side have a sense also for $N \leq 0$ and has zeros at $N = 0, -1/2, -2$.

For general case $E_k(N)$ we have

$$x_k(N) = detV_l(N)/detW_k(N),$$

$$detW_k(N) = det \begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ . & . & . & . \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix},$$

$$detV_l(N) = det \begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ \bar{E}_l(N_1) & \bar{E}_l(N_2) & . & \bar{E}_l(N_{k+1}) \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix}, \ X = \begin{pmatrix} x_0 \\ x_1 \\ . \\ x_k \end{pmatrix}$$

$$\bar{E}_k(N_l) = E_k(N_l) - \frac{N_l^{k+1}}{k+1}, \ E = \begin{pmatrix} \bar{E}_k(N_1) \\ \bar{E}_k(N_2) \\ . \\ \bar{E}_k(N_{k+1}) \end{pmatrix},$$

$$WX = E, \ X = W^{-1}E \tag{79}$$

As a numbers N_n we can take any different integers, but the simplest choice is: $N_{n+1} = N_n + 1,\ N_1 = 1$, as in considered explicit calculation for E_2. In this case, $E_k(N+1) = E_k(N) + (N+1)^k$.

We propose the following compact form for E_k

$$E_k(N) = \frac{d^k}{dx^k} P(x,N)|_{x \Rightarrow 0} \equiv D^k P = P^{(k)}(0,N),$$
$$P(x,N) = \sum_{n=1}^{N} e^{nx} = \frac{e^{(N+1)x} - e^x}{e^x - 1} \quad (80)$$

We take also the following slightly simpler form of $P(x,N)$, for $k = 1, 2, 3, ...$

$$P(x,N) = \sum_{n=0}^{N} e^{nx} = \frac{e^{(N+1)x} - 1}{e^x - 1} \quad (81)$$

As an example, let us calculate $E_1(N)$,

$$E_1(N) = \frac{(N+1)e^{(N+1)x}}{e^x - 1} - \frac{(e^{(N+1)x} - 1)e^x}{(e^x - 1)^2}$$
$$= \frac{(N+1)e^{(N+1)x}(e^x - 1) - (e^{(N+1)x} - 1)e^x}{(e^x - 1)^2} \Downarrow$$
$$\frac{(N+1)(1+(N+1)x...)(x+\frac{x^2}{2}...) - ((N+1)x + \frac{(N+1)^2x^2}{2}...)(1+x...)}{(x+...)^2}$$
$$= (N+1)^2 + (N+1)/2 - (N+1) - (N+1)^2/2 = N(N+1)/2 \quad (82)$$

We can present the derivative operator in the complex integral form

$$f^{(k)}(0) = \frac{k!}{2\pi i} \oint \frac{dz f(z)}{z^{k+1}} \tag{83}$$

In this form the calculation gives

$$\begin{aligned} S(1,N) &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1)z + (N+1)^2 z^2/2}{z + z^2/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1) + (N+1)^2 z/2}{1 + z/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z} \frac{(N+1)}{z}(1 - z/2) + (N+1)^2/2 = N(N+1)/2 \end{aligned} \tag{84}$$

By this example we see that the second form of calculation is easier.

Why supersymmetry is
So universal?

Supermathematics unifies discrete and continual aspects of mathematics.
Boson oscillator hamiltonian is

$$H_b = \hbar\omega(b^+b + bb^+)/2 = \hbar\omega(b^+b + a),\ a = 1/2. \tag{85}$$

corresponding energy spectrum E_{bn} and eigenfunctions $|n_b>$ are

$$H_b|n_b >= E_{bn}|n_b >,\ E_{bn} = \hbar\omega(n_b + a),\ n_b = 0, 1, 2, ... \tag{86}$$

Fermion oscillator hamiltonian, eigenvectors and energies are

$$\begin{aligned} &H_f = \hbar\omega(f^+f - ff^+)/2 = \hbar\omega(f^+f - a), \\ &H_f = |n_f >= E_{fn}|n_f >,\ E_{fn} = \hbar\omega(n_f - a),\ n_f = 0, 1. \end{aligned} \tag{87}$$

For supersymmetric oscillator we have

$$\begin{aligned} &H = H_b + H_f,\ H|n_b, n_f >= \hbar\omega(n_b + n_f)|n_b, n_f >, \\ &|n_b, n_f >= |n_b > |n_f >,\ E_{n_b,n_f} = \hbar\omega(n_b + n_f) \end{aligned} \tag{88}$$

For background-vacuum $|0, 0 >$, energy $E_{0,0} = 0$. For higher energy states $|n - 1, 1 >,\ |n, 0 >,\ E_{n-1,1} = E_{n,0}$. Supersymmetry needs not only the same frequency for boson and fermion oscillators, but also that $2a = 1$.

A minimal realization of the algebra of supersymmetry

$$\{Q, Q^+\} = H,\ \{Q, Q\} = \{Q^+, Q^+\} = 0, \tag{89}$$

is given by a point particle dynamics in one dimension, [Witten 1981]

$$Q = f(-iP + W)/\sqrt{2},\ Q^+ = f^+(iP + W)/\sqrt{2},\ P = -i\partial/\partial x \tag{90}$$

where the superpotential $W(x)$ is any function of x, and spinor operators f and f^+ obey the anticommuting relations

$$\{f, f^+\} = 1,\ f^2 = (f^+)^2 = 0. \tag{91}$$

There is a following representation of operators f, f^+ and σ by Pauli spin matrices

$$f = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, f^+ = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$
$$\sigma = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{92}$$

Supermatematics

From formulae (89) and (90) then we have

$$H = (P^2 + W^2 + \sigma W_x)/2. \tag{93}$$

The simplest nontrivial case of the superpotential $W = \omega x$ corresponds to the supersimmetric oscillator with Hamiltonian

$$H = H_b + H_f, \quad H_b = (P^2 + \omega^2 x^2)/2, \quad H_f = \omega\sigma/2, \tag{94}$$

The ground state energies of the bosonic and fermionic parts are

$$E_{b0} = \omega/2, \quad E_{f0} = -\omega/2, \tag{95}$$

so the vacuum energy of the supersymmetric oscillator is

$$< 0|H|0 >= E_0 = E_{b0} + E_{f0} = 0, \quad |0 >= |n_b, n_f >= |n_b > |n_f > . \tag{96}$$

Let us see on this toy - solution of the cosmological constant problem from the quantum statistical viewpoint. The statistical sum of the supersymmetric oscillator is

$$Z(\beta) = Z_b Z_f, \tag{97}$$

where

$$Z_b = \sum_n e^{-\beta E_{bn}} = e^{-\beta\omega/2} + e^{-\beta\omega(1+1/2)} + ... = e^{-\beta\omega/2}/(1 - e^{-\beta\omega})$$
$$Z_f = \sum_n e^{-\beta E_{fn}} = e^{\beta\omega/2} + e^{-\beta\omega/2}. \tag{98}$$

In the low temperature limit,

$$Z(\beta) = 1 + O(e^{-\beta\omega}) \to 1, \quad \beta = T^{-1}, \tag{99}$$

so cosmological constant $\lambda \sim lnZ \to 0$. From observable values of β and the cosmological constant we estimate ω.

The Riemann zeta function (RZF) can be interpreted in thermodynamic terms as a statistical sum of a system with energy spectrum:
$E_n = \ln n,\ n = 1, 2, ...$:

$$\zeta(s) = \sum_{n \geq 1} n^{-s} = Z(\beta) = \sum_{n \geq 1} \exp(-\beta E_n),$$
$$\beta = s,\ E_n = \ln n,\ n = 1, 2, ... \qquad (100)$$

After formulation of the mathematical framework of quantum mechanics (QM), operatorial formulation of QM, Koopman and von Neumann gave operatorial approach to classical Hamiltonian mechanics [Koopman 1931], [von Neumann 1932]. After Wiener introduction of the functional integrals, Dirac and Feynman gave formal functional integral formulation of the quantum theory [Feynman, Hibbs 1965]. Recently Gozzi invented functional integral formulation of the classical theory [Gozzi et al 1989]. The path-integral formulation of Hamiltonian classical mechanics.
For supersymmetric gauge theories stochastic quantization appears to have one definite advantage: since a gauge fixing term is unnecessary, supersymmetry will not be broken at any step. This holds both for the Abelian and non-Abelian case. It appears at the moment as if stochastic regularization is the only viable candidate for a regularization scheme which manifestly conserves both supersymmetry, chiral symmetry and gauge invariance. However, supersymmetry is related to stochastic quantization also at a much deeper level. As an example, even purely scalar field theories will, when quantized stochastically, display a 'hidden' supersymmetry. This issue, is intimately connected with the existence of a so-called 'Nicolai map' for supersymmetric field theories [Nicolai 1980].

Parisi-Sourlas 'dimensional reduction' of scalar field theories in external random fields [Parisi, Sourlas 1979], is closely related to both supersymmetry and stochastic quantization. This becomes apparent when one establishes the connection to the Nicolai map.
The phenomenon of dynamical 'dimensional reduction' was first noted within the context of critical phenomena associated with spin systems in random external fields. Systems very close to such a situation can in fact be created and studied in the laboratory. From renormalization group theory, the detailed long-distance behaviour of, for example, Ising spin systems can, sufficiently close to a critical point, be understood from the behaviour of a scalar field theory

$$S = \int d^D x (\frac{1}{2}\varphi(-\partial^2 + m^2)\varphi + V(\varphi)),\ V(\varphi) = a\varphi^3 + g\varphi^4 \qquad (101)$$

We start in the simplest possible way by considering the Langevin equation associated with a point particle being subjected to random background noise. This corresponds to the very real physical problem of the Brownian motion of a (classical) particle in a heat bath. Surprisingly, this problem turns out to be equivalent to a supersymmetric quantum mechanical problem. Let us now see why. The Langevin equation for the particle reads

$$\frac{dx}{dt} \equiv \dot{x} = -\frac{\delta S}{\delta x} + \eta(t) \tag{102}$$

where x represents the space coordinate of the particle. Expectation values are, as usual, evaluated as the path integral

$$< x(t_1)\dots\, x(t_n) >= \int d\eta\; x(t_1)\dots\, x(t_n) \exp(-\frac{1}{4}\int dt \eta(t)^2) \tag{103}$$

over a Gaussian noise, i.e.

$$< \eta(t_1)\eta(t_2) >= 2\delta(t_1 - t_2) \tag{104}$$

we now attempt to make a change of variables: $\eta \to x$. This involves the Jacobian

$$det(\delta\eta(t)/\delta x(t')) = det((d/dt + V')\delta(t - t')) \tag{105}$$

where we have introduced $V = \delta S/\delta x$.
For partition function Z,

$$\begin{aligned}
Z &= \int d\eta \exp(-\frac{1}{4}\int dt\eta(t)^2) \\
&= \int d\eta dx det(d/dt + V')\delta(\dot{x} + V - \eta(t)) \exp(-\frac{1}{4}\int dt\eta(t)^2) \\
&= \int dx det(d/dt + V') \exp(-\frac{1}{4}\int dt(\dot{x} + V)^2) \\
&= \int dx d\psi d\bar{\psi} \exp(-S), \\
S &= \int dt(\frac{1}{4}(\dot{x} + V)^2 - \bar{\psi}(d/dt + V')\psi)
\end{aligned} \tag{106}$$

This system is recognized as Witten's example of supersymmetric quantum mechanics.

The theory of analytic functions of a complex variable occupies a central place in analysis. Riemann considered the unique continuation property to be the most characteristic feature of analytic functions. GPF do possess the unique continuation property, and each class of GPF has almost as much structure as the class of analytic functions. In particular, the operations of complex differentiation and complex integration have meaningful counterparts in the theory of GPF and this theory generalizes not only the Cauchy-Riemann approach to function theory but also that of Weierstrass. Such functions were considered by Picard and by Beltrami, but the first significant result was obtained by Carleman in 1933, and a systematic theory was formulated by Lipman Bers [Bers 1952] and Ilia Vekua (1907-1977) [Vekua 1962]. For more resent results see [Giorgadze 2011].

Analytic function $f = u + iv$ satisfy the partial differential equation $\partial_{\overline{z}} f = 0$, where complex differential operators are defined as

$$\partial_{\overline{z}} = \frac{\partial}{\partial \overline{z}} := \frac{1}{2}(\partial_x + i\partial_y),\ \partial_z = \frac{\partial}{\partial z} := \frac{1}{2}(\partial_x - i\partial_y) \tag{107}$$

Generalized analytic functions $f = u + iv$ satisfy the following generalized Cauchy-Riemann equation [Vekua 1962]

$$\partial_{\overline{z}} f = Af + B\bar{f} + J,\ A = A_0 + iA_1,\ B = B_0 + iB_1,\ J = j_1 + ij_2 \tag{108}$$

or in terms of the real u and imaginary v components canonical form of the elliptic systems of partial differential equations of the first order

$$\begin{aligned} u_x - v_y &= au + bv + j_1,\ a = A_0 + B_0,\ b = -A_1 + B_1, \\ u_y + v_x &= cu + dv + j_2,\ c = A_1 + B_1,\ d = A_0 - B_0, \end{aligned} \tag{109}$$

or in matrix form

$$\begin{aligned} &D\psi = E\psi + J,\ D = \begin{pmatrix} \partial_x & -\partial_y \\ \partial_y & \partial_x \end{pmatrix} = \partial_x - i\sigma_2\partial_y, \\ &E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ \psi = \begin{pmatrix} u \\ v \end{pmatrix},\ J = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}. \end{aligned} \tag{110}$$

In the classical sense by a solution of the system of equations (119) we understand a pair of real continuously differentiable functions $u(x,y),\ v(x,y)$ of the real variables x and y which satisfy this system everywhere in a domain G. Such solutions, however, exist only for a comparatively narrow class of equations.
The formal solution of the canonical equation for GPF (119) is

$$\psi = \psi_0 + RJ,\ R = (D-E)^{-1},\ (D-E)\psi_0 = 0. \tag{111}$$

Let us introduce a length parameter $l = h^{-1}$, which is of order of the source J size, $x_n \to l x_n$. Then, for the resolvent R, we will have the longwave and shortwave expansions,

$$R_{LW} := (lD-E)^{-1} = -E^{-1} \sum_{n \geq 0} l^n (DE^{-1})^n,$$
$$R_{ShW} := (lD-E)^{-1} = hD^{-1} \sum_{n \geq 0} h^n (ED^{-1})^n, \tag{112}$$

$$E^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} / \Delta_E, \ \Delta_E = ad - bc,$$
$$D^{-1} = \Delta_D^{-1} \begin{pmatrix} \partial_x & \partial_y \\ -\partial_y & \partial_x \end{pmatrix} = \Delta_D^{-1}(\partial_x + i\sigma_2\partial_y), \ \Delta_D = \partial_x^2 + \partial_y^2 \quad (113)$$

There is a fairly complete theory of generalized analytic functions; it represents an essential extension of the classical theory preserving at the same time its principal features [Vekua 1962].
From the previous consideration it is natural to make the following four dimensional extention

$$\begin{aligned} &D = \partial_x - i\sigma_2\partial_y \Rightarrow D_4 = \partial_t - i\sigma_n\nabla_n = -i(\partial_\tau + \sigma_n\nabla_n) = -iD_{13}, \\ &D^{-1} = \Delta_4^{-1}(\partial_t + i\sigma_n\nabla_n), \ \Delta_4 = \partial_t^2 + \Delta_3, \ \Delta_3 = \partial_x^2 + \partial_y^2 + \partial_z^2, \ t = i\tau \\ &D_{13}^{-1} = \Delta_{13}^{-1}(\partial_\tau - \sigma_n\nabla_n), \ \Delta_{13} = \partial_\tau^2 - \Delta_3, \\ &\sigma_n\sigma_m = \delta_{nm} + i\varepsilon_{nmk}\sigma_k \end{aligned} \quad (114)$$

In matrix form

$$\begin{aligned}
&D_4 = \left(\begin{array}{cc} \partial_t - i\partial_z & -i\partial_x - \partial_y \\ -i\partial_x + \partial_y & \partial_t + i\partial_z \end{array} \right) = 2 \left(\begin{array}{cc} \partial_\zeta & -\partial_{\bar\eta} \\ \partial_\eta & \partial_{\bar\zeta} \end{array} \right), \\
&\zeta = t + iz,\ \eta = y + ix, \\
&D_{13} = \left(\begin{array}{cc} \partial_\tau + \partial_z & \partial_x - i\partial_y \\ \partial_x + i\partial_y & \partial_\tau - \partial_z \end{array} \right) = 2 \left(\begin{array}{cc} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{array} \right), \\
&\Delta_4 = 4(\partial^2_{\zeta\bar\zeta} + \partial^2_{\eta\bar\eta}),\ \Delta_{13} = 4(\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma}), \\
&\pm = \tau \pm z,\ \varsigma = x + iy,
\end{aligned} \tag{115}$$

In the Minkowski spacetime for analytic functions in matrix form $D_{13}\psi = 0$ or in components

$$\begin{aligned}
&\partial_- u + \partial_\varsigma v = 0,\ \partial_+ v + \partial_{\bar\varsigma} u = 0 \Rightarrow (\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma}) u_n = 0, \\
&u_1 = u,\ u_2 = v
\end{aligned} \tag{116}$$

In euclidian space $D_4\psi = 0$,

$$\partial_\zeta u - \partial_{\bar\eta} v = 0,\ \partial_{\bar\zeta} v + \partial_\eta u = 0 \Rightarrow (\partial^2_{\zeta\bar\zeta} + \partial^2_{\eta\bar\eta}) u_n = 0, \tag{117}$$

So, u_n are harmonic or wave functions.

$$\begin{aligned}\partial_- u + \partial_\varsigma v = au + bv + j_1,\\ \partial_+ v + \partial_{\bar\varsigma} u = cu + dv + j_2,\end{aligned} \tag{118}$$

or in matrix form

$$D\psi = E\psi + J,\ D = \begin{pmatrix} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{pmatrix} = \partial_\tau + \sigma_n \nabla_n,$$
$$E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ \psi = \begin{pmatrix} u \\ v \end{pmatrix},\ J = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}. \tag{119}$$

It is curious to imagine that Hamilton knew about neutrinos equation a hundred years before Weil :) In the extended version, to the $E-$terms corresponds neutrinos mass.

Now SM is well established theory of fundamental physics with several indications, aesthetical and theoretical, on father developments, on physics beyond SM (BSM), on new physics. One well established step toward BSM is neutrino masses. In SM the neutrinos are massless. In SM we have three type of lefthanded neutrinos $\nu_n,\ n = e, \mu, \tau$ which interacts weakly with corresponding leptons, lepton number is conserved. Corresponding part of the SM lagrangian is

$$\bar{l}_n \gamma^\mu \nu'_n W_\mu + \bar{\nu}'(\gamma\partial - M)\nu',\ \bar{\nu}' = (\bar{\nu}'_e \bar{\nu}'_\mu \bar{\nu}'_\tau) \tag{120}$$

where M is a 3×3 matrix in flavor space. If the matrix is nondiagonal, we diagonalize it by an unitary transformation:

$$\begin{aligned} &\bar{\nu}(\gamma\partial - M)\nu = \bar{\nu}_n(\gamma\partial - m_n)\nu_n, \\ &U^{-1} M U = diag(m_1, m_2, m_3),\ \nu_n = U^{-1}_{nk} \nu'_k, \\ &\bar{l}_n \gamma^\mu \nu'_n W_\mu = \bar{l}_n \gamma^\mu U_{nk} \nu_k W_\mu \end{aligned} \tag{121}$$

If the Koide formula works for lepton masses, may be it works also for neutrino masses. If the lepton masses are an unique solution of the Koide formula, than neutrino masses are proportional to the lepton masses: $m_n = qM_n,\ n = e, \mu, \tau.$
(Super)symmetry, stochastic dynamics and kaleidoscope effect. Time reflection invariance and dynamical origin of spin.
The meromorphic functions form a field, in fact a field extension of the complex numbers.
Weyl proposed the following 2-component equations for the zero mass spin 1/2 particles in 1929,

$$(\partial_0 - s_n \partial_n)W = 0,\ W = (u, v)^t \tag{122}$$

for the wave functions of the left-neutrino-right-antineutrino pairs. At that time they were rejected by Pauli because of their lack of invariance with respect to space inversion. Indeed, it was always a basic principle that the wave equations should be invariant under all Lorentz transformations, not just those in the connected component. In particular, invariance under space inversion, also called parity conservation, was demanded.

In the mid 1950s, in experiments performed by Wu following a famous suggestion by Yang and Lee that the neutrinos did not have the parity conservation property, it was found that the neutrinos emitted during beta decay had a preferred orientation. Experimental evidence further indicated that the spin is always antiparallel to the momentum for the neutrinos so that the neutrinos are always left-handed. After Wu's experiment, Landau and Salam proposed that the Weyl equation for the left-handed neutrino-right-handed antineutrino pairs be restored as the equation satisfied by the neutrino. It is this equation that now governs massless particles, not only in Minkowski spacetime but also in curved spacetime.

Let us consider a general dynamical system described by the following system of the ordinary differential equations [Arnold, 1978]

$$\dot{x}_n = v_n(x),\ 1 \leq n \leq N, \tag{123}$$

$\dot{x}_n$ stands for the total derivative with respect to the parameter t.
When the number of the degrees of freedom is even, and

$$v_n(x) = \varepsilon_{nm} \frac{\partial H_0}{\partial x_m},\ 1 \leq n, m \leq 2M, \tag{124}$$

the system (123) is Hamiltonian one and can be put in the form

$$\dot{x}_n = \{x_n, H_0\}_0, \tag{125}$$

where the Poisson bracket is defined as

$$\{A, B\}_0 = \varepsilon_{nm} \frac{\partial A}{\partial x_n} \frac{\partial B}{\partial x_m} = A \frac{\overleftarrow{\partial}}{\partial x_n} \varepsilon_{nm} \frac{\overrightarrow{\partial}}{\partial x_m} B, \tag{126}$$

and summation rule under repeated indices has been used.

Let us consider the following Lagrangian

$$L = (\dot{x}_n - v_n(x))\psi_n \tag{127}$$

and the corresponding equations of motion

$$\dot{x}_n = v_n(x), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n}\psi_m. \tag{128}$$

The system (128) extends the general system (123) by linear equation for the variables ψ. The extended system can be put in the Hamiltonian form [Makhaldiani, Voskresenskaya, 1997]

$$\dot{x}_n = \{x_n, H_1\}_1, \dot{\psi}_n = \{\psi_n, H_1\}_1, \tag{129}$$

where first level (order) Hamiltonian is

$$H_1 = v_n(x)\psi_n \tag{130}$$

and (first level) bracket is defined as

$$\{A, B\}_1 = A(\frac{\overleftarrow{\partial}}{\partial x_n}\frac{\overrightarrow{\partial}}{\partial \psi_n} - \frac{\overleftarrow{\partial}}{\partial \psi_n}\frac{\overrightarrow{\partial}}{\partial x_n})B. \tag{131}$$

Note that when the Grassmann grading [Berezin, 1987] of the conjugated variables x_n and ψ_n are different, the bracket (131) is known as Buttin bracket[Buttin, 1996].

In the Faddeev-Jackiw formalism [Faddeev, Jackiw, 1988] for the Hamiltonian treatment of systems defined by first-order Lagrangians, i.e. by a Lagrangian of the form

$$L = f_n(x)\dot{x}_n - H(x), \tag{132}$$

motion equations

$$f_{mn}\dot{x}_n = \frac{\partial H}{\partial x_m}, \tag{133}$$

for the regular structure function f_{mn}, can be put in the explicit hamiltonian (Poisson; Dirac) form

$$\dot{x}_n = f^{-1}_{nm}\frac{\partial H}{\partial x_m} = \{x_n, x_m\}\frac{\partial H}{\partial x_m} = \{x_n, H\}, \tag{134}$$

where the fundamental Poisson (Dirac) bracket is

$$\{x_n, x_m\} = f^{-1}_{nm},\ f_{mn} = \partial_m f_n - \partial_n f_m. \tag{135}$$

The system (128) is an important example of the first order regular hamiltonian systems. Indeed, in the new variables,

$$y_n^1 = x_n, y_n^2 = \psi_n, \tag{136}$$

lagrangian (127) takes the following first order form

$$\begin{aligned} L &= (\dot{x}_n - v_n(x))\psi_n \Rightarrow \frac{1}{2}(\dot{x}_n\psi_n - \dot{\psi}_n x_n) - v_n(x)\psi_n \\ &= \frac{1}{2} y_n^a \varepsilon^{ab} \dot{y}_n^b - H(y) \\ &= f_n^a(y)\dot{y}_n^a - H(y), f_n^a = \frac{1}{2} y_n^b \varepsilon^{ba}, H = v_n(y^1) y_n^2, \\ f_{nm}^{ab} &= \frac{\partial f_m^b}{\partial y_n^a} - \frac{\partial f_n^a}{\partial y_m^b} = \varepsilon^{ab}\delta_{nm}; \end{aligned} \tag{137}$$

corresponding motion equations and the fundamental Poisson bracket are

$$\dot{y}_n^a = \varepsilon_{ab}\delta_{nm}\frac{\partial H}{\partial y_m^b} = \{y_n^a, H\}, \{y_n^a, y_m^b\} = \varepsilon_{ab}\delta_{nm}. \tag{138}$$

To the canonical quantization of this system corresponds

$$[\hat{y}^a_n, \hat{y}^b_m] = i\hbar\varepsilon_{ab}\delta_{nm},\ \ \hat{y}^1_n = y^1_n,\ \ \hat{y}^2_n = -i\hbar\frac{\partial}{\partial y^1_n} \tag{139}$$

In this quantum theory, classical part, motion equations for y^1_n, remain classical.

Note that the procedure of reduction of the higher order dynamical system, e.g. second order Euler-Lagrange motion equations, to the first order dynamical systems, in the case to the Hamiltonian motion equations, can be continued using fractal calculus. E.g. first order system can be reduced to the half order one,

$$\begin{aligned} &D^{1/2}q = \psi, \\ &D^{1/2}\psi = p \Leftrightarrow \dot{q} = p. \end{aligned} \tag{140}$$

Nabu – Babylonian God
of Wisdom and Writing.

The Hamiltonian mechanics (HM) is in the fundamentals of mathematical description of the physical theories [Faddeev, Takhtajan, 1990]. But HM is in a sense blind; e.g., it does not make a difference between two opposites: the ergodic Hamiltonian systems (with just one integral of motion) [Sinai, 1993] and (super)integrable Hamiltonian systems (with maximal number of the integrals of motion).
Nabu mechanics (NM) [Nambu, 1973, Whittaker, 1927] is a proper generalization of the HM, which makes the difference between dynamical systems with different numbers of integrals of motion explicit (see, e.g.[Makhaldiani, 2007]).

In the canonical formulation, the equations of motion of a physical system are defined via a Poisson bracket and a Hamiltonian, [Arnold, 1978]. In Nambu's formulation, the Poisson bracket is replaced by the Nambu bracket with $n+1, n \geq 1$, slots. For $n = 1$, we have the canonical formalism with one Hamiltonian. For $n \geq 2$, we have Nambu-Poisson formalism, with n Hamiltonians, [Nambu, 1973], [Whittaker, 1927].

The system of N vortexes can be described by the following system of differential equations, [Aref, 1983, Meleshko,Konstantinov, 1993]

$$\dot{z}_n = i \sum_{m \neq n}^{N} \frac{\gamma_m}{z_n^* - z_m^*}, \quad 1 \leq n \leq N, \tag{141}$$

where $z_n = x_n + iy_n$ are complex coordinate of the centre of n-th vortex, for $N = 3$, and the quantities

$$\begin{aligned} u_1 &= ln|z_2 - z_3|^2, \\ u_2 &= ln|z_3 - z_1|^2, \\ u_3 &= ln|z_1 - z_2|^2 \end{aligned} \tag{142}$$

reduce to the following system

$$\begin{aligned} \dot{u}_1 &= \gamma_1(e^{u_2} - e^{u_3}), \\ \dot{u}_2 &= \gamma_2(e^{u_3} - e^{u_1}), \\ \dot{u}_3 &= \gamma_3(e^{u_1} - e^{u_2}), \end{aligned} \tag{143}$$

The system (143) has two integrals of motion

$$H_1 = \sum_{i=1}^{3} \frac{e^{u_i}}{\gamma_i}, H_2 = \sum_{i=1}^{3} \frac{u_i}{\gamma_i}$$

and can be presented in the Nambu–Poisson form, [Makhaldiani, 1997,2]

$$\dot{u}_i = \omega_{ijk} \frac{\partial H_1}{\partial u_j} \frac{\partial H_2}{\partial u_k} = \{x_i, H_1, H_2\} = \omega_{ijk} \frac{e^{u_j}}{\gamma_j} \frac{1}{\gamma_k},$$

where

$$\omega_{ijk} = \epsilon_{ijk}\rho, \rho = \gamma_1\gamma_2\gamma_3$$

and the Nambu–Poisson bracket of the functions A, B, C on the three-dimensional phase space is

$$\{A, B, C\} = \omega_{ijk} \frac{\partial A}{\partial u_i} \frac{\partial B}{\partial u_j} \frac{\partial C}{\partial u_k}. \tag{144}$$

This system is superintegrable: for $N = 3$ degrees of freedom, we have maximal number of the integrals of motion $N - 1 = 2$.

The reduction of the dimensionless couplings in GUTs is achieved by searching for RD integrals of motion-renormdynamic invariant (RDI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exist RDI relations among dimensional couplings that guarantee the vanishing of all beta-functions in certain GUTs even to all orders. In this case the number of the independent motion integrals N is equal to the number of the coupling constants. Note that in superintegrable dynamical systems the number of the integrals is $\leq N-1$, so the RD of the finite field theories is trivial, coupling constants do not run, they have fixed values, the renormdynamics is more than superintegrable, it is hyperintegrable. Developments in the soft supersymmetry breaking sector of GUTs and FUTs lead to exact RDI relations, i.e. reduction of couplings, in this dimensionful sector of the theory, too. Based on the above theoretical framework phenomenologically consistent FUTs have been constructed. The main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the SM in terms of a few fundamental ones. In other words, to achieve reduction of couplings at a more fundamental level.

As an example of the infinite dimensional Nambu-Poisson dynamics, let me conside the following extension of Schrödinger quantum mechanics [Makhaldiani, 2000]

$$iV_t = \Delta V - \frac{V^2}{2}, \tag{145}$$

$$i\psi_t = -\Delta\psi + V\psi. \tag{146}$$

An interesting solution to the equation for the potential (145) is

$$V = \frac{4(4-d)}{r^2}, \tag{147}$$

where d is the dimension of the spase. In the case of $d = 1$, we have the potential of conformal quantum mechanics.
The variational formulation of the extended quantum theory, is given by the following Lagrangian

$$L = (iV_t - \Delta V + \frac{1}{2}V^2)\psi. \tag{148}$$

The momentum variables are

$$P_v = \frac{\partial L}{\partial V_t} = i\psi, P_\psi = 0. \qquad (149)$$

As Hamiltonians of the Nambu-theoretic formulation, we take the following integrals of motion

$$\begin{aligned} H_1 &= \int d^d x(\Delta V - \frac{1}{2}V^2)\psi, \\ H_2 &= \int d^d x(P_v - i\psi), \\ H_3 &= \int d^d x P_\psi. \end{aligned} \qquad (150)$$

We invent unifying vector notation, $\phi = (\phi_1, \phi_2, \phi_3, \phi_4) = (\psi, P_\psi, V, P_v)$. Then it may be verified that the equations of the extended quantum theory can be put in the following Nambu-theoretic form

$$\phi_t(x) = \{\phi(x), H_1, H_2, H_3\}, \qquad (151)$$

where the bracket is defined as

$$\{A_1, A_2, A_3, A_4\} = i\varepsilon_{ijkl} \int \frac{\delta A_1}{\delta \phi_i(y)} \frac{\delta A_2}{\delta \phi_j(y)} \frac{\delta A_3}{\delta \phi_k(y)} \frac{\delta A_4}{\delta \phi_l(y)} dy$$

$$= i \int \frac{\delta(A_1, A_2, A_3, A_4)}{\delta(\phi_1(y), \phi_2(y), \phi_3(y), \phi_4(y))} dy = i det(\frac{\delta A_k}{\delta \phi_l}). \quad (152)$$

The basic building blocks of M theory are membranes and $M5-$branes. Membranes are fundamental objects carrying electric charges with respect to the 3-form C-field, and $M5$-branes are magnetic solitons. The Nambu-Poisson 3-algebras appear as gauge symmetries of superconformal Chern-Simons nonabelian theories in 2 + 1 dimensions with the maximum allowed number of $N = 8$ linear supersymmetries.
The Bagger and Lambert [Bagger, Lambert, 2007] and, Gustavsson [Gustavsson, 2007] (BLG) model is based on a 3-algebra,

$$[T^a, T^b, T^c] = f_d^{abc} T^d \tag{153}$$

where T^a, are generators and f_{abcd} is a fully anti-symmetric tensor.

Given this algebra, a maximally supersymmetric Chern-Simons lagrangian is:

$$L = L_{CS} + L_{matter},$$
$$L_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f_{abcd}A^{ab}_{\mu}\partial_{\nu}A^{cd}_{\lambda} + \frac{2}{3}f_{cdag}f^{g}_{efb}A^{ab}_{\mu}A^{cd}_{\nu}A^{ef}_{\lambda}), \quad (154)$$

$$L_{matter} = \frac{1}{2}B^{Ia}_{\mu}B^{\mu I}_{a} - B^{Ia}_{\mu}D^{\mu}X^{I}_{a}$$
$$+\frac{i}{2}\bar{\psi}^{a}\Gamma^{\mu}D_{\mu}\psi_{a} + \frac{i}{4}\bar{\psi}^{b}\Gamma_{IJ}x^{I}_{c}x^{J}_{d}\psi_{a}f^{abcd}$$
$$-\frac{1}{12}tr([X^{I},X^{J},X^{K}][X^{I},X^{J},X^{K}]),\ I = 1,2,...,8, \quad (155)$$

where A^{ab}_{μ} is gauge boson, ψ^{a} and $X^{I} = X^{I}_{a}T^{a}$ matter fields. If $a = 1,2,3,4$, then we can obtain an $SO(4)$ gauge symmetry by choosing $f_{abcd} = f\varepsilon_{abcd}, f$ being a constant. It turns out to be the only case that gives a gauge theory with manifest unitarity and $N = 8$ supersymmetry.

The action has the first order form so we can use the formalism of the first section. The motion equations for the gauge fields

$$f^{nm}_{abcd}\dot{A}^{cd}_m(t,x) = \frac{\delta H}{\delta A^{ab}_n(t,x)}, f^{nm}_{abcd} = \varepsilon^{nm} f_{abcd} \tag{156}$$

take canonical form

$$\begin{aligned} &\dot{A}^{ab}_n = f^{abcd}_{nm}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, A^{cd}_m\}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, H\}, \\ &\{A^{ab}_n(t,x), A^{cd}_m(t,y)\} = \varepsilon_{nm} f^{abcd}\delta^{(2)}(x-y) \end{aligned} \tag{157}$$

Nambu-Poisson dynamics of an extended particle with spin in an accelerator

The quasi-classical description of the motion of a relativistic (nonradiating) point particle with spin in accelerators and storage rings includes the equations of orbit motion

$$\begin{array}{l} \dot{x}_n = f_n(x),\ f_n(x) = \varepsilon_{nm}\partial_m H,\ n,m = 1,2,...,6; \\ x_n = q_n,\ x_{n+3} = p_n,\ \varepsilon_{n,n+3} = 1,\ n = 1,2,3; \\ H = e\Phi + c\sqrt{\wp^2 + m^2c^2},\ \wp_n = p_n - \frac{e}{c}A_n \end{array} \tag{158}$$

and Thomas-BMT equations
[Tomas, 1927, Bargmann, Michel,Telegdi, 1959] of classical spin motion

$$\begin{array}{l} \dot{s}_n = \varepsilon_{nmk}\Omega_m s_k = \{H_1, H_2, s_n\},\ H_1 = \Omega \cdot s,\ H_2 = s^2, \\ \{A, B, C\} = \varepsilon_{nmk}\partial_n A \partial_m B \partial_k C, \end{array} \tag{159}$$

$$\Omega_n = \frac{-e}{m\gamma c}((1+k\gamma)B_n - k\frac{(B\cdot\wp)\wp_n}{m^2c^2(1+\gamma)} + \frac{1+k(1+\gamma)}{mc(1+\gamma)}\varepsilon_{nmk}E_m\wp_k) \tag{160}$$

where, parameters e and m are the charge and the rest mass of the particle, c is the velocity of light, $k=(g-2)/2$ quantifies the anomalous spin g factor, γ is the Lorentz factor, p_n are components of the kinetic momentum vector, E_n and B_n are the electric and magnetic fields, and A_n and Φ are the vector and scalar potentials;

$$B_n = \varepsilon_{nmk}\partial_m A_k,\ \ E_n = -\partial_n\Phi - \frac{1}{c}\dot{A}_n,$$
$$\gamma = \frac{H-e\Phi}{mc^2} = \sqrt{1+\frac{\wp^2}{m^2c^2}} \tag{161}$$

The spin motion equations we put in the Nambu-Poisson form. Hamiltonization of this dynamical system according to the general approach of the previous sections we will put in the ground of the optimal control theory of the accelerator.

The general method of Hamiltonization of the dynamical systems we can use also in the spinning particle case. Let us invent unified configuration space $q = (x, p, s),\ x_n = q_n,\ p_n = q_{n+3},\ s_n = q_{n+6},\ n = 1, 2, 3$; extended phase space, (q_n, ψ_n) and hamiltonian

$$H = H(q, \psi) = v_n\psi_n,\ n = 1, 2, ...9; \tag{162}$$

motion equations

$$\begin{aligned} \dot{q}_n &= v_n(q), \\ \dot{\psi}_n &= -\frac{\partial v_m}{\partial q_n}\psi_m \end{aligned} \tag{163}$$

where the velocities v_n depends on external fields as in previous section as control parameters which can be determined according to the optimal control criterium.

EDM are one of the keys to understand the origin of our Universe [Sakharov, 1967]. Andrei Sakharov formulated three conditions for baryogenesis:

1. Early in the evolution of the universe, the baryon number conservation must be violated sufficiently strongly,
2. The C and CP invariances, and T invariance thereof, must be violated, and
3. At the moment when the baryon number is generated, the evolution of the universe must be out of thermal equilibrium.

CP violation in kaon decays is known since 1964, it has been observed in B-decays and charmed meson decays. The Standard Model (SM) accommodates CP violation via the phase in the Cabibbo-Kobayashi-Maskawa matrix.

CP and P violation entail nonvanishing P and T violating electric dipole moments (EDM) of elementary particles $\vec{d} = d\vec{s}$.

Although extremely successful in many aspects, the SM has at least two weaknesses: neutrino oscillations do require extensions of the SM and, most importantly, the SM mechanisms fail miserably in the expected baryogenesis rate.
Simultaneously, the SM predicts an exceedingly small electric dipole moment of nucleons $10^{-33} < d_n < 10^{-31} e \cdot cm$, way below the current upper bound for the neutron EDM, $d_n < 2.9 \times 10^{-26} e \cdot cm$. In the quest for physics beyond the SM one could follow either the high energy trail or look into new methods which offer very high precision and sensitivity.
Supersymmetry is one of the most attractive extensions of the SM and S. Weinberg emphasized [Weinberg, 1993]: "Endemic in supersymmetric (SUSY) theories are CP violations that go beyond the SM. For this reason it may be that the next exciting thing to come along will be the discovery of a neutron electric dipole moment."

The SUSY predictions span typically $10^{-29} < d_n < 10^{-24} e \cdot cm$ and precisely this range is targeted in the new generation of EDM searches [Roberts, Marciano, 2010]. There is consensus among theorists that measuring the EDM of the proton, deuteron and helion is as important as that of the neutron. Furthermore, it has been argued that T-violating nuclear forces could substantially enhance nuclear EDM [Flambaum, Khriplovich, Sushkov, 1986]. At the moment, there are no significant direct upper bounds available on d_p or d_d. Non-vanishing EDMs give rise to the precession of the spin of a particle in an electric field. In the rest frame of a particle

$$\dot{s}_n = \varepsilon_{nmk}(\Omega_m s_k + d_m E_k),\ \Omega_m = -\mu B_m, \tag{164}$$

where in terms of the lab frame fields

$$\begin{aligned} B_n &= \gamma(B^l_n - \varepsilon_{nmk}\beta_m E^l_k), \\ E_n &= \gamma(E^l_n + \varepsilon_{nmk}\beta_m B^l_k) \end{aligned} \tag{165}$$

Now we can apply the Hamiltonization and optimal control theory methods to this dynamical system.

The idea of computations on quanputers is in finding of the needed (value of the) state (wave function $\psi(t,x)$) from the initial, easy constructible, state ($\psi(0,x),$) which is superposition of different states, including interesting one, with the same weight. During the computation the weight of the interesting state is growing till the value when we can guess the solution of the problem and then test it, which is much more easier then to find it.
Let us consider the following nonlinear evolution equation

$$iV_t = \Delta V - \frac{1}{2}V^2 + J, \tag{166}$$

extended Lagrangian and Hamiltonian

$$L = \int dx^D (iV_t - \Delta V + \frac{1}{2}V^2 - J)\psi,$$
$$H = \int dx^D (\Delta V - \frac{1}{2}V^2 + J)\psi \tag{167}$$

and corresponding Hamiltonian motion equations

$$iV_t = \Delta V - \frac{1}{2}V^2 + J = \{V, H\},$$
$$i\psi_t = -\Delta\psi + V\psi = \{\psi, H\},$$
$$\{V(t,x), \psi(t,y)\} = \delta^D(x-y) \tag{168}$$

The solution of the problem is given in the form

$$|T) = U(T)|0),\ \psi(t,x) =< x|t),\ U(T) = Pexp(-i\int_0^T dtH(t)) \tag{169}$$

Under the programming of the quanputer we understand construction of the potential V, or the corresponding Hamiltonian. For the given potential, we calculate corresponding source J.
The discrete version of the system can be put in the form

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$
$$\Psi_m(n-1) = A_{mk}(S(n))\Psi_k(n),\ A_{mk}(S(n)) = \frac{\partial\Phi_k(S(n))}{\partial S_m(n)} \tag{170}$$

or, in the regular case, when the matrix A is regular, we obtain explicit form of the corresponding discrete dynamics

$$\begin{aligned} S_m(n+1) &= \Phi_n(S(n)) + J_m(n), \\ \Psi_m(n+1) &= A^{-1}_{mk}(S(n+1))\Psi_k(n), \end{aligned} \tag{171}$$

Now the state vector $S(n)$ and wave vector $\Psi_m(n)$ may correspond not only to the discrete values of the potential $V(n,m) = S_m(n)$, and wave function $\psi(n,m)$

GRID and Quanputing

As an example of GRID we take LHC Computing Grid. The LHC Computing Grid (LCG), is an international collaborative project that consists of a grid-based computer network infrastructure incorporating over 170 computing centers in 36 countries. It was designed by CERN to handle the prodigious volume of data produced by Large Hadron Collider (LHC) experiments. The Large Hadron Collider at CERN was designed to prove or disprove the existence of the Higgs boson, an important but elusive piece of knowledge that had been sought by particle physicists for over 40 years. A very powerful particle accelerator was needed, because Higgs bosons might not be seen in lower energy experiments, and because vast numbers of collisions would need to be studied. Such a collider would also produce unprecedented quantities of collision data requiring analysis. Therefore, advanced computing facilities were needed to process the data. A design report was published in 2005. It was announced to be ready for data on 3 October 2008. It incorporates both private fiber optic cable links and existing high-speed portions of the public Internet. At the end of 2010, the Grid consisted of some 200,000 processing cores and 150 petabytes of disk space, distributed across 34 countries.

The data stream from the detectors provides approximately 300 GByte/s of data, which after filtering for "interesting events", results in a data stream of about 300 MByte/s. The CERN computer center, considered "Tier 0" of the LHC Computing Grid, has a dedicated 10 Gbit/s connection to the counting room. The project was expected to generate 27 TB of raw data per day, plus 10 TB of "event summary data", which represents the output of calculations done by the CPU farm at the CERN data center. This data is sent out from CERN to eleven Tier 1 academic institutions in Europe, Asia, and North America, via dedicated 10 Gbit/s links. This is called the LHC Optical Private Network. More than 150 Tier 2 institutions are connected to the Tier 1 institutions by general-purpose national research and education networks. The data produced by the LHC on all of its distributed computing grid is expected to add up to 10-15 PB of data each year.

Today, without big efforts, we can modify (some) GRID elements in time-invertible form. After development of the quanputer technologies, we can modify (some) GRID elements in quanputer forms.

Social profit of big collaborations

Nowadays there are several big collaborations in science, e.g. LHC. Scientific value of LHC depends on three components, the highest quality of accelerator, highest quality of detectors and distributed data processing. The first two components need good mathematical and physical modeling. Third component and the collaboration as a social structure are not under (anther) the control by scientific methods and corresponding modeling. By definition, scientific collaborations (SC) have a main scientific aim: to obtain answer on the important scientific question(s) and maybe gain extra scientific bonus: new important questions and discoveries. SC is more open information system than e.g. finance or military systems. So, it is possible to describe and optimize SC by scientific methods. Profit from scientific modeling of SC maybe also for other information systems and social structures.

Let us consider the following **discrete dynamics**:

$$S_{n+1} + S_{n-1} = \Phi(S_n), \tag{172}$$

which is obviously a (discrete) time (n) invertible in this implicit form. In the explicit form

$$S_{n+1} = F(S_n, S_{n-1}) = \Phi(S_n) - S_{n-1} \tag{173}$$

it is not obviously time invertible. If we take two step time lattice-make simplest discrete RD step and from one component-scalar S(n) construct two component-spinor $\Psi(n)$, we obtain explicit time invertible dynamics

$$\Psi_{n+1} = \Omega(\Psi_n),\ \Psi_{n+1} = \begin{pmatrix} S_{n+2} \\ S_{n+1} \end{pmatrix},\ \Psi_n = \begin{pmatrix} S_n \\ S_{n-1} \end{pmatrix} \tag{174}$$

This **dynamical mechanism of origin spin** which connects time inversion symmetry and spin was invented when was constructed the theory of quanputers [Makhaldiani, 2011.2].

This mechanism indicates that with time inversion symmetry we can have only composed scalar fields. With the discovery of the Higgs particle with mass $125\ GeV$, a nice number $m_W/m_H \simeq 2/3$ appear, which, at least for me, indicates for composed nature of W and H, with a same mass of about $40\ GeV$ two and three valence constituents correspondingly. The fermion constituents ψ_n^a of W and scalar constituents φ_n^a of H compose scalar super multiplet (φ_n^a, ψ_n^a) with a flavor index n and color index a. Another notation is (h, sh)-(He, She:).

With exact SUSY we have cofinement by dimentional counting: superspace dimension is zero on the hadronic scale, hadrons are pointlike, color is confined inside hadrons. For SM QCD this picture indicates that at the hadronic scale we have effective SQCD, which contains scalar quarks.

The 40 GeV constituents may be good candidates in dark matter particles.
Coupling constant unification at $\alpha_u^{-1} = 29.0$ and scale $10^{16} GeV$ in MSSM [Makhaldani, 2014] has a relict on the SM scale: $\alpha_2^{-1}(m) = 29.0$ at $m = 41 GeV$.
If we extrapolate the SM value of $\alpha^{-1}(m_Z)$ to electron masse scale, we find $\alpha^{-1}(m_e) = 137.0$
Recent (missing) discovery of the second Higgs particle with mass $M_H = 750 GeV$ indicates an interesting structures. It is curious that $M_H/m_h = 750/125 = 6!$

In the Standard Model of Particle Physics (SM), the values of the coupling constants and masses of particles depends on the scale according to the Renormdynamic motion equations. One charge a, one mass m RD equations are

$$\begin{aligned} \dot{\alpha} &= \beta(\alpha), \\ \dot{m} &= \gamma(\alpha)m \end{aligned} \tag{175}$$

For the electron and nucleon masses, electrodynamic and pion-nucleon fine structure constants we have an empirical relation:

$$m_e/\alpha \simeq m_N/\alpha_{\pi N} \tag{176}$$

We take the relation $m/\alpha = const$ as an integral of renormdynamic motion equations for m and α, find exact form of the β function in the minimal mass parametrization

$$\begin{aligned} &\gamma(\alpha) = \gamma_1\alpha + \gamma_2\alpha^2 + ... = \gamma_1 A, \\ &A = f^{-1}(\alpha) = \alpha + \gamma_2/\gamma_1\alpha^2 + \gamma_3/\gamma_1\alpha^3 + ..., \\ &\alpha = f(A) = A + f_2A^2 + f_3A^3 + ... \end{aligned} \tag{177}$$

From the integral of motion, in the minimal mass parametrization: $\gamma(\alpha) = \gamma_1 \alpha$, we obtain

$$\begin{aligned} &(\ln \alpha)^{\cdot} = (\ln m)^{\cdot} \Rightarrow \beta(\alpha)/\alpha = \gamma(\alpha) \\ &= \gamma_1 \alpha \Rightarrow \beta(\alpha) = \beta_2 \alpha^2, \ \beta_2 = \gamma_1 \end{aligned} \tag{178}$$

so, we have the following algebraic-diofant equations for the flavor and color content of the theory

$$\begin{aligned} &\beta_n = 0, \ n \geq 3, \\ &\beta_2 = \gamma_1 \end{aligned} \tag{179}$$

and prediction for the dimension of space-time: $D = 4$. Solution of the motion equations are

$$\begin{aligned} &\alpha(t) = \frac{\alpha_0}{1 - \alpha_0 \beta_2 t}, \\ &m(t) = m_0 |\alpha_0^{-1} - \beta_2 t|^{-\gamma_1/\beta_2} = \frac{m_0}{\alpha_0} \alpha(t) \end{aligned} \tag{180}$$

In the multidimentional renormdynamics, when we have several (N) coupling constants and masses, we assume that there are maximal number $(N-1)$ integrals of motion H_n. If the number of integrals is N, we not have dynamics, we have only statics - finite field theory,
$\alpha_n = const,\ n = 1, ..., N.$
The idea of reduction to the one dimensional renormdynamics is simple:

$$\frac{d\alpha_n}{dt} = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha_N) \Rightarrow \frac{d\alpha_n}{d\alpha} = B_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha),\ \alpha = \alpha_N,$$
$$B_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha) = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha)/\beta_N(\alpha_1, ..., \alpha_{(N-1)}, \alpha),$$
$$\alpha_n = \sum_{k\geq 1} f_{nk}\alpha^k, \quad n = 1, 2, ..., N-1 \tag{181}$$

Solitons are particlelike states, solutions of motion equations and they quantum extensions. Examples are solitons of SinGordon motion equation or barions-skirmions of Skyrme model. In particle theory, the skyrmion was described by Tony Skyrme in 1962 and consists of a quantum superposition of baryons and resonance states.
Skyrmions as topological objects are important in solid state physics. Researchers could read and write skyrmions using scanning tunneling microscopy. The topological charge, representing the existence and non-existence of skyrmions, can represent the bit states "1" and "0".

QCD consists of quarks and gluons. Quarks possess both color (r, g, b) and flavor $(u, d, s,\text{etc.})$, while gluons possess color (r, g, b) and anti-color $(\bar{r}, \bar{g}, \bar{b})$, but not flavor. An open **string** (a string with two endpoints) is ideally suited to account for such quantum numbers at its two ends. For quarks, one end represents color and the other end flavor. For gluons, one end represents color and the other anti-color. In string theory, there are **branes** (higher dimensional extended objects that are generalized membranes) to which the endpoints of an open string are confined. Applying this idea to QCD, we introduce N_c colored branes and N_f flavored branes at which open strings corresponding to quarks and gluons terminate. The energy of a string is given by the sum of the classical energy stored inside the string and the excitation energies of vibration and rotation. Because the classical energy of a string is proportional to its length and because gluons are massless, N_c colored branes should lie on top of one another. On the other hand, quarks possess intrinsic masses, and therefore the endpoints of a quark string, namely, a flavored brane and a colored brane should be separated from each other by a nonvanishing distance U. Then, the intrinsic quark mass m_q can be represented as $m_q = U \times$(string tension), where the string tension is the energy stored inside a unit length of string and is represented: string tension$=1/(2\pi\alpha')$ in terms of α' , historically called the Regge slope.

To describe QCD, we have to prepare Dp-branes and Dq-branes with p, q$\geq$3 for colored branes and flavored branes, respectively, and these branes should be located in the space of more than five dimensions. To evaluate the amplitude for a certain process to occur in the above picture, we have to sum up all the possible two-dimensional world sheets with the weight $\exp(iS)$, where the action S is given by S=(energy)$\times$(time)=(area of the strings world sheet)$/2\pi\alpha'$, following the Feymann path integral formulation.

Cumulative Effect: Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons is connected to the existence of **Fluctons** - droplets of dense cold nuclear matter.
Classical fields have canonical, rational for integer D, (mass)dimensions e.g. in electrodynamics

$$L = \int d^D x(\bar{\psi}(\gamma(\partial - eA) - m)\psi - \frac{1}{4}F^2),$$
$$d_\psi = [\psi] = (D-1)/2,\ d_A = (D-2)/2,\ d_e = (4-D)/2 \quad (182)$$

Quantum corrections introduce (anomaly) corrections to the canonical dimensions, so the fields and coupling constants become **fractals.** At fixed points of RD, the fractals are self similar and their compositions present at low energies **unparticles.**

Qualitative picture of the (un)particle(like) objects we will illustrate with the simplest model of scalar field given by the following lagrangian

$$L = L(\Phi, M, \lambda, n) = \frac{1}{2}(\partial_\mu \Phi) - \frac{1}{2}M^2\Phi^2 - V(\Phi),\ \mu = 0, 1, 2, ..., D \quad (183)$$

where self interaction usually we take in the form

$$V(\Phi) = \lambda\Phi^n,\ n = -2, 1, 2, 3, 4, 6 \quad (184)$$

In renormalisible case,

$$n = \frac{2D}{D-2} = 2 + \varepsilon(D),\ \varepsilon(D) = \frac{4}{D-2},$$
$$D = \frac{2n}{n-2} = 2 + \varepsilon(n),\ \varepsilon(n) = \frac{4}{n-2}, \quad (185)$$

sometimes we consider also intermediate values of n and D and other forms of V.

In the free (self non interacting) field (particle) approximation: $\lambda = 0$, but in external gravitational field we have

$$L(g, \Phi, M) = \sqrt{-g} L(\Phi, M, 0), \ g = det g_{\mu\nu}(x) \tag{186}$$

Now we will see a nice composite particle mechanism :) Let us take a substitute: $\Phi = \varphi^k$, than we find

$$L(g, \Phi, M) = L((k\varphi^{k-1})^4 g, \varphi, M/k), \ g_{\mu\nu}(x) \Rightarrow (k\varphi^{k-1})^{4/D} g_{\mu\nu}(x) \tag{187}$$

Indeed

$$L(g, \Phi, M) = \sqrt{-g}(k^2\varphi^{2(k-1)}\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}M^2\varphi^{2k})$$
$$= \sqrt{-g(k\varphi^{k-1})^4}(\frac{1}{2}(\partial_\mu\varphi)^2) - \frac{1}{2}(\frac{M}{k})^2\varphi^2) \tag{188}$$

Now, having an experience with constituent - composite particle relation, we turn attention on the self-interaction therm,

$$L = \sqrt{-g(k\varphi^{k-1})^4}(... - \frac{\lambda}{k^2}\varphi^N), \ N = kn - 2(k-1) \qquad (189)$$

Most natural value of n for stable systems $(1+1 \to 1+1, \ 2 \to 2)$ is $n = 4$. In this case, $N = 2k + 2$ and only natural value of constituents for which we have a renormalizable interaction is $k = 2 \Rightarrow N = 6$ with corresponding spacetime dimension $D = 3$. The most natural value for fission-fusion interaction $(1 \leftrightarrow 2)$ is $n = 3 \Rightarrow N = k + 2$, for which we have realistic values $k = 2$ and $N = 4, \ D = 4$:) Other interesting values of naturally interpretable monomial (polynomial) interactions generally corresponds to the non-integer, fractional-fractal dimensions of space(time) D, with fractal-flucton-unparticle interpretations of the corresponding states of matter.

The size of particle-like states (solutions of the motion equations) are defined as $l \sim M^{-1}$, because at the boundary region, the linear part of the motion equations dominates and the Yukawa-like asymptotic $\Phi(r) \sim e^{-Mr}$ acts. In a pion-nucleon model for nucleon size we have $l_N \sim m_\pi^{-1} \simeq 1.43$ fm. The amplitude of the state (at maximum) $A \sim \lambda^{-\alpha},\ \alpha = 1/(n-2)$. Indeed, the motion equation do not contains the coupling constant after a scaling substitution $\Phi = \lambda^{-\alpha}\phi$, so a particle-like solution ϕ dos not contains λ and corresponding solution $\Phi = \lambda^{-\alpha}\phi \sim \lambda^{-\alpha}$,

$$\triangle\Phi + M^2\Phi + \lambda n\Phi^{n-1} = \lambda^{-\alpha}(\triangle\phi + M^2\phi + \lambda^{1-(n-2)\alpha}n\phi^{n-1}) = 0. \quad (190)$$

At not so low energies from string theory we may extract the following scalar field theory

$$L = \sqrt{-g}(\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi^3),\ \mu = 0, 1, ..., D-1,\ D = 6 + \varepsilon \quad (191)$$

where $\varepsilon \in [0, 20]$. The one loop $\beta-$function is

$$\beta(a) = (D-6)a - \beta_2 a^2,\ a \sim \lambda^2 \quad (192)$$

and it has stable UV fixed point at $a = (D-6)/\beta_2$ and IR fixed point $a = 0$. Beyond this point we have an unparticle $\Phi = \phi^2$ with lagrangian

$$L = \sqrt{-g'}(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\frac{M}{2})^2\phi^2 - \frac{\lambda}{4}\phi^4),\ \mu = 0, 1, ..., D-1, \\ d = 4 - \varepsilon,\ \varepsilon \in [0, 1]. \quad (193)$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-4)\lambda + b\lambda^2 \tag{194}$$

and it has stable IR fixed point at $\lambda = (4-d)/b$. The UV fixed point is $\lambda = 0$. At this point we have reduction from higher dimentional Φ^3 to lower dimentional ϕ^4.
Another possibilities is an unparticle $\Phi = \varphi^4$ with lagrangian

$$L = \sqrt{-g''}(\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}(\frac{M}{4})^2\varphi^2 - \frac{\lambda}{16}\varphi^6), \ \mu = 0, 1, ..., d-1. \tag{195}$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-3)\lambda + c\lambda^2, \ d = 3-\varepsilon, \ \varepsilon \in [0, 2]. \tag{196}$$

The IR fixed point is $\lambda = \varepsilon/c$. UV fixed point is $\lambda = 0$.

Similar consideration gives reduction from higher energy ϕ^4 model to lower energy φ^6 one. Some technical questions remain. One of them concern to the substitution $\Phi = \phi^2$. It restricts Φ as $\Phi \geq 0$. OK, we already have a constraint, that the fields are real valued, we have a restriction

$$\phi^*(x) = \phi(x) = \frac{1}{(2\pi)^D} \int d^D p \exp(ipx) \hat{\phi}(p) \Rightarrow \hat{\phi}^*(p) = \hat{\phi}(-p) \quad (197)$$

To formulate positivity condition is not so easy. We will take another path, we define the interaction as $\Phi^3 = (\Phi^2)^{3/2} \geq 0$. Then the substitution $\Phi^2 = \phi^4$ will works. Bytheway by this definition we made also another improvement: the potential become bounded from below. For the reduction the substitution $\Phi^2 = \phi^4$ also works,

$$L = \sqrt{-g}(\frac{1}{8\Phi^2})(\partial_\mu \Phi^2)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda (\Phi^2)^{n/2}, \; n = 3, 4. \quad (198)$$

Note that by substitution

$$(\frac{\Phi}{\Phi_0})^2 = \phi^{2k},\ \phi^2 = \exp(\ln(\Phi^2/\Phi_0^2)/k)$$
$$= 1 + \frac{1}{k}\ln(\frac{\Phi}{\Phi_0})^2 + O(K^{-2}),\ \phi = \pm 1 + O(k^{-1}) \quad (199)$$

we reduce the field theory to a discrete theory, to a system of bits. Also, changing dimension of space D and nonlinearity n restricted by condition

$$n = \frac{2D}{D-2},\ D = \frac{2n}{n-2},\ \frac{1}{n} + \frac{1}{D} = \frac{1}{2} \quad (200)$$

we assume that they are functions of scale or coupling constant, due to monotonic property of the coupling constant. We have the following relation

$$\beta_n = -\frac{4}{(D-2)^2}\beta_D,$$
$$\beta_n = \mu\frac{dn}{d\mu} = \frac{dn}{d\lambda}\beta_\lambda,\ \beta_D = \mu\frac{dD}{d\mu} = \frac{dD}{d\lambda}\beta_\lambda. \quad (201)$$

Computers are physical devices and their behavior is determined by physical laws. The Quantum Computations [Benenti, Casati, Strini, 2004 , Nielsen, Chuang, 2000], Quantum Computing, Quanputing [Makhaldiani, 2007.2], is a new interdisciplinary field of research, which benefits from the contributions of physicists, computer scientists, mathematicians, chemists and engineers. Contemporary digital computer and its logical elements can be considered as a spatial type of discrete dynamical systems [Makhaldiani, 2001]

$$S_n(k+1) = \Phi_n(S(k)), \tag{202}$$

where

$$S_n(k), \quad 1 \leq n \leq N(k), \tag{203}$$

is the state vector of the system at the discrete time step k. Vector S may describe the state and Φ transition rule of some Cellular Automata [Toffoli, Margolus, 1987].The systems of the type (202) appears in applied mathematics as an explicit finite difference scheme approximation of the equations of the physics [Samarskii, Gulin, 1989].

Definition: *We assume that the system (202) is time-reversible if we can define the reverse dynamical system*

$$S_n(k) = \Phi_n^{-1}(S(k+1)). \tag{204}$$

In this case the following matrix

$$M_{nm} = \frac{\partial \Phi_n(S(k))}{\partial S_m(k)}, \tag{205}$$

is regular, i.e. has an inverse. If the matrix is not regular, this is the case, for example, when $N(k+1) \neq N(k)$, we have an irreversible dynamical system (usual digital computers and/or corresponding irreversible gates).

Let us consider an extension of the dynamical system (202) given by the following action function

$$A = \sum_{kn} l_n(k)(S_n(k+1) - \Phi_n(S(k))) \tag{206}$$

and corresponding motion equations

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)) = \frac{\partial H}{\partial l_n(k)}, \\ l_n(k-1) &= l_m(k)\frac{\partial \Phi_m(S(k))}{\partial S_n(k)} = l_m(k)M_{mn}(S(k)) = \frac{\partial H}{\partial S_n(k)}, \end{aligned} \tag{207}$$

where

$$H = \sum_{kn} l_n(k)\Phi_n(S(k)), \tag{208}$$

is discrete Hamiltonian. In the regular case, we put the system (207) in an explicit form

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)), \\ l_n(k+1) &= l_m(k)M^{-1}_{mn}(S(k+1)). \end{aligned} \tag{209}$$

From this system it is obvious that, when the initial value $l_n(k_0)$ is given, the evolution of the vector $l(k)$ is defined by evolution of the state vector $S(k)$. The equation of motion for $l_n(k)$ - Elenka is linear and has an important property that a linear superpositions of the solutions are also solutions.

Statement: *Any time-reversible dynamical system (e.g. a time-reversible computer) can be extended by corresponding linear dynamical system (quantum - like processor) which is controlled by the dynamical system and has a huge computational power,*
[Makhaldiani, 2001, Makhaldiani, 2002, Makhaldiani, 2007.2, Makhaldiani, 2011.2].

For motion equations (207) in the continual approximation, we have

$$\begin{aligned} &S_n(k+1) = x_n(t_k+\tau) = x_n(t_k) + \dot{x}_n(t_k)\tau + O(\tau^2), \\ &\dot{x}_n(t_k) = v_n(x(t_k)) + O(\tau), \quad t_k = k\tau, \\ &v_n(x(t_k)) = (\Phi_n(x(t_k)) - x_n(t_k))/\tau; \\ &M_{mn}(x(t_k)) = \delta_{mn} + \tau\frac{\partial v_m(x(t_k))}{\partial x_n(t_k)}. \end{aligned} \tag{210}$$

(de)Coherence criterion: *the system is reversible, the linear (quantum, coherent, soul) subsystem exists, when the matrix M is regular,*

$$detM = 1 + \tau\sum_n \frac{\partial v_n}{\partial x_n} + O(\tau^2) \neq 0. \tag{211}$$

For the Nambu - Poisson dynamical systems (see e.g. [Makhaldiani, 2007])

$$\begin{aligned} &v_n(x) = \varepsilon_{nm_1m_2...m_p}\frac{\partial H_1}{\partial x_{m_1}}\frac{\partial H_2}{\partial x_{m_2}}...\frac{\partial H_p}{\partial x_{m_p}}, \quad p = 1,2,3,...,N-1, \\ &\sum_n \frac{\partial v_n}{\partial x_n} \equiv divv = 0. \end{aligned} \tag{212}$$

Let me motivate an idea of construction of the reversible dynamical systems by simple example from field theory. There are renormalizable models of scalar field theory of the form (see, e.g. [Makhaldiani, 1980])

$$L = \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) - g\varphi^n, \tag{213}$$

with the constraint

$$n = \frac{2d}{d-2} \ , \tag{214}$$

where d is dimension of the space-time and n is degree of nonlinearity. It is interesting that if we define d as a function of n, we find

$$d = \frac{2n}{n-2} \tag{215}$$

the same function !
Thing is that, the constraint can be put in the symmetric implicit form [Makhaldiani, 1980]

$$\frac{1}{n} + \frac{1}{d} = \frac{1}{2} \tag{216}$$

Now it is natural to consider the following symmetric function

$$f(y) + f(x) = c \tag{217}$$

and define its solution

$$y = f^{-1}(c - f(x)). \tag{218}$$

This is the general method, that we will use in the following construction of the reversible dynamical systems. In the simplest case,

$$f(x) = x, \tag{219}$$

we take

$$y = S(k+1), \quad x = S(k-1), \quad c = \tilde{\Phi}(S(k)) \tag{220}$$

and define our reversible dynamical system from the following symmetric, implicit form (see also [Toffoli, Margolus, 1987])

$$S(k+1) + S(k-1) = \tilde{\Phi}(S(k)), \tag{221}$$

explicit form of which is

$$\begin{aligned} S(k+1) &= \Phi(S(k), S(k-1)) \\ &= \tilde{\Phi}(S(k)) - S(k-1). \end{aligned} \tag{222}$$

This dynamical system defines given state vector by previous two state vectors. We have reversible dynamical system on the time lattice with time steps of two units,

$$\begin{aligned} &S(k+2,2) = \Phi(S(k,2)), \\ &S(k+2,2) \equiv (S(k+2), S(k+1)), \\ &S(k,2) \equiv (S(k), S(k-1))). \end{aligned} \tag{223}$$

Internal, spin, degrees of freedom

Starting from a general discrete dynamical system, we obtained reversible dynamical system with internal(spin,bit) degrees of freedom

$$\begin{aligned} S_{ns}(k+2) &\equiv \begin{pmatrix} S_n(k+2) \\ S_n(k+1) \end{pmatrix} = \begin{pmatrix} \Phi_n(\Phi(S(k)) - S(k-1)) - S(k)) \\ \Phi_n(S(k)) - S_n(k-1) \end{pmatrix} \\ &\equiv \Phi_{ns}(S(k)), \quad s = 1, 2 \end{aligned} \tag{224}$$

where

$$S(k) \equiv (S_{ns}(k)), \quad S_{n1}(k) \equiv S_n(k), \quad S_{n2}(k) \equiv S_n(k-1) \tag{225}$$

For the extended system we have the following action

$$A = \sum_{kns} l_{ns}(k)(S_{ns}(k+2) - \Phi_{ns}(S(k))) \tag{226}$$

and corresponding motion equations

$$
\begin{aligned}
S_{ns}(k+2) &= \Phi_{ns}(S(k)) = \frac{\partial H}{\partial l_{ns}(k)}, \\
l_{ns}(k-2) &= l_{mt}(k)\frac{\partial \Phi_{mt}(S(k))}{\partial S_{ns}(k)} \\
&= l_{mt}(k) M_{mtns}(S(k)) = \frac{\partial H}{\partial S_{ns}(k)},
\end{aligned}
\tag{227}
$$

By construction, we have the following reversible dynamical system

$$
\begin{aligned}
S_{ns}(k+2) &= \Phi_{ns}(S(k)), \\
l_{ns}(k+2) &= l_{mt}(k) M^{-1}_{mtns}(S(k+2)),
\end{aligned}
\tag{228}
$$

with classical S_{ns} and quantum l_{ns}(in the external, background S) string bit dynamics.

We can also consider p-point generalization of the previous structure,

$$\begin{aligned}
&f_p(S(k+p)) + f_{p-1}(S(k+p-1)) + ... + f_1(S(k+1)) \\
&+f_1(S(k-1)) + ... + f_p(S(k-p)) = \tilde{\Phi}(S(k)), \\
&S(k+p) = \Phi(S(k), S(k+p-1), ..., S(k-p)) \\
&\equiv f_p^{-1}(\tilde{\Phi}(S(k)) - f_{p-1}(S(k+p-1)) - ... - f_p(S(k-p)))
\end{aligned} \tag{229}$$

and corresponding reversible p-oint cluster dynamical system

$$\begin{aligned}
&S(k+p,p) \equiv \Phi(S(k,p)), \\
&S(k+p,p) \equiv (S(k+p), S(k+p-1), ..., S(k+1)), \\
&S(k,p) \equiv (S(k), S(k-1), ..., S(k-p+1)), \quad S(k,1) = S(k).
\end{aligned} \tag{230}$$

So we have general method of construction of the reversible dynamical systems on the time (tame) scale p. The method of linear extension of the reversible dynamical systems (see [Makhaldiani, 2001] and previous section) defines corresponding Quanputers,

$$\begin{aligned}
&S_{ns}(k+p) = \Phi_{ns}(S(k)), \\
&l_{ns}(k+p) = l_{mt}(k) M^{-1}_{mtns}(S(k+p)),
\end{aligned} \tag{231}$$

p-point cluster and higher spin states reversible dynamics, or pit string dynamics

This case the quantum state function $l_{ns},\ s = 1, 2, ...p$ will describes the state with spin $(p-1)/2$.

Note that, in this formalism for reversible dynamics minimal value of the spin is $1/2$. There is not a place for a scalar dynamics, or the scalar dynamics is not reversible. In the Standard model (SM) of particle physics, [Beringer et al, 2012], all of the fundamental particles, leptons, quarks and gauge bosons have spin. Only scalar particles of the SM are the Higgs bosons. Perhaps the scalar particles are composed systems or quasiparticles like phonon, or Higgs dynamics is not reversible (a mechanism for 'time arrow').

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

The $NP \stackrel{?}{=} P$ problem will be solved if for some $NP-$ complete problem, e.g. TSP, a polynomial algorithm find; or show that there is not such an algorithm; or show that it is impossible to find definite answer to that question.
TSP means to find minimal length path between N fixed points on a surface, which attends any point ones. We consider a system where N points with quenched positions $x_1, x_2, ..., x_N$ are independently distributed on a finite domain D with a probability density function $p(x)$. In general, the domain D is multidimensional and the points x_n are vectors in the corresponding Euclidean space. Inside the domain D we consider a polymer chain composed of N monomers whose positions are denoted by $y_1, y_2, ..., y_N$. Each monomer y_n is attached to one of the quenched sites x_m and only one monomer can be attached to each site. The state of the polymer is described by a permutation $\sigma \in \Sigma_N$ where Σ_N is the group of permutations of N objecs.

The Hamiltonian for the system is given by

$$H = \sum_{n=1}^{N} V(|y_n - y_{n-1}|) \tag{232}$$

Here V is the interaction between neighboring monomers on the polymer chain. For convenience the chain is taken to be closed, thus we take the periodic boundary condition $x_0 = x_N$. A physical realization of this system is one where the x_n are impurities where the monomers of a polymer loop are pinned. In combinatorial optimization, if one takes $V(x)$ to be the norm, or distance, of the vector x then $H(\sigma)$ is the total distance covered by a path which visits each site x_n exactly once. The problem of finding σ_0 which minimizes $H(\sigma)$ is known as the traveling salesman problem (TSP) [Gutin, Pannen, 2002].

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

In field theory language to the TSP we correspond the calculation of the following correlator

$$
\begin{aligned}
& G_{2N}(x_1, x_2, ..., x_N) = Z_0^{-1} \int d\varphi(x)\varphi^2(x_1)\varphi^2(x_2)...\varphi^2(x_N)e^{-S(\varphi)} \\
& = \frac{\delta^{2N} F(J)}{\delta J(x_1)^2 ... \delta J(x_N)^2}, \ F(J) = \ln Z(J), \\
& Z(J) = \int d\varphi e^{-\frac{1}{2}\varphi \cdot A \cdot \varphi + J \cdot \varphi} = e^{\frac{1}{2} J \cdot A^{-1} \cdot J}, \ A^{-1}(x, y; m) = e^{-m|x-y|}, \\
& L_{min}(x_1, ..., x_N) = -\frac{d}{dm} \ln G_{2Ns} + O(e^{-am}) \\
& < A^{-1} > \equiv \frac{1}{\Gamma(s)} \int_0^\infty dm m^{s-1} A^{-1}(x, y; m) = \frac{1}{|x-y|^s} \\
& = L_s A^{-1}(x-y; s) \\
& k(d)\Delta_d L_s A^{-1}(x; s) = \delta^d(x) \Rightarrow A(x; s) = k(d)\Delta_d L_s, \\
& s = d-2; \varphi = \varphi(x, m).
\end{aligned} \tag{233}
$$

If we take relativistic massive scalar field, then $A = \Delta_d + m^2$,

$$A^{-1}(x) \sim |x|^{2-d} e^{-m|x|}, \tag{234}$$

and for $d = 2$, we also have the needed behaviour. Note that G_{2N} is symmetric with respect to its arguments and contains any paths including minimal length one.

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