

# Self-consistent analysis of hadron production in pp and AA collisions at mid-rapidity



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# Content

- 1. Self-similarity approach for A-A collisions and its further development.**
- 2. Self-consistent description of inclusive spectra of hadrons in p-p and A-A collisions in the mid-rapidity region.**
- 3. Checking of our suggested gluon distribution by description of different kind of data.**
- 4. Conclusion.**

The inclusive spectrum of the produced particle **1** in AA collision can be presented as the universal function dependent of the self-similarity parameter which was chosen, for example, as the Gaussian

function:

$$E \cdot \frac{d^3\sigma}{dp^3} = C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp(-\Pi/C_2)$$

where  $\alpha(N_I) = 1/3 + N_I/3$ ,  $\alpha(N_{II}) = 1/3 + N_{II}/3$ ,  $C_1 = 1.9 \cdot 10^4 \text{mb} \cdot \text{GeV}^{-2} \cdot \text{c}^3 \cdot \text{st}^{-1}$ ,  $C_2 = 0.125$ .

For reaction with the production of the inclusive particle **1** we can write the conservation law of four-momentum in the following form:

$$(N_I P_I + N_{II} P_{II} - p_1)^2 = (N_I m_0 + N_{II} m_0 + M)^2$$

where  $N_I$  and  $N_{II}$  the number of nucleons involved in the interaction;  $P_I$ ,  $P_{II}$ ,  $p_1$  are four momenta of the nuclei I and II and particle **1**, respectively;  $m_0$  is the mass of the nucleon;  $M$  is the mass of the particle providing the conservation of the baryon number, strangeness, and other quantum numbers.

In ***A. M. Baldin, A. A. Baldin. Phys. Particles and Nuclei, 29 (3), (1998) 232*** the parameter of self-similarity is introduced, which allows one to describe the differential cross section of the yield of a large class of particles in relativistic nuclear collisions:

$$\Pi = \min \frac{1}{2} \sqrt{(u_I N_I + u_{II} N_{II})^2}$$

where  $u_I$  and  $u_{II}$  are four velocities of the nuclei I and II.

The question arises, what is a relation of the similarity parameter  $\Pi$  to the relativistic invariant variables  $s, p_t^2$  ? This relation can be found from Eqs.(5-8) using  $ch(Y) = \sqrt{s}/(2m_0)$ . Then, we have the following form for  $\Pi$ :

$$\Pi = \left\{ \frac{m_{1t}}{2m_0\delta} + \frac{M}{\sqrt{s}\delta} \right\} \left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{m_{1t}^2} \delta} \right\}$$

where  $\delta = 1 - 4m_0^2/s$ ;  $m_{1t} = \sqrt{p_t^2 + m_1^2}$  is the transverse mass of the produced hadron  $h$ . At large initial energies  $\sqrt{s} \gg 1$  GeV the similarity parameter  $\Pi$  becomes

$$\Pi = \frac{m_{1t}}{2m_0(1 - 4m_0^2/s)} \left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{m_{1t}^2} (1 - 4m_0^2/s)} \right\}$$

***For produced pions  $M=0$  and we have at  $p_t > m_1 = \mu_\pi$  approximately***

$$\Pi \simeq \frac{m_{1t}}{m_0(1 - 4m_0^2/s)}$$

# Further development of this to hadron production in N-N collision at high energies

$$E(d^3\sigma/dp^3)_q = \rho_q(y=0, p_t) = \varphi_q(y=0, p_t) \cdot \sum_{n=1}^{\infty} [n \cdot \sigma_n(s)] =$$

$$= \varphi_q(y=0, p_t) \cdot g(s/s_0)^\Delta$$

K.A.Ter-Martirosyan.  
Sov.J.Nucl.Phys., 44, 817  
(1986).

Inclusive hadron production in central region and the **AGK (Abramovsky, Gribov, Kanchelly)** cancellation

$$E(d^3\sigma/dp^3)_g \leftarrow \rho_g(y=0, p_t) = \varphi_g(y=0, p_t) \cdot \sum_{n=2}^{\infty} [(n-1)\sigma_n(s)] =$$

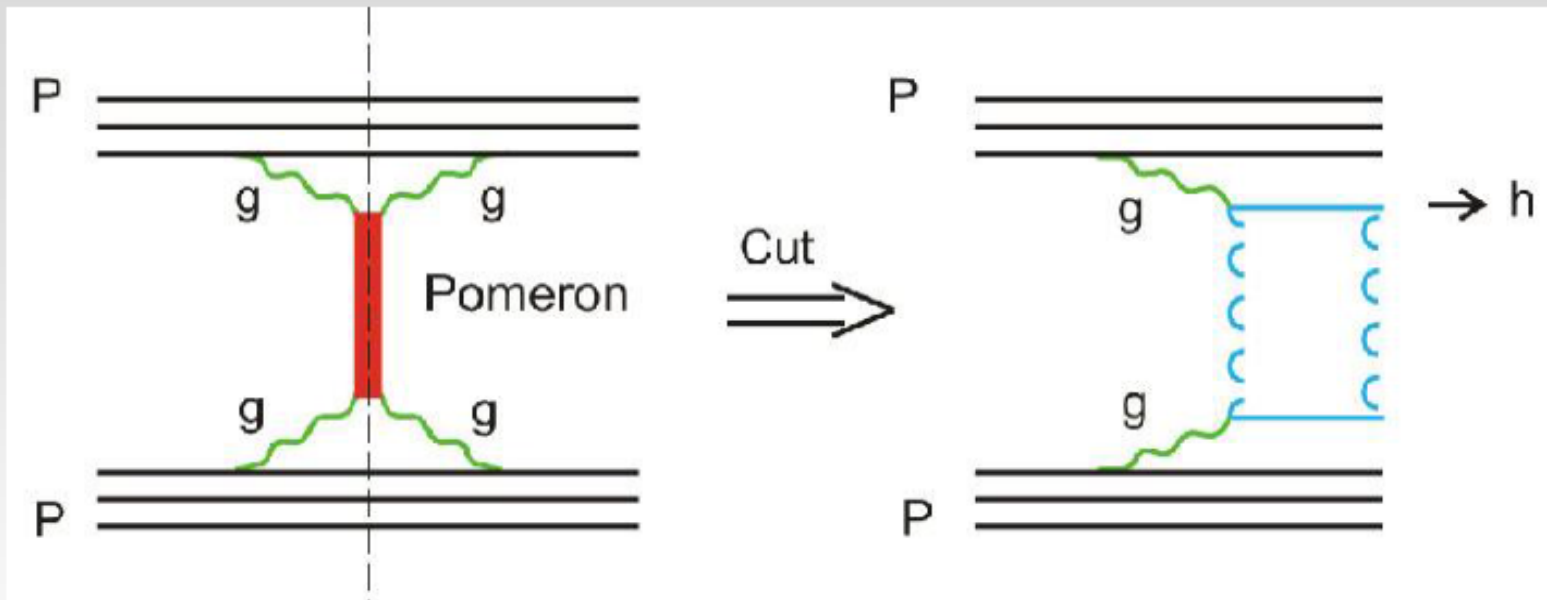
$$= \varphi_g(y=0, p_t) \cdot (g(s/s_0)^\Delta - \sigma_{nd})$$

V.A. Bednyakov, A.A. Grinyuk, G.I. Lykasov, M. Poghosyan, Int.J.Mod.Phys., A27, (2012) 1250042; A.A. Grinyuk, G.I. Lykasov, A.V. Lipatov, N.P. Zotov, Phys.Rev.D87 (2013) 074017.

$$E(d^3\sigma/dp^3) = [\varphi_q(y, p_t) + \varphi_g(y, p_t) \cdot (1 - \sigma_{nd}/g(s/s_0)^\Delta)] \cdot g(s/s_0)^\Delta$$

$\sigma_n$  – cross-section of hadron production by the exchange of n-pomerons.

$\phi = \phi(\Pi)$ ,  $g$  – constant ( $\sim 20$  mbarn),  $S_0 \sim 1$  GeV<sup>2</sup>,  $\Delta = [\alpha_p(0)-1] \sim 0,08$



One-Pomeron exchange (left) and the cut one-Pomeron exchange (right); P-proton, g-gluon, h-hadron produced in PP

In the light cone dynamics the proton has a general decomposition:

$|uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle, \dots$  S.J.Brodsky, C.Peterson, N.Sakai,  
Phys.Rev. D 23 (1981) 2745.

$$E \frac{d^3 \sigma}{d^3 p} = \frac{1}{\pi} \frac{d\sigma}{dp_t^2 dy} \equiv \frac{1}{\pi} \frac{d\sigma}{dm_{1t}^2 dy}$$

$$\frac{1}{\pi} \frac{d\sigma}{dm_{1t}^2 dy} = [\phi_q(y=0, \Pi) + \phi_g(y=0, \Pi) \cdot (1 - \sigma_{nd}/g((s/s_0)^\Delta))] \cdot g(s/s_0)^\Delta .$$

The first part of the inclusive spectrum (Soft QCD (quarks)) is related to the function  $\phi_q(y=0, \Pi)$ , which is fitted by the following form [★]:

$$\phi_q(y=0, \Pi) = A_q \exp(-\Pi/C_q) ,$$

$$\text{where } A_q = 3.68 (GeV/c)^{-2}, C_q = 0.147$$

The function  $\phi_g(y=0, \Pi)$  related to the second part (Soft QCD (gluons)) of the spectrum is fitted by the following form [30]:

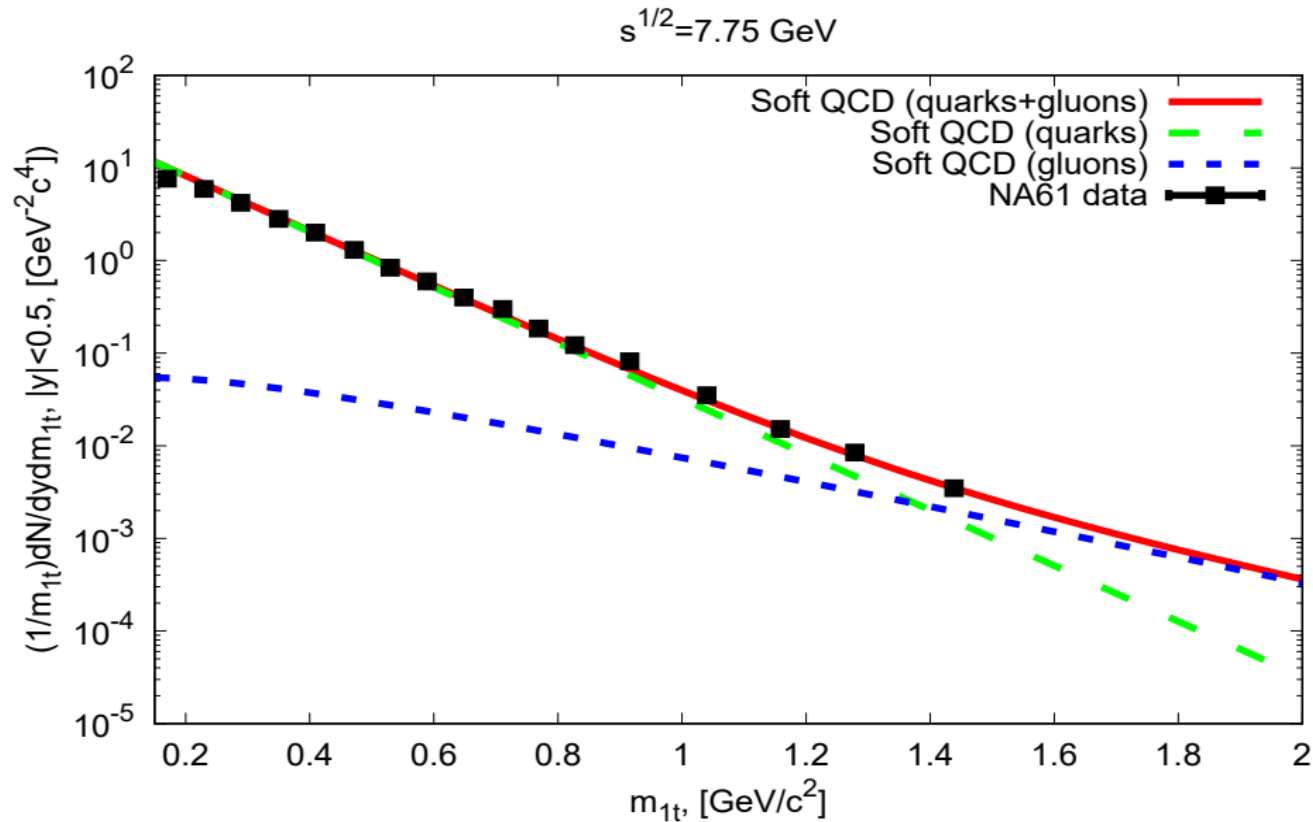
$$\phi_g(y=0, \Pi) = A_g \sqrt{m_{1t}} \exp(-\Pi/C_g) ,$$

$$\text{where } A_g = 1.7249 (GeV/c)^{-2}, C_g = 0.289$$

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★ **V. A. Bednyakov, A. A. Grinyuk, G. I. Lykasov, M. Pogosyan.**  
**Int.J.Mod.Phys., A27 (2012) 1250042.**

# PP $\rightarrow \pi + X$ at initial momentum about 31 GeV/c



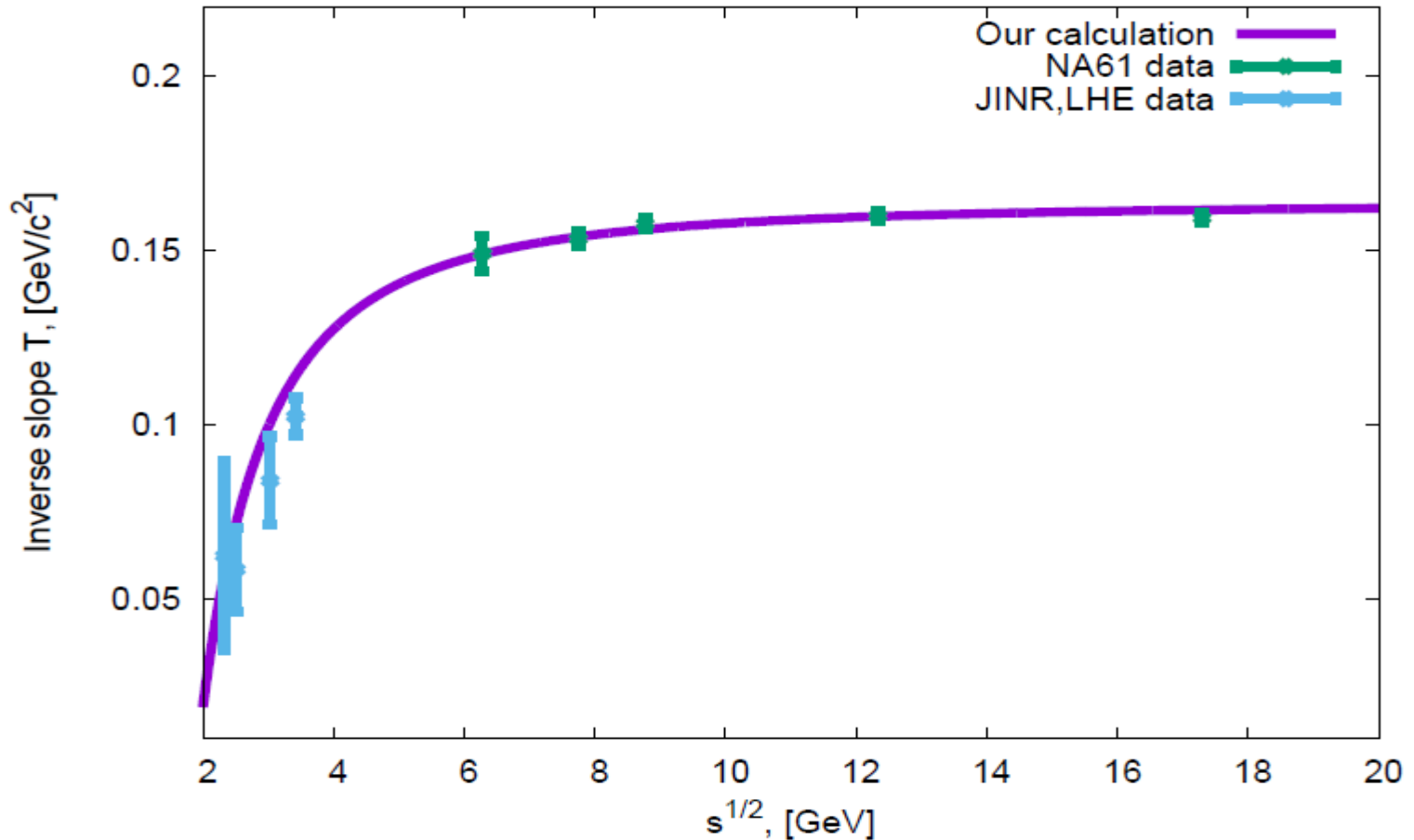
Transverse mass distribution of negative pions produced in p-p

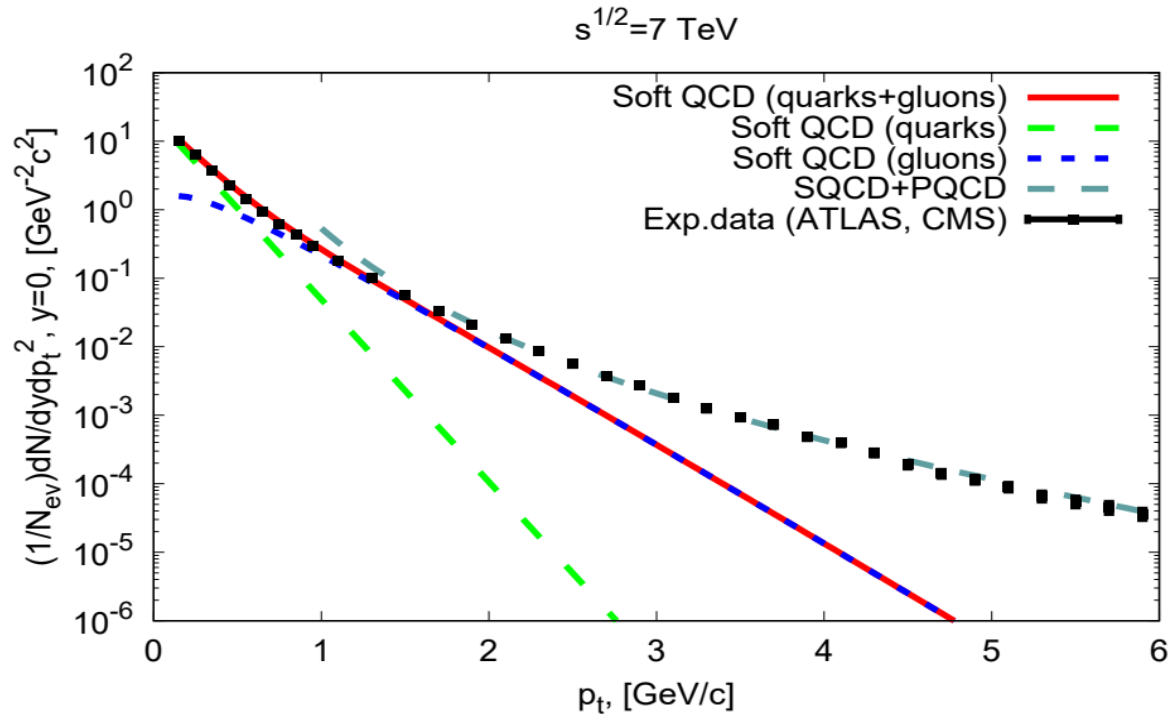


$$E(d^3\sigma/dp^3) \sim \exp(-m_t/T), \quad T = \text{Const}$$

$$E(d^3\sigma/dp^3) \sim \exp(-\Pi/C_1) = \exp(-m_t/[C_1 m_0(1-4m_0^2/s)])$$

$$T = C_1 m_0(1-4m_0^2/s)$$



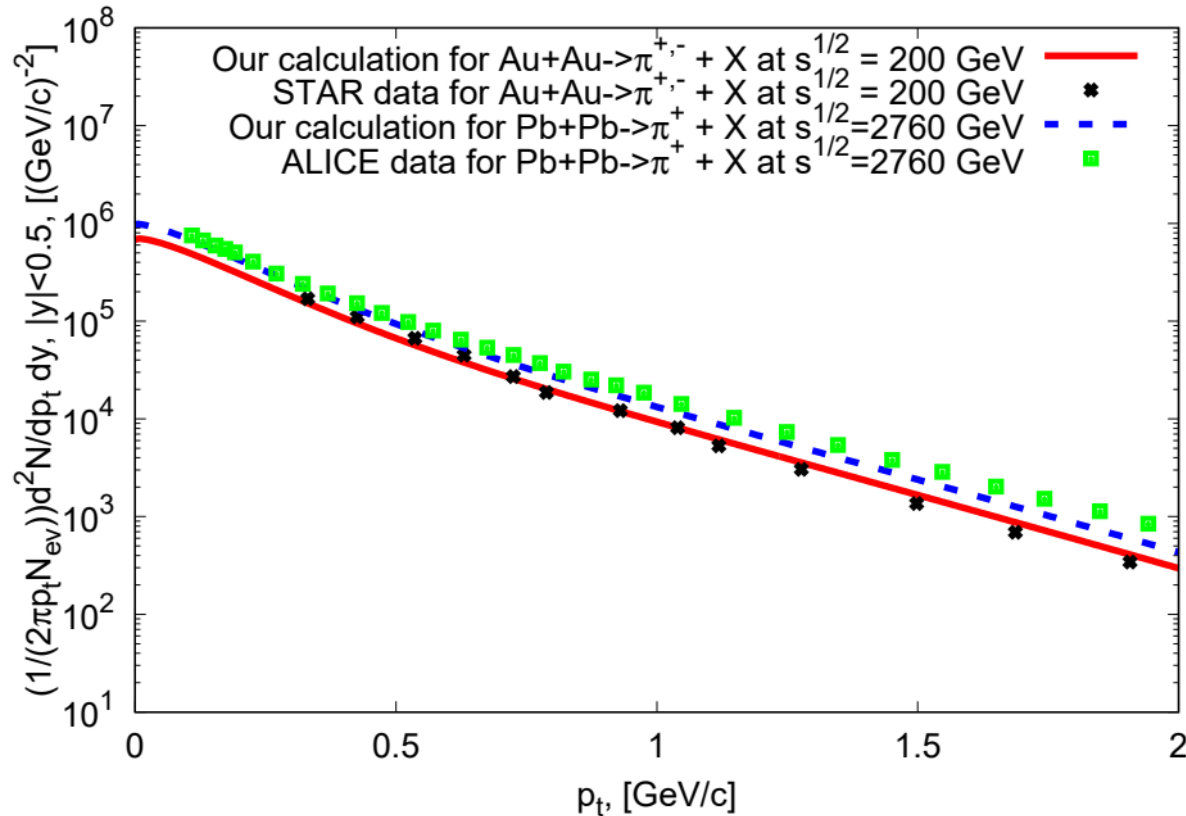


Results of the calculations of the inclusive cross section of charge hadrons produced in pp collisions at the LHC energies as a function of their transverse momentum  $p_t$  at  $\sqrt{s} = 7$  TeV. The points are the LHC experimental data: ***G. Aad, et al. (ATLAS Collaboration), New J. Phys. 13, 053033 (2011) and V. Khachatryan, et al. (CMS Collaboration), Phys. Rev. Lett. 105, 022002 (2010).***

**A.A. Grinyuk, A.V.Lipatov, G.L., N.P.Zotov, *Phys.Rev. D93, 014035 (2016)***

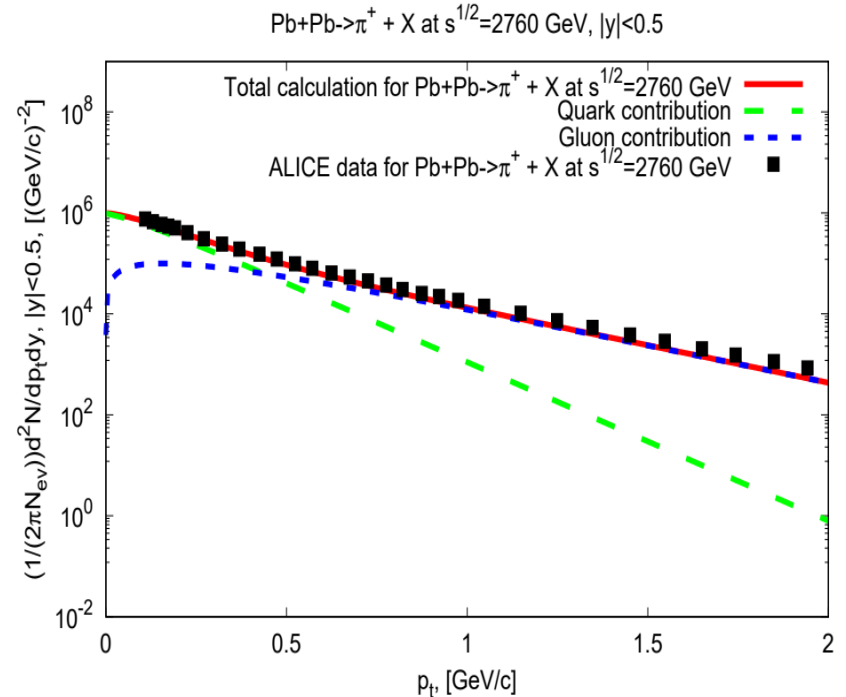
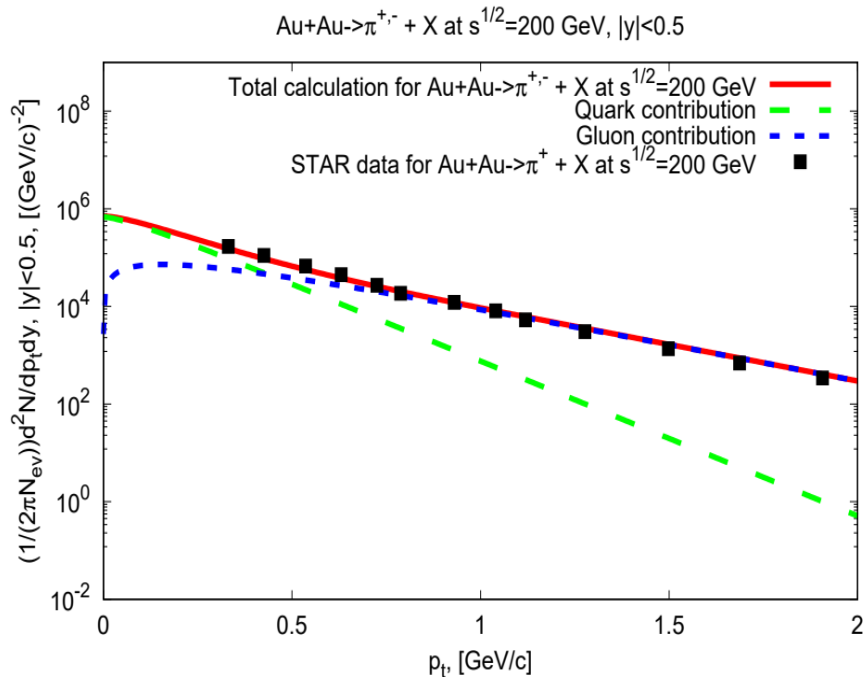
# Pion production in A-A collisions

$$Ed^3\sigma/dp^3 = C_1 A_1^{\alpha(NI)} A_{II}^{\alpha(NII)} F(\Pi), F = F_q + F_g$$



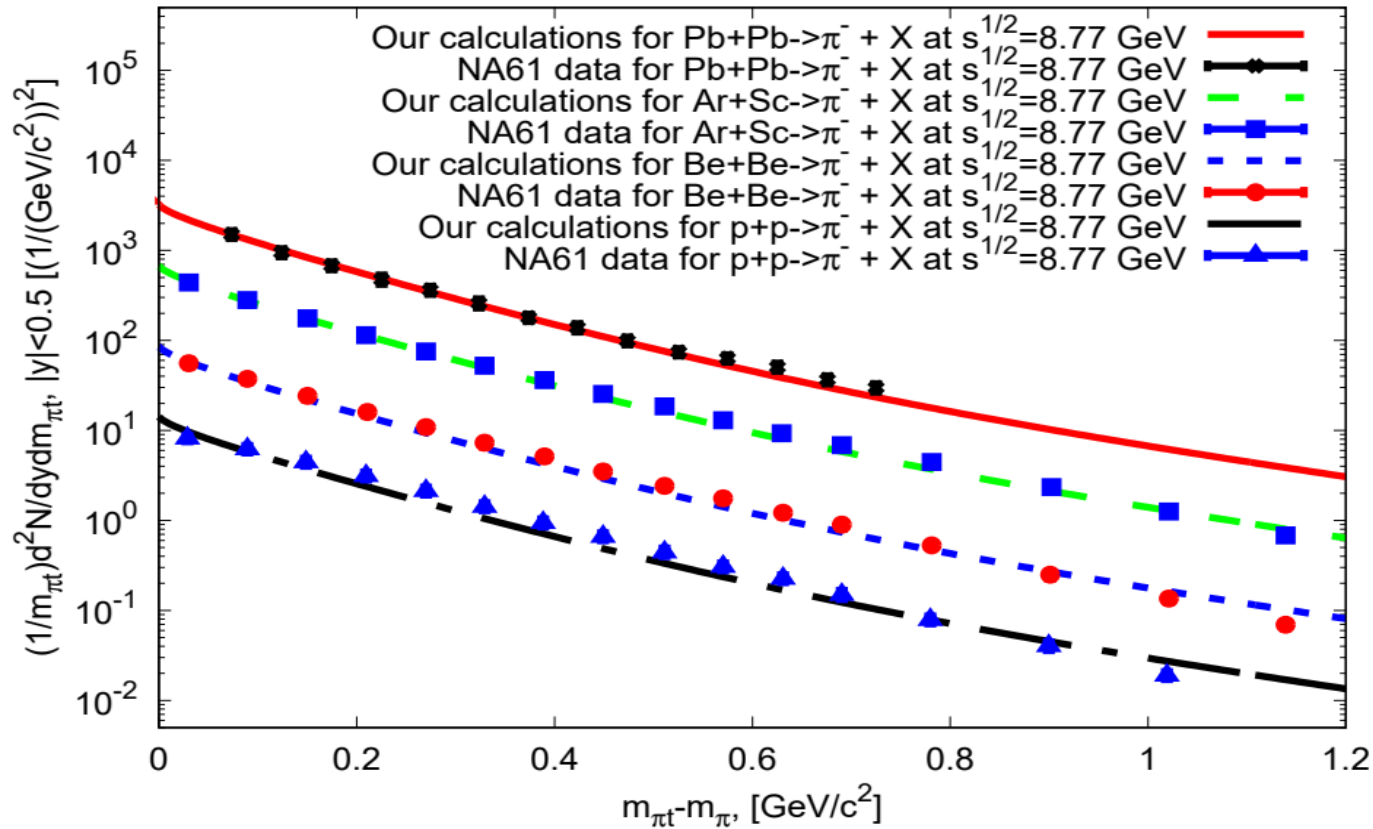
G.I.Lykasov, A.I. Malakhov, arXiv: 1801.07250 [hep-ph], EPJ A in press

# Description of A-A data in detail



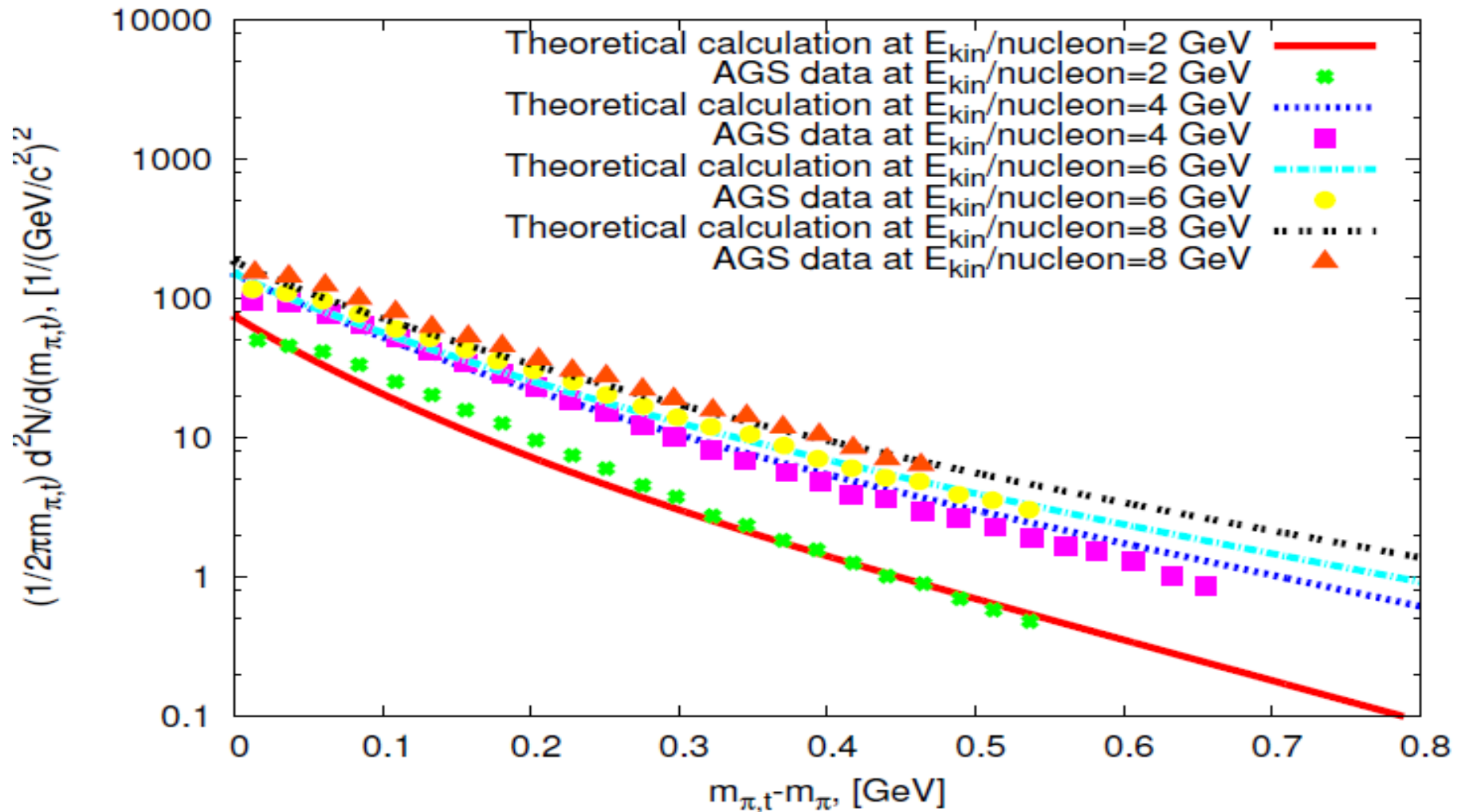
Left: STAR data description, solid red line is our total calculation; dashed green curve is the quark contribution; short dash blue line is the gluon contribution; black squares are STAR data for Au+Au  $\rightarrow \pi + X$  at 200 GeV  
Right panel is the similar as the left plot but for Pb+Pb  $\rightarrow \pi + X$  at 2.76 TeV, points are ALICE data .

# Pion production in A-A collisions at 40 GeV/c/nucleon

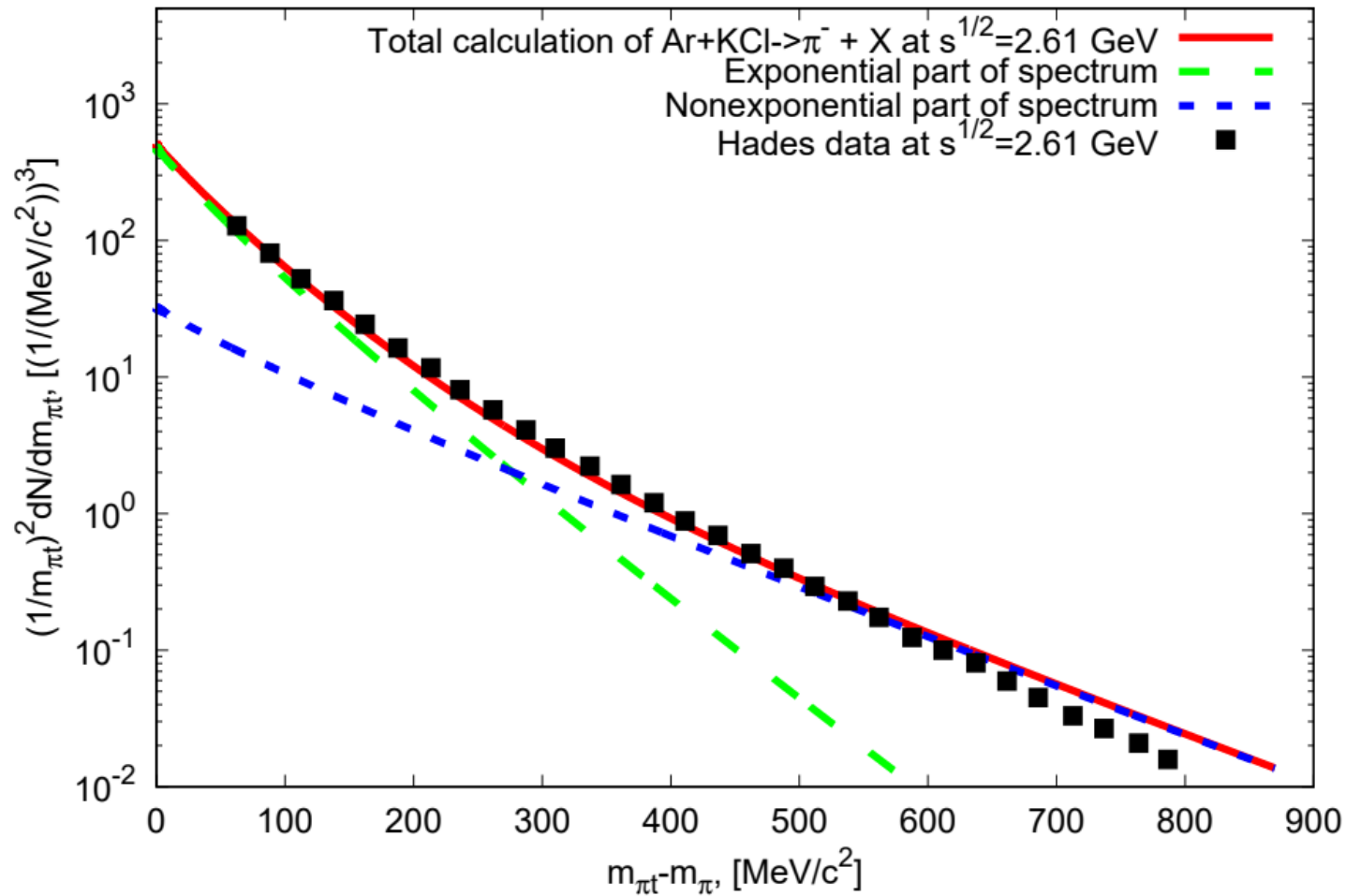


G.I.Lykasov, A.I. Malakhov, arXiv: 1801.07250 [hep-ph], EPJ A in press

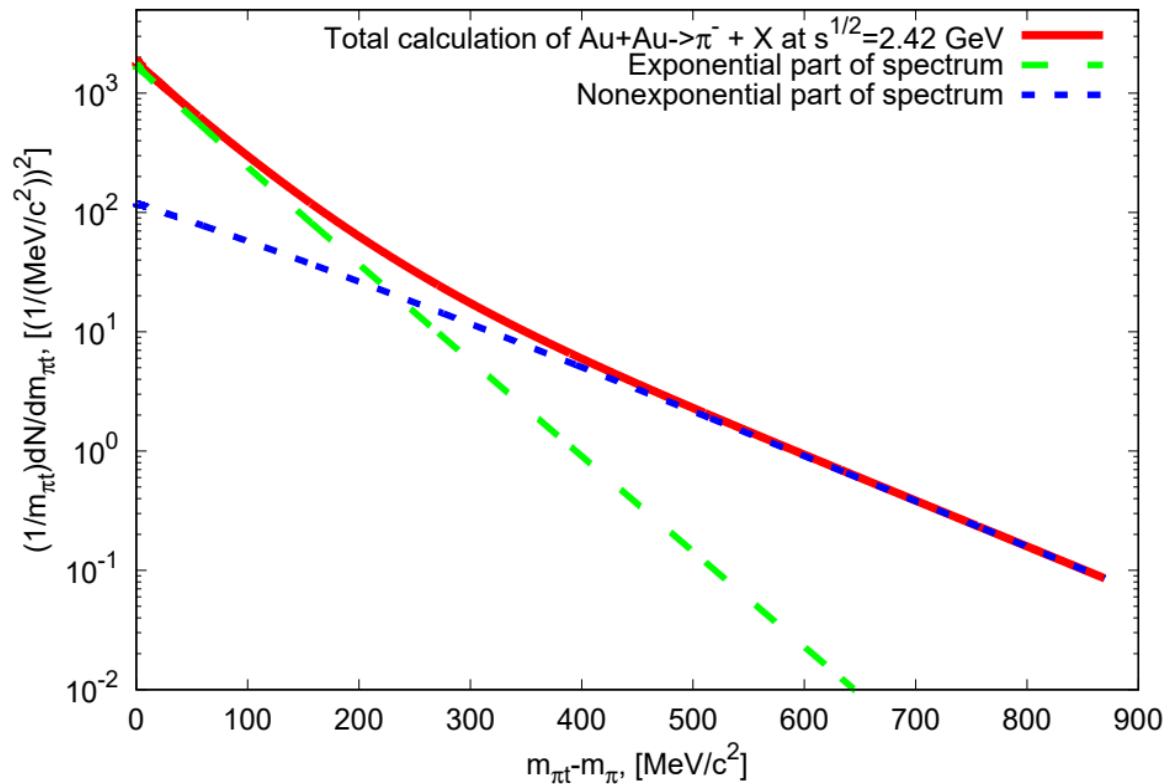
# Pion production in Au+Au, AGS data



# Pion production in Ar+KCl, HADES data at 1.75 GeV/n

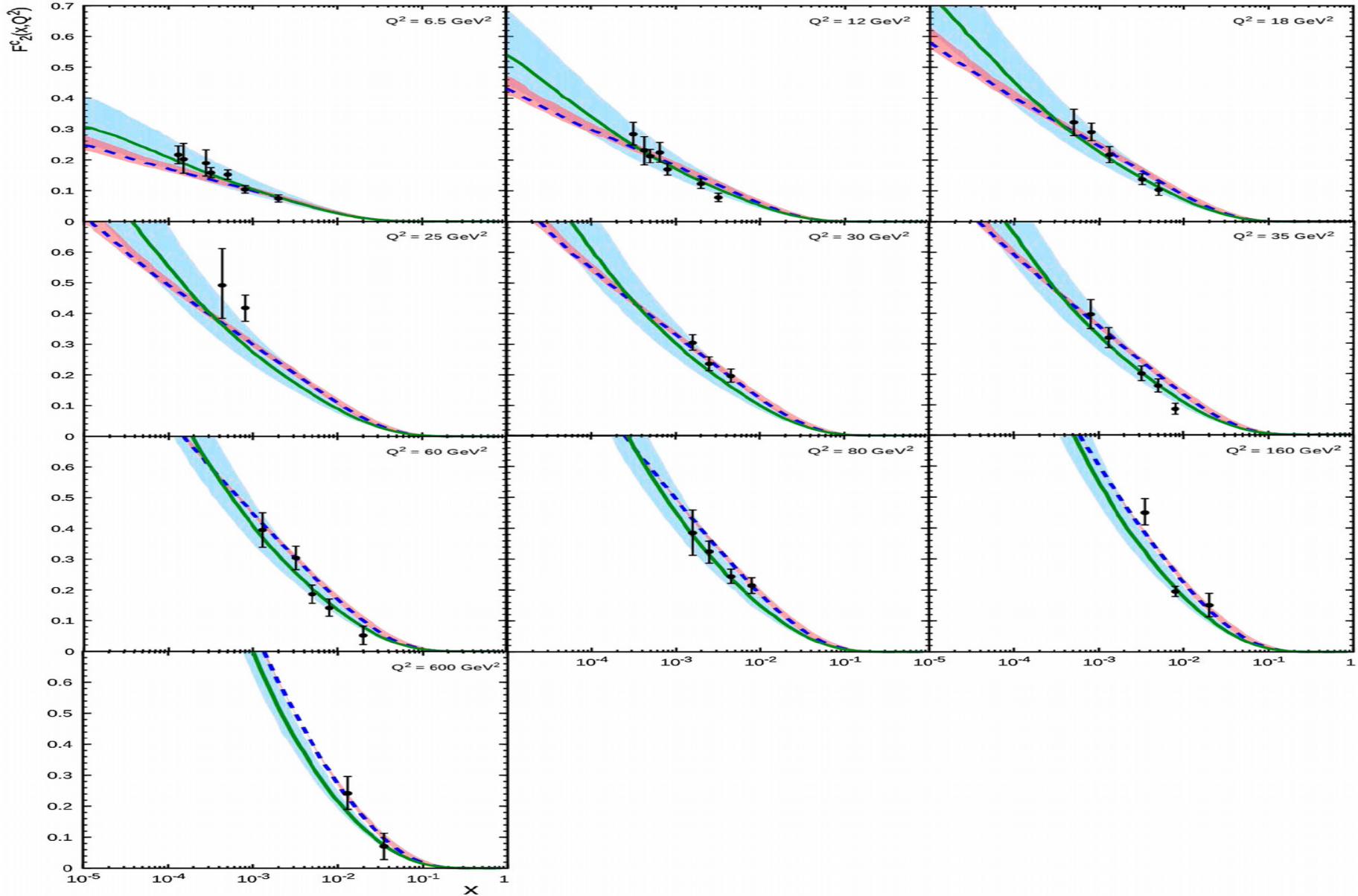


**Pion production in Au+Au  
at  $E_{kin/nucl} = 1.25$  GeV  
Prediction for HADES**



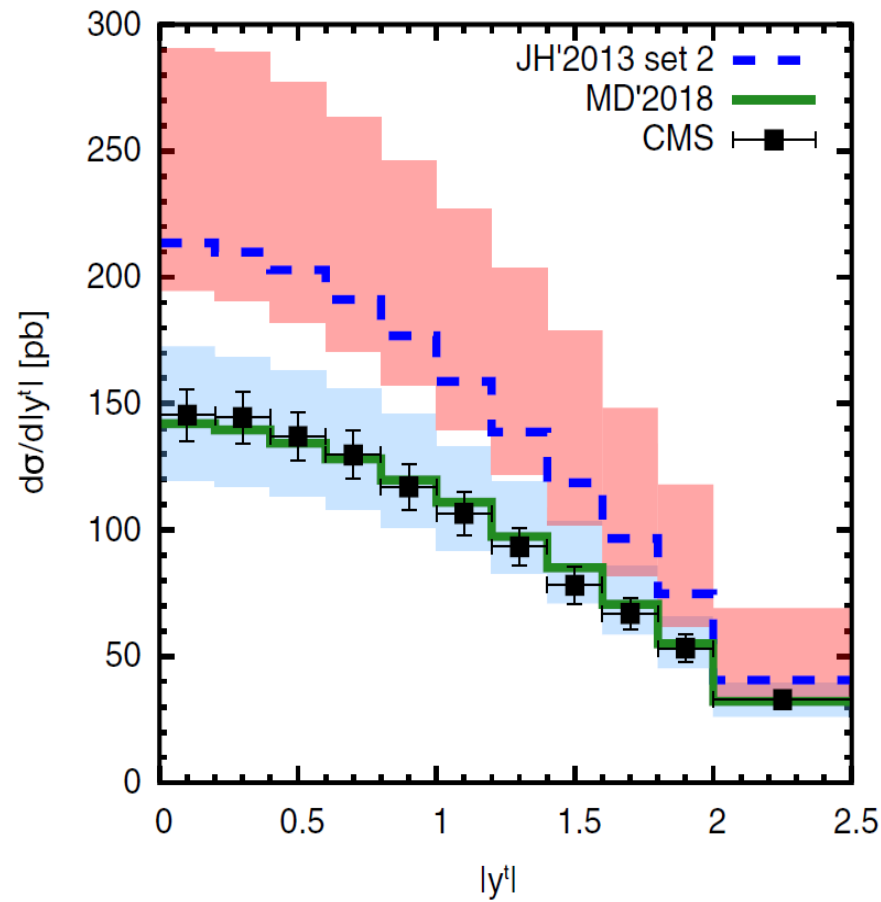
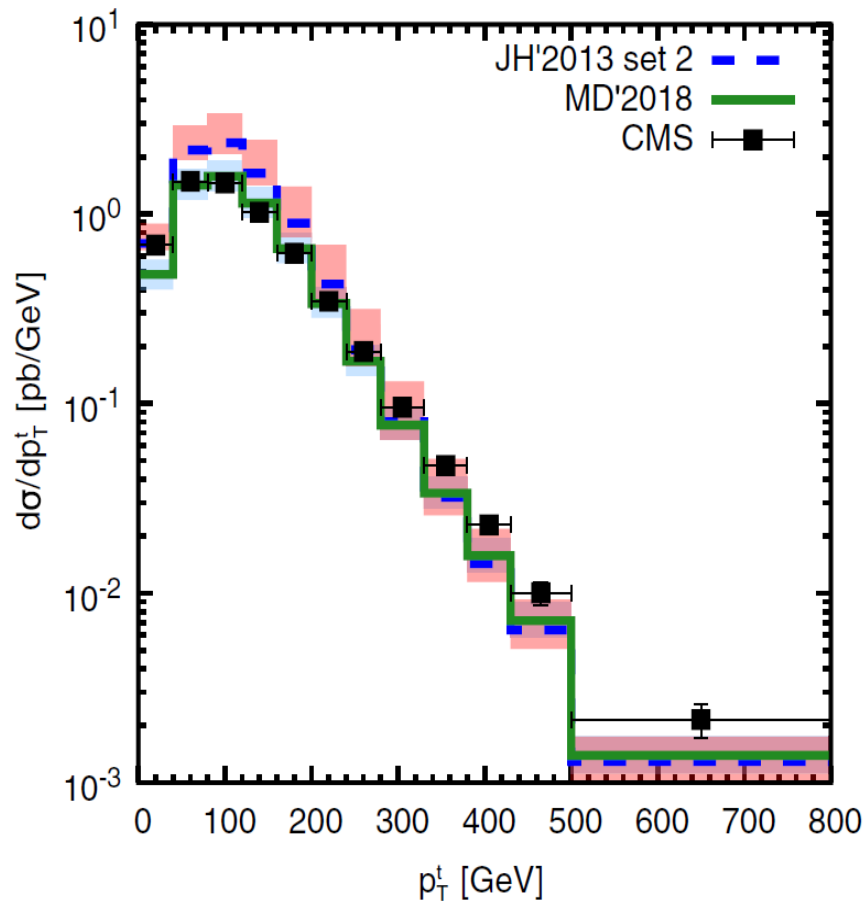
G.I.Lykasov, A.I. Malakhov, arXiv: 1801.07250 [hep-ph], EPJ A in press





The charm structure function of proton as a function of  $x$  compared to ZEUS and H1 of e-p experiment

*N.Abdulov, H.Jung, A.Lipatov, G.L., M Malyshev; Phys.Rev.98 054010 (2018)*



The transverse momentum and rapidity distributions of inclusive  $t\bar{t}$  production in  $pp$  collision at 13 TeV. The experimental data are from CMS.

*N.Abdulov, H.Jung, A.Lipatov, G.L., M Malyshev; Phys.Rev.98 054010 (2018)*

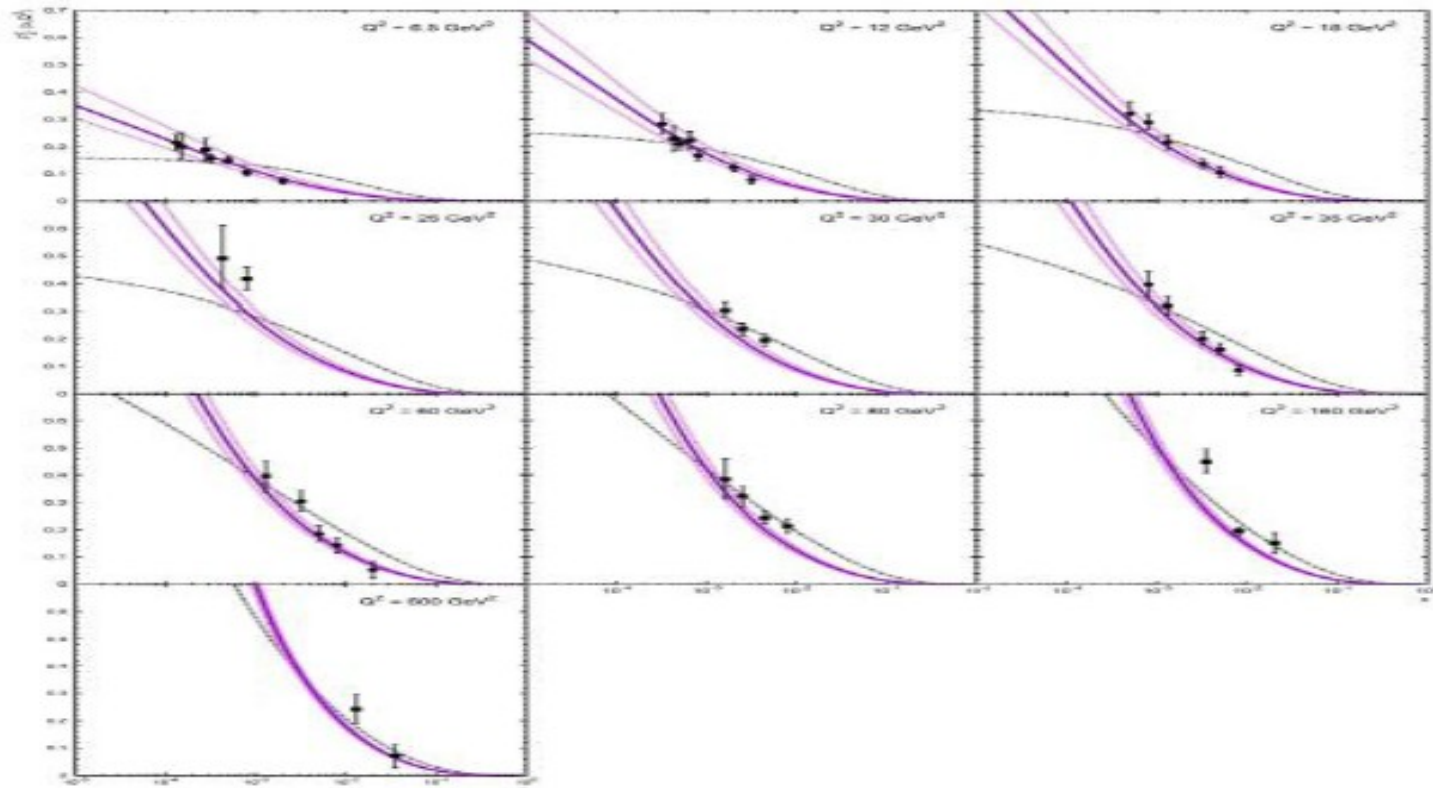
# Conclusion

1. We have shown that the energy dependence of the self-similarity parameter is very significant at low energies. The inverse slope of  $p_T$  – spectra  $T$  starting from the threshold of hadron production increases as a function of  $s^{1/2}$ . This is the main advantage of approach used the four-momentum velocities of particles.
2. It is also shown that the  $s$ -dependence is not enough to describe the inclusive spectra of hadrons produced in the mid-rapidity region in self-similarity approach at LHC energy.
3. To describe the data in the mid-rapidity region and at  $p_t$  up to 2-3 GeV/c, we modify the simple exponential form of the spectrum and present it in two parts due to the contribution of quarks and gluons, each of them has different energy dependence.
4. Applying the suggested approach to the pion production in p-p and A-A collisions at the mid-rapidity region we got a satisfactory description of data at not large transverse momenta  $p_t$  up to 1 GeV/c.
5. Our gluon distribution is verified by satisfactory description of charmed and beauty structure functions and hard processes of heavy quark production, for example,  $t\bar{t}$ , single top-quark and Higgs boson.
6. Our gluon distribution depended on  $x$  and  $k_T$  can be found on the web as MD-2018.

***Thank you very much  
for your attention!***

**BACK UP**

# CHARM STRUCTURE FUNCTION $F_2^c(x, Q^2)$



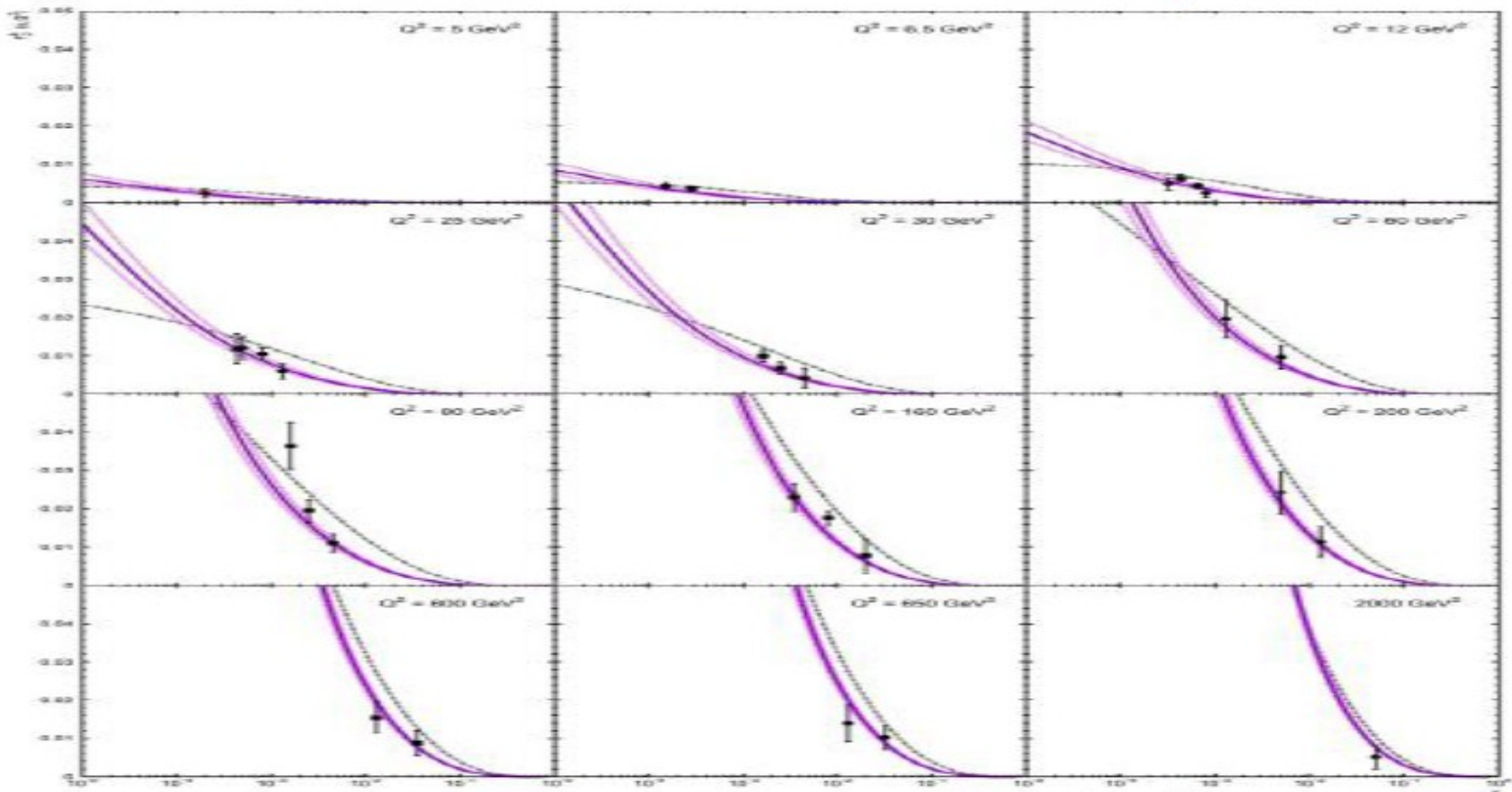
The charm contribution to the structure function  $F_2(x, Q^2)$  as a function of  $x$  calculated at different  $Q^2$ . The experimental data are from ZEUS and H1

**A.A. Grinyuk, A.V.Lipatov, G.L., N.P.Zotov, *Phys.Rev. D93, 014035 (2016)*.**

The solid lines correspond to our gluon density, the dash lines are results of H.Jung (DESY)

***N.Abdulov, H.Jung, A.Lipatov, G.L., M Malyshev; Phys.Rev.98 054010 (2018)***

# BOTTOM STRUCTURE FUNCTION $F_2^b(x, Q^2)$



The beauty contribution to the structure function  $F_2(x, Q^2)$  as a function of  $x$  calculated at different  $Q^2$ .

A.A. Grinyuk, A.V.Lipatov, G.L., N.P.Zotov, *Phys.Rev.* D93, 014035 (2016).

The solid lines correspond to our gluon density, the dash lines are results of H.Jung (DESY)

N.Abdulov, H.Jung, A.Lipatov, G.L., M Malyshev; *Phys.Rev.*98 054010 (2018)

In this case  $N_I$  and  $N_{II}$  are equal to each other:  $N_I = N_{II} = N$ .

$$N = [1 + (1 + \Phi_\delta / \Phi^2)^{1/2}] \Phi,$$

Where  $\Phi = (m_{1t} \text{ch}Y + M) / (2m_0 \text{sh}^2Y)$ ,  $\Phi_\delta = (M^2 - m_1^2) / (4m_0^2 \cdot \text{sh}^2Y)$ .

Here  $m_{1t}$  is the transverse mass of the particle **1**,  $m_{1t} = (m_1^2 + p^2)^{1/2}$ ,  $Y$  - rapidity of interacting nuclei.

And then  $\Pi = N \cdot \text{ch}Y$

The details for the case, when  $N_I$  and  $N_{II}$  are different, are presented in [G.I. Lykasov, A.I. Malakhov, 1801.07250 \(2018\)](#)