ON THE PAIR CORRELATIONS OF NEUTRAL K, D, B AND B_s MESONS WITH CLOSE MOMENTA PRODUCED IN INCLUSIVE MULTIPARTICLE PROCESSES

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The phenomenological structure of inclusive cross-sections of the production of two neutral K mesons in hadron–hadron, hadron–nucleus and nucleus–nucleus collisions is theoretically investigated taking into account the strangeness conservation in strong and electromagnetic interactions. Relations describing the dependence of the correlations of two short-lived and two long-lived neutral kaons $K_S^0 K_S^0$, $K_L^0 K_L^0$ and the correlations of "mixed" pairs $K_S^0 K_L^0$ at small relative momenta upon the space-time parameters of the generation region of K^0 and $\bar{K}^0$ mesons have been obtained. It is shown that under the strangeness conservation the correlation functions of the pairs $K_S^0 K_S^0$ and $K_L^0 K_L^0$, produced in the same inclusive process, coincide, and the difference between the correlation functions of the pairs $K_S^0 K_S^0$ and $K_S^0 K_L^0$ is conditioned exclusively by the production of the pairs of non-identical neutral kaons $K^0 \bar{K}^0$.

For comparison, analogous correlations for the pairs of neutral heavy mesons D^0, B^0 and B_s^0 , generated in multiple inclusive processes with charm (beauty) conservation, are also theoretically analyzed . These correlations are described by quite similar expressions: in particular, just as for K^0 mesons, the correlation functions for the pairs of states with the same CP parity $(R_{SS} = R_{LL})$ and with different CP parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0 \bar{D}^0$, $B^0 \bar{B}^0$ and $B_s^0 \bar{B}_s^0$. However, contrary to the case of K^0 mesons, here the distinction of CP-even and CP-odd states encounters difficulties – due to the insignificant differences of their lifetimes and the relatively small probability of purely CP-even and CP-odd decay channels. Nevertheless, one may hope that it will become possible at future colliders.

1 Consequences of the strangeness conservation for neutral kaons

In the work [1] the properties of the density matrix of two neutral K mesons, following from the strangeness conservation in strong and electromagnetic interactions, have been investigated. By definition, the diagonal elements of the non-normalized two-particle density matrix coincide with the two-particle structure functions, which are proportional to the double inclusive cross-sections.

Strangeness is the additive quantum number. Taking into account the strangeness conservation, the pairs of neutral kaons $K^0 K^0$ (strangeness $S = +2$), $\bar{K}^0 \bar{K}^0$ (strangeness $S = -2$) and $K^0 \bar{K}^0$ (strangeness $S = 0$) are produced incoherently. This means that in the K^0-$\bar{K}^0$- representation the non-diagonal elements of the density matrix between the states $K^0 K^0$ and $\bar{K}^0 \bar{K}^0$, $K^0 K^0$ and $K^0 \bar{K}^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$ are equal to zero. However, the non-diagonal elements of the two-kaon density matrix between the two states $|K^0\rangle^{(\mathbf{p}_1)} |\bar{K}^0\rangle^{(\mathbf{p}_2)}$ and $|\bar{K}^0\rangle^{(\mathbf{p}_1)} |K^0\rangle^{(\mathbf{p}_2)}$ with the zero strangeness are not equal to zero, in general. Here $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the first and second kaons.

The internal states of K^0 meson ($S = 1$) and $\bar{K}^0$ meson ($S = -1$) are the superpositions of the states $|K^0_S\rangle$ and $|K^0_L\rangle$, where K^0_S is the short-lived neutral kaon and K^0_L is the long-lived one. Neglecting the small effect of CP non-invariance, the CP-parity of the state K^0_S is equal to $(+1)$, and the CP-parity of the state K^0_L is equal to (-1); in doing so,

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K^0_S\rangle + |K^0_L\rangle), \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(|K^0_S\rangle - |K^0_L\rangle).$$

It is clear that both the quasistationary states of the neutral kaon have no definite strangeness.

It is easy to show that

$$|K^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)} = \frac{1}{2}(|K^0_S\rangle^{(\mathbf{p}_1)} \otimes |K^0_S\rangle^{(\mathbf{p}_2)} + |K^0_L\rangle^{(\mathbf{p}_1)} \otimes |K^0_L\rangle^{(\mathbf{p}_2)} +$$

$$+|K^0_S\rangle^{(\mathbf{p}_1)} \otimes |K^0_L\rangle^{(\mathbf{p}_2)} + |K^0_L\rangle^{(\mathbf{p}_1)} \otimes |K^0_S\rangle^{(\mathbf{p}_2)}), \tag{1}$$

$$|\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} = \frac{1}{2}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)} + |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)} -$$

$$-|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)}). \qquad (2)$$

It follows from the Bose-symmetry of the wave function of two neutral kaons with respect to the total permutation of internal states and momenta that the CP-parity of the system $K^0\bar{K}^0$ is always positive [2] (the C-parity is $(-1)^L$, the space parity is $P = (-1)^L$, where L is the orbital momentum).

The system of two non-identical neutral kaons $K^0\bar{K}^0$ in the symmetric internal state, corresponding to even orbital momenta, is decomposed into the schemes $|K_S^0\rangle|K_S^0\rangle$ and $|K_L^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} + |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)}); \qquad (3)$$

meantime, the system $K^0\bar{K}^0$ in the antisymmetric internal state, corresponding to odd orbital momenta, is decomposed into the scheme $|K_S^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} - |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)}). \qquad (4)$$

The strangeness conservation leads to the fact that all the double inclusive cross-sections of production of pairs $K_S^0 K_S^0$, $K_L^0 K_L^0$ and $K_S^0 K_L^0$ (two-particle structure functions) prove to be symmetric with respect to the permutation of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{SS}(\mathbf{p}_2, \mathbf{p}_1); \quad f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_2, \mathbf{p}_1);$$

$$f_{SL}(\mathbf{p}_1, \mathbf{p}_2) = f_{SL}(\mathbf{p}_2, \mathbf{p}_1). \qquad (5)$$

Besides, due to the strangeness conservation, the structure functions of neutral K mesons produced in inclusive processes are invariant with respect

to the replacement of the short-lived state K_S^0 by the long-lived state K_L^0, and *vice versa* [1]:

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4}[f_{K^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) +$$

$$+ f_{K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2)] + \frac{1}{2} \operatorname{Re} \rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2), \qquad (6)$$

$$f_{SL}(\mathbf{p}_1, \mathbf{p}_2) = f_{LS}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4}[f_{K^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) +$$

$$+ f_{K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2)] - \frac{1}{2} \operatorname{Re} \rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2), \qquad (7)$$

where $\rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) = (\rho_{\bar{K}^0 K^0 \to K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2))^*$ are the non-diagonal elements of the two-kaon density matrix. The difference between the two-particle structure functions f_{SS} and f_{SL} is connected just with the contribution of these non-diagonal elements.

It is evident that the one-particle structure functions for the production of K_S^0 and K_L^0 are equal to each other. After integrating the relations (6) over the momentum distribution of neutral kaons one can obtain the mutual equality of the average multiplicities of the K_S^0 and K_L^0 states, as well as the mutual equality of the average squares of multiplicities:

$$\langle n_S \rangle = \langle n_L \rangle, \qquad \langle n_S^2 \rangle = \langle n_L^2 \rangle. \qquad (8)$$

2 Structure of pair correlations of identical and non-identical neutral kaons with close momenta

Now let us consider, within the model of one-particle sources [2-7], the correlations of pairs of neutral K mesons with close momenta (see also our respective papers, e.g. [8-13]). In the case of the identical states $K_S^0 K_S^0$ and $K_L^0 K_L^0$ we obtain the following expressions for the correlation functions R_{SS}, R_{LL} (proportional to the structure functions), normalized to unity at large relative momenta:

$$R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0 K^0} \left[1 + F_{K^0}(2\mathbf{k}) + 2\, b_{\mathrm{int}}(\mathbf{k}) \right] +$$
$$+ \lambda_{\bar{K}^0 \bar{K}^0} \left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\, \tilde{b}_{\mathrm{int}}(\mathbf{k}) \right] +$$
$$+ \lambda_{K^0 \bar{K}^0} \left[1 + F_{K^0 \bar{K}^0}(2\mathbf{k}) + 2\, B_{\mathrm{int}}(\mathbf{k}) \right]. \tag{9}$$

Here $\mathbf{k}$ is the momentum of one of the kaons in the c.m. frame of the pair, and the quantities $\lambda_{K^0 K^0}$, $\lambda_{\bar{K}^0 \bar{K}^0}$ and $\lambda_{K^0 \bar{K}^0}$ are the relative fractions of the average numbers of produced pairs $K^0 K^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$, respectively $(\lambda_{K^0 K^0} + \lambda_{\bar{K}^0 \bar{K}^0} + \lambda_{K^0 \bar{K}^0} = 1)$. The "formfactors" $F_{K^0}(2\mathbf{k})$, $F_{\bar{K}^0}(2\mathbf{k})$ and $F_{K^0 \bar{K}^0}(2\mathbf{k})$ appear due to the contribution of Bose-statistics:

$$F_{K^0}(2\mathbf{k}) = \int W_{K^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r}, \qquad F_{\bar{K}^0}(2\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r},$$

$$F_{K^0 \bar{K}^0}(2\mathbf{k}) = \int W_{K^0 \bar{K}^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r}. \tag{10}$$

where $W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$ and $W_{K^0 \bar{K}^0}(\mathbf{r})$ are the probability distributions of distances between the sources of emission of two K^0 mesons, between the sources of emission of two $\bar{K}^0$ mesons and between the sources of emission of the K^0 meson and $\bar{K}^0$ meson, respectively, in the c.m. frame of the kaon pair. Meantime, the quantity $b_{\mathrm{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of two K^0 mesons, the quantity $\tilde{b}_{\mathrm{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of two $\bar{K}^0$ mesons and the quantity $B_{\mathrm{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of the K^0 meson with the $\bar{K}^0$ meson. Due to the CP invariance, the quantities $b_{\mathrm{int}}(\mathbf{k})$ and $\tilde{b}_{\mathrm{int}}(\mathbf{k})$ can be expressed by means of averaging the same function $b(\mathbf{k}, \mathbf{r})$ over the different distributions:

$$b_{\mathrm{int}}(\mathbf{k}) = \int W_{K^0}(\mathbf{r}) b(\mathbf{k}, \mathbf{r}) d^3\mathbf{r}, \quad \tilde{b}_{\mathrm{int}}(\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r}) b(\mathbf{k}, \mathbf{r}) d^3\mathbf{r}.$$

The quantity $B_{\mathrm{int}}(\mathbf{k})$ has the structure

$$B_{\mathrm{int}}(\mathbf{k}) = \int W_{K^0 \bar{K}^0}(\mathbf{r}) B(\mathbf{k}, \mathbf{r}) d^3\mathbf{r},$$

where $B(\mathbf{k}, \mathbf{r}) \neq b(\mathbf{k}, \mathbf{r})$.

Let us emphasize that when the pair of non-identical neutral kaons $K^0 \bar{K}^0$ is produced but the pair of identical quasistationary states $K_S^0 K_S^0$ (or $K_L^0 K_L^0$)

is registered over decays, the two-particle correlations at small relative momenta have the same character as in the case of usual identical bosons with zero spin [2].

For the pairs of non-identical kaon states $K_S^0 K_L^0$ the correlation functions at small relative momenta have the form:

$$R_{SL}(\mathbf{k}) = R_{LS}(\mathbf{k}) = \lambda_{K^0 K^0} \left[1 + F_{K^0}(2\mathbf{k}) + 2\, b_{\text{int}}(\mathbf{k}) \right] +$$
$$+ \lambda_{\bar{K}^0 \bar{K}^0} \left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\, \tilde{b}_{\text{int}}(\mathbf{k}) \right] +$$
$$+ \lambda_{K^0 \bar{K}^0} \left[1 - F_{K^0 \bar{K}^0}(2\mathbf{k}) \right]. \tag{11}$$

In accordance with Eq.(11), at the production of the pair of non-identical neutral kaons $K^0 \bar{K}^0$ and the registration of the two-particle state $K_S^0 K_L^0$ over decays the pair correlations are analogous to the correlations of two identical fermions with the same spin projections. This is connected with the fact that in the considered case the pair $K_S^0 K_L^0$ has odd orbital momenta [2].

It follows from Eqs.(9) and (11) that the correlation functions of pairs of neutral K mesons with close momenta, which are created in inclusive processes, satisfy the relation

$$R_{SS}(\mathbf{k}) + R_{LL}(\mathbf{k}) - R_{SL}(\mathbf{k}) - R_{LS}(\mathbf{k}) = 2\left[R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) \right] =$$
$$= 4\lambda_{K^0 \bar{K}^0} \left[F_{K^0 \bar{K}^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k}) \right]. \tag{12}$$

We see that the difference between the correlation functions of the pairs of identical neutral kaons $K_S^0 K_S^0$ and pairs of non-identical neutral kaons $K_S^0 K_L^0$ is conditioned exclusively by the generation of $(K^0 \bar{K}^0)$ pairs.

The relations connecting the contribution of the S-wave strong interaction into the pair correlations of particles at small relative momenta with the parameters of low-energy scattering were obtained earlier in the papers [4-7]. It is essential that the "formfactors" (10) and the functions $b_{\text{int}}(\mathbf{k})$, $\tilde{b}_{\text{int}}(\mathbf{k})$ and $B_{\text{int}}(\mathbf{k})$ depend on the space-time parameters of the generation region of neutral kaons and tend to zero at high values of the relative momentum $q = 2|\mathbf{k}|$ of two neutral kaons. [1]

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[1] Let us note that, in principle, the P-wave resonance (ϕ-meson, $M = 1021$ MeV/c^2, $\Gamma = 4$ MeV) may influence the $K_S^0 K_L^0$–correlations. But this influence manifests itself only in the narrow region of comparatively large relative momenta $2k \approx 220$ MeV/c, and it is strongly suppressed at small relative momenta which are discussed here.

3   Contribution of the S-wave $K^0\bar{K}^0-$ interaction

The function $B(\mathbf{k}, \mathbf{r})$, describing the contribution of the final-state interaction between K^0 and $\bar{K}^0$ mesons into the $K_S^0 K_S^0-$correlations and into the difference of the correlation functions $(R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}))$, may be calculated analytically using the approximation of the superposition of the plane and spherical waves, if characteristic distances r_0 between sources of K^0 and $\bar{K}^0$ mesons are: $r_0 \gg d_0$, where d_0 is the radius of action of short-range forces between the K^0 meson and $\bar{K}^0$ meson (really, already at $r_0 > d_0$) [4]. In so doing,

$$B(\mathbf{k}, \mathbf{r}) = |A_{K^0\bar{K}^0}(k)|^2 \frac{1}{r^2} + 2\,\mathrm{Re}\left(A_{K_0\bar{K}_0}(k)\frac{\exp(ikr)\cos\mathbf{kr}}{r}\right), \qquad (13)$$

where $A_{K^0\bar{K}^0}(k) \equiv A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k)$ is the amplitude of the S-wave $K^0\bar{K}^0-$ scattering, $k = |\mathbf{k}|$, $r = |\mathbf{r}|$.

Now let us take into account the effect of the possible transition $K^+K^- \to K^0\bar{K}^0$ between the pairs of oppositely charged and neutral kaons on the pair correlations of two identical kaons with small relative momenta. We can use here the theory of the S-wave multichannel scattering [7]. Assuming that pairs K^+K^- and $K^0\bar{K}^0$ are emitted with equal probabilities by the same pairs of isotopically unpolarized sources, the function $B(\mathbf{k}, \mathbf{r})$ should be replaced by $\widetilde{B}(\mathbf{k}, \mathbf{r}) = B(\mathbf{k}, \mathbf{r}) + \Delta B(\mathbf{k}, \mathbf{r})$, where [7]

$$\Delta B(\mathbf{k}, \mathbf{r}) = \left|A^{(c)}_{K^+K^- \to K^0\bar{K}^0}(k)\right|^2 \frac{\cos^2\tilde{k}r + C(\tilde{k}a_c)\sin^2\tilde{k}r}{r^2}. \qquad (14)$$

Here $\tilde{k} = \sqrt{k^2 + 2M_K\Delta M_K}$ and k are the moduli of the momentum of each of the charged kaons and of each of the neutral kaons, respectively, in the c.m. frame of the pair $K^0\bar{K}^0$,

$M_K = (M_{K^0} + M_{K^+})/2 \approx 495.6$ MeV/c^2,

$\Delta M_K = M_{K^0} - M_{K^+} \approx 4$ MeV/c^2,

$a_c = 2\hbar^2/(M_{K^+}e^2) \approx 108.5$ Fm is the Bohr radius of the (K^+K^-) system,

$$C(\tilde{k}a_c) = \frac{2\pi/\tilde{k}a_c}{1 - \exp(-2\pi/\tilde{k}a_c)}$$

is the Coulomb factor corresponding to the attraction of the oppositely charged kaons, $A^{(c)}_{K^+K^- \to K^0\bar{K}^0}(k)$ is the effective amplitude of the reaction $K^+K^- \to K^0\bar{K}^0$, renormalized by the Coulomb interaction. [2]

At $k = 0$ the modulus of the momentum of the K^+ (K^-)-meson in the c.m. frame of the pair $K^0\bar{K}^0$ is equal to $\tilde{k}_0 = \sqrt{2M_K\Delta M_K} \approx 62.8$ MeV/c. As a result, the Coulomb factor incorporated in Eq.(14) is close to unity:

$$1 < C(\tilde{k}a_c) \le C(\tilde{k}_0 a_c) \approx 1.0934.$$

Thus, with the precision of the order of 10% we obtain

$$\Delta B(\mathbf{k}, \mathbf{r}) = |A_{K^+K^- \to K^0\bar{K}^0}(k)|^2 \frac{1}{r^2} \,. \tag{15}$$

It is known that the amplitudes $A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k)$ and $A_{K^+K^- \to K^0\bar{K}^0}(k)$ are determined by the contribution of the sub-threshold S-wave resonances $f_0(980)$ (isotopic spin $T = 0$) and $a_0(980)$ (isotopic spin $T = 1$) [4,7]:

$$A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k) = \frac{1}{2}[A^{(T=0)}(k) + A^{(T=1)}(k)],$$

$$A_{K^+K^- \to K^0\bar{K}^0}(k) = \frac{1}{2}[A^{(T=0)}(k) - A^{(T=1)}(k)]\,. \tag{16}$$

According to Eqs. (13), (15) and (16), we come to the following approximate relation:

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[2] The S-wave amplitude, determining directly the effective cross-section of the process $K^+K^- \to K^0\bar{K}^0$ according to the standard formula

$$\sigma_{K^+K^- \to K^0\bar{K}^0}(k) = 4\pi \, |A_{K^+K^- \to K^0\bar{K}^0}(k)|^2 \frac{k}{\tilde{k}},$$

is $A_{K^+K^- \to K^0\bar{K}^0}(k) = \sqrt{C(\tilde{k}a_c)} A^{(c)}_{K^+K^- \to K^0\bar{K}^0}(k)$.

$$\widetilde{B}(\mathbf{k}, \mathbf{r}) = B(\mathbf{k}, \mathbf{r}) + \Delta B(\mathbf{k}, \mathbf{r}) = [\, |A^{(T=0)}(k)|^2 + |A^{(T=1)}(k)|^2 \,] \frac{1}{2r^2} +$$

$$+ \operatorname{Re} \left([A^{(T=0)}(k) + A^{(T=1)}(k)] \, \frac{\exp(ikr) \cos(\mathbf{kr})}{r} \right). \qquad (17)$$

Let us remark that the first statistically meaningful experimental results on the Bose-correlations of two K_S^0 mesons, produced in collisions of relativistic heavy ions, were presented in 2006 by the international STAR Collaboration (see [14]) .

4   Correlations of neutral heavy mesons

Formally, analogous relations are valid also for the neutral heavy mesons D^0, B^0 and B_s^0 . In doing so, the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons . In these cases the quasistationary states are also states with definite CP parity, neglecting the effects of CP nonconservation .

For example,

$$|B_S^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle),\ CP \text{ parity } (+1);$$

$$|B_L^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle),\ CP \text{ parity } (-1).$$

In accordance with the mechanism of mixing a particle with the respective antiparticle due to weak interaction through the exchange of two virtual W bosons, states with CP parity (-1) have the greater mass and the larger lifetime than states with CP parity $(+1)$. The difference of masses between the respective CP-odd and CP-even states is very insignificant in all the cases, ranging from 10^{-12} MeV for K^0 mesons   up to 10^{-8} MeV for B_s^0 mesons . Concerning the lifetimes of these states, in the case of K^0 mesons they differ by 600 times, but for D^0, B^0 and B_s^0 mesons the respective lifetimes are almost the same. In connection with this, it is practically impossible to distinguish the states of D^0, B^0 and B_s^0 mesons with definite CP

parity by the difference in their lifetimes. These states, in principle, can be identified through the purely CP-even and purely CP-odd decay channels ; however, in fact the branching ratio for such decays is very small . For example,

$$Br\left(D^0 \to \pi^+\pi^-\right) = 1.62 \cdot 10^{-3} \quad (CP = +1);$$

$$Br\left(D^0 \to K^+K^-\right) = 4.25 \cdot 10^{-3} \quad (CP = +1);$$

$$Br\left(B_s^0 \to J/\Psi \ \pi^0\right) < 1.2 \cdot 10^{-3} \quad (CP = +1);$$

$$Br\left(B^0 \to J/\Psi \ K_S^0\right) = 9 \cdot 10^{-4} \quad (CP = -1).$$

Just as in the case of neutral K mesons, the correlation functions for the pairs of states of neutral D, B and B_s mesons with the same CP parity ($R_{SS} = R_{LL}$) and for the pairs of states with different CP parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0\bar{D}^0$, $B^0\bar{B}^0$ and $B_s^0\bar{B}_s^0$, respectively (see also, e.g., [11-13]). In particular, for B_s^0 mesons the following relation holds:

$$R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) = 2\lambda_{B_s^0\bar{B}_s^0}\left[F_{B_s^0\bar{B}_s^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\right]; \qquad (18)$$

here $\lambda_{B_s^0\bar{B}_s^0}$ is the relative fraction of generated pairs $B_s^0\bar{B}_s^0$,

$$F_{B_s^0\bar{B}_s^0}(2\mathbf{k}) = \int W_{B_s^0\bar{B}_s^0}(\mathbf{r}) \ \cos(2\mathbf{kr}) \ d^3\mathbf{r}, \quad B_{\text{int}}(\mathbf{k}) = \int W_{B_s^0\bar{B}_s^0}(\mathbf{r}) \ B(\mathbf{k},\mathbf{r}) \ d^3\mathbf{r},$$

$$B(\mathbf{k},\mathbf{r}) = |A_{B_s^0\bar{B}_s^0}(k)|^2 \frac{1}{r^2} + 2\,\text{Re}\left(A_{B_s^0\bar{B}_s^0}(k)\frac{\exp(ikr)\cos\mathbf{kr}}{r}\right),$$

where $A_{B_s^0\bar{B}_s^0}(k) \equiv A_{B_s^0\bar{B}_s^0 \to B_s^0\bar{B}_s^0}(k)$ is the amplitude of S-wave $B_s^0\bar{B}_s^0-$ scattering, $k = |\mathbf{k}|$, $r = |\mathbf{r}|$. Let us remark that the B_s^0 and $\bar{B}_s^0$ mesons do not have charged partners (the isotopic spin equals zero) and, hence, in the given case the transition similar to $K^+K^- \to K^0\bar{K}^0$ is absent .

5   Summary

1. It is shown that, taking into account the strangeness conservation, the double inclusive cross-sections of the production of two short-lived neutral K mesons and two long-lived neutral K mesons are equal to each other. This result is the direct consequence of the strangeness conservation.

2. Within the model of one-particle sources the formulas for the correlation functions $R_{SS} = R_{LL}$ and $R_{SL} = R_{LS}$ are obtained, which involve the contributions of Bose-statistics, of the S-wave final-state interaction of two K^0 ($\bar{K}^0$) mesons as well as of a K^0 meson with a $\bar{K}^0$ meson, and also the contribution of transitions $K^+ K^- \to K^0 \bar{K}^0$, and depend upon the relative fractions of produced pairs $K^0 K^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$.

3. It is shown that the production of $(K^0 \bar{K}^0)$ pairs with the zero strangeness leads to the difference between the correlation functions R_{SS} and R_{SL} of two neutral kaons.

4. The character of analogous correlations for neutral heavy mesons D^0, B^0, B_s^0 with nonzero charm and beauty is discussed . Contrary to the case of K^0 mesons, here the distinction of respective CP-even and CP-odd states encounters difficulties, which are connected with the insignificant difference of their lifetimes and the relatively small probability of purely CP-even and purely CP-odd decay channels .

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