EMT correlators at positive gradient flow time: a way to calculate viscosity?

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How to define viscosity on a lattice

Kubo formula:

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Spectral density:

$$\rho(\omega) = \frac{1}{\pi} \mathrm{Im} \langle T_{12} T_{12} \rangle_{ret}$$

The EMT correlator in Euclidean time $C_{12,12} = \int \langle T_{12}(x_0, x) T_{12}(0,0) \rangle d^3x$ is an analytic continuation of retarded Green function.
Relation to $\rho(\omega)$:

$$C_{12,12}(x_0) = \int d\omega \rho(\omega) \frac{\cosh \omega(\frac{1}{2}\beta - x_0)}{\sinh \frac{\beta\omega}{2}}$$

The perturbative formula for $\rho(\omega)$:

$$\rho(\omega) = \frac{1}{10}\frac{N_c^2 - 1}{(4\pi)^2}\frac{\omega^4}{\tanh\frac{1}{4}\omega\beta}$$

Higher perturbative expansion can be made.
However, that gives no information about low-energy structure of the theory.

Gradient flow on a lattice

Gradient flow is a flow transformation of fields given by equations

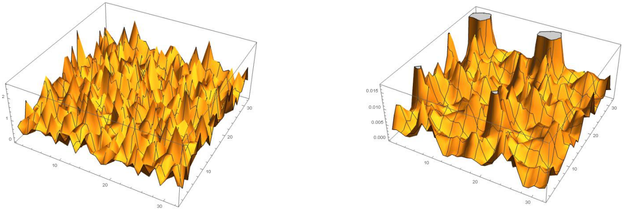
$$\frac{\partial A_\mu}{\partial t} = D_\nu F_{\nu\mu},$$

where $t \neq x_0$ is a fifth (flow) coordinate.

Properties of gradient flow

- Minimizes the action
- Smoothes field configurations
- Eliminates high frequencies
- Makes quantities cut-off independent (but t dependent)
- Statistical errors at $t > 0$ are small

Smoothing the field configuration



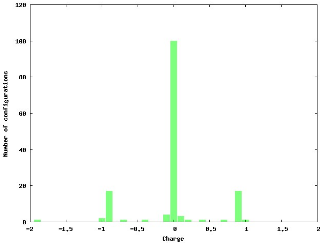
(a) Flow time = 0 **(b)** Flow time = 30

Average spacial plaquette for a single configuration

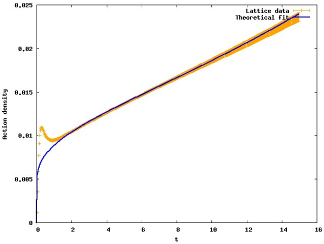
It can be shown that the field variebles are averaged over the sphere with radius $\approx \sqrt{8t}$.

Gradient flow applications

- Smooth configurations have well-defined topological properties
- Allows to define renormalized physical quantities on a lattice



(a) Topological charge distribution at $t = 1.5$



(b) $t^2 S$

Example: the entropy density

Perturbative expansion for entropy:

$$s^R/T = \frac{1}{\alpha_U(t)} \left(-\langle T_{00} \rangle + \frac{1}{3} \sum_k \langle T_{kk} \rangle \right)$$

$\alpha_U(t)$ above depends on running coupling only.

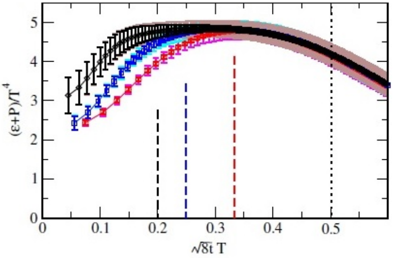
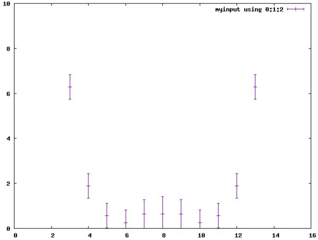
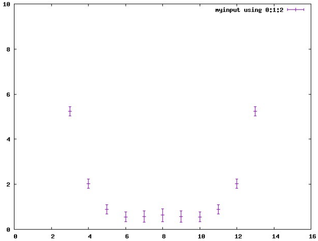


Figure: Entropy depending on t from Phys. Rev. D 90 (2014) 1, 011501

EMT correlators at t > 0

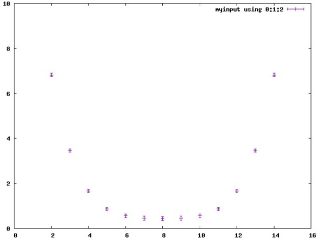


(a) t = 5

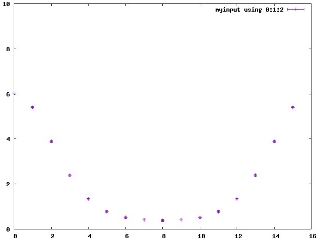


(b) $t = 10$

EMT correlators at $t > 0$



(a) $t = 20$



(b) $t = 30$

Fitting the correlators

A reasonable fit that unfortunately doesn't work:

$$\rho(\omega) = \eta\frac{\omega}{\pi} + B\frac{1}{10}\frac{N_c^2-1}{(4\pi)^2}\frac{\omega^4}{\tanh\frac{1}{4}\omega\beta}$$

Conclusion

- The EMT correlator under gradient flow can be measured with a good precision
- We still don't know how to extract viscosity from it

Thank you for your attention!