Heavy mesons in hadronic medium: interaction and transport coefficients

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Subatech – École des Mines de Nantes

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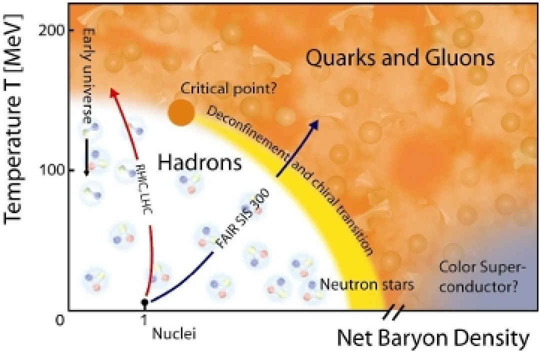
Introduction: QCD phase diagram



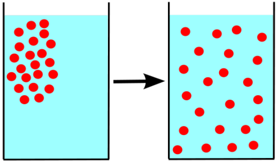
- Hadronic stage of a RHIC (low T, low μ_B)
- Open charm and open bottom mesons (D and $\overline{B}$ mesons)
- Hadron interaction $\rightarrow$ Unitarized EFTs
- $\rightarrow$ Transport Coefficients for Heavy Mesons

Heavy Mesons

- Heavy particles: produced at the initial stage, not thermal distribution (see plenary talks by M. Nahrgang and A. Beraudo)
- Heavy mesons interact with light species: π, K, $\overline{K}$,η...
- Charm and bottom quantum numbers are conserved by strong force $\rightarrow$ Transport Coefficients
- Diffusion coefficient D_x of Heavy mesons

$$\vec{j}_H = -D_x \vec{\nabla} n_H$$

$$H = \{charm, bottom\} = \{D, \overline{B}\}$$



Heavy meson – light meson interaction

$D, \overline{B}$ mesons are much heavier than (l)ight mesons!

$$m_H \gg m_l$$

In addition, their mass dominates over all other scales:

$$m_H \gg \Lambda_{QCD}, T, k \text{ (exchanged momentum in collisions)}$$

$\rightarrow$ H is a Brownian particle propagating in a thermalized bath composed by the light hadrons

Idealized Case:

- π, K, η as massless Goldstone bosons: **chiral symmetry**
- D, $\overline{B}$ with infinite heavy mass: **heavy-quark symmetry**

These symmetries (and how they are broken) help to construct an EFT in a systematic way

F.-K., Guo et al. (2009), L.S. Geng et al. (2010)

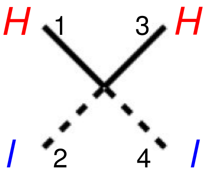
Heavy meson – light meson interaction

At NLO (χ) and LO (*HQ*) the contribution to the amplitude

Perturbative amplitude

$$V = \frac{C_0}{4F^2}(s-u) + \frac{2C_1}{3F^2}h_1 + \frac{2C_2}{F^2}h_3(p_2\cdot p_4) + \frac{2C_3}{F^2}h_5[(p_1\cdot p_2)(p_3\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)]$$

F is the pion decay constant
Isospin coefficients: fixed by symmetry
Low-energy constants: fixed by experiment



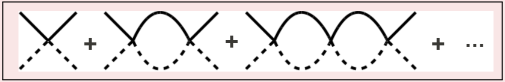
This amplitude describes elastic scatterings:
$H\pi$, HK, $H\bar{K}$, $H\eta$
$H_s\pi$, H_sK, $H_s\bar{K}$, $H_s\eta$ and their inelastic channels.

Unitarization

Caveat: Exact S-matrix unitarity is lost in the truncation of the EFT
Solution: We accommodate a unitarization method

Two-body equation for the scattering amplitude

$$T = V + VGV + VGVGV + \cdots = V + VGT$$



The equation can be algebraically solved using the "on-shell" method see J.A.Oller and E. Oset (1997), L. Roca et al. (2005)

Unitarized scattering amplitude

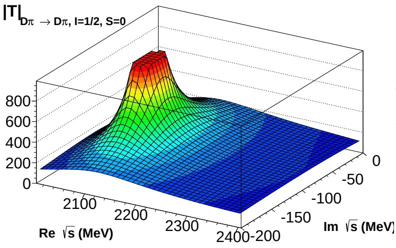
$$T = \frac{V}{1 - GV} \qquad (\text{with } |T|^2 \propto \text{Im}\, T)$$

Unitarization increases the domain of validity of the EFT!

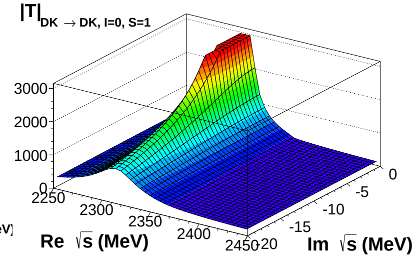
Resonances

Unitarized amplitude (several channels)

$$T_{ij} = [1 - GV]^{-1}_{ik} V_{kj}$$



D_0 (2400)



$D^*_{s0}(2317)$

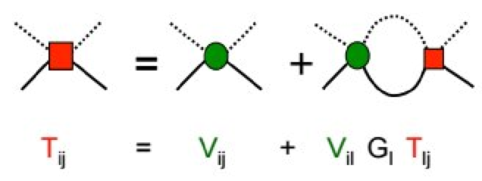
Important: Both resonances (2nd Riemann sheet) and bound states (1st Riemann sheet) will affect transport coefficients!

Meson-baryon interaction (in collaboration with L. Tolos and O. Romanets)

EFT for the interaction between $D, \bar{B}$ mesons and baryons:

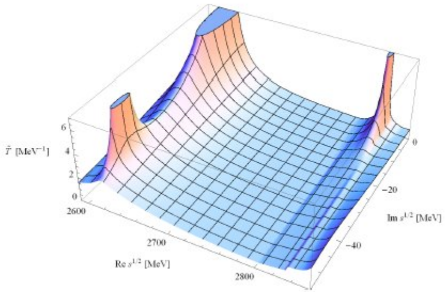
$$V_{ij} = \frac{D_{ij}}{4 f_i f_j}(2\sqrt{s} - M_i - M_j)\sqrt{\frac{M_i + E_i}{2M_i}}\sqrt{\frac{M_j + E_j}{2M_j}}$$

T.Mizutani, A.Ramos (2006), C.Garcia-Recio et al. (2012), O.Romanets et al. (2012)

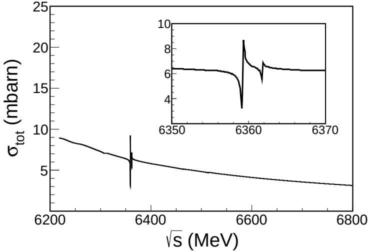


See Laura Tolos' talk!

Meson-baryon interaction (in collaboration with L. Tolos and O. Romanets)



Λ_c (2595) resonance, contributes to the elastic $DN \to DN$ cross section.



$\bar{B}N \to \bar{B}N$ total cross section

See Laura Tolos' talk!

Kinetic Equation

Already introduced (and discussed) in the **talks by M. Nahrgang** (Wed.), **A. Beraudo** (Wed.) and **S.K. Das** (Tues.)

Fokker-Planck equation

$$\frac{\partial f_H(t,p)}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(p) f_H(t,p) + \frac{\partial}{\partial p_j} \left[\Gamma_{ij}(p) f_H(t,p) \right] \right\}$$

For an isotropic medium, 3 coefficients: Drag Force

$$F(p) = \int d\mathbf{k} \; w(\mathbf{p},\mathbf{k}) \frac{\mathbf{k}\cdot\mathbf{p}}{p^2}$$

Diffusion coefficients

$$\Gamma_0(p) = \frac{1}{4} \int d\mathbf{k} \; w(\mathbf{p},\mathbf{k}) \left[\mathbf{k}^2 - \frac{(\mathbf{k}\cdot\mathbf{p})^2}{p^2} \right] \; ; \; \Gamma_1(p) = \frac{1}{2} \int d\mathbf{k} \; w(\mathbf{p},\mathbf{k}) \frac{(\mathbf{k}\cdot\mathbf{p})^2}{p^2}$$

Related by the fluctuation-dissipation theorem.

Einstein relation ($p \to 0$)

$$\Gamma_0 = \Gamma_1 = F m_H T$$

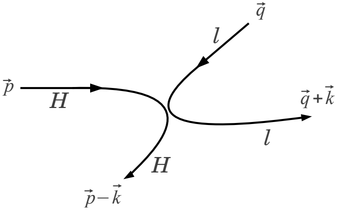
Fokker-Planck equation

Fokker-Planck equation

$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p_i}\left\{ F_i(p) f(t,p) + \frac{\partial}{\partial p_j}\left[\Gamma_{ij}(p) f(t,p)\right]\right\}$$

$w(\mathbf{p},\mathbf{k})$ represents the probability of a particle with momentum $\mathbf{p}$ to have a collision and lose a momentum $\mathbf{k}$.

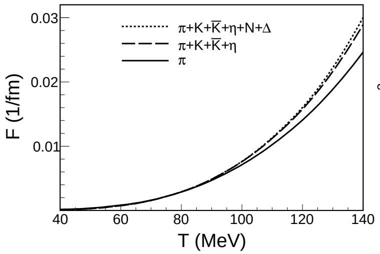
$$w(\mathbf{p},\mathbf{k}) \propto \int d\mathbf{q} \ldots \overline{|T|^2}(p,k,q)$$



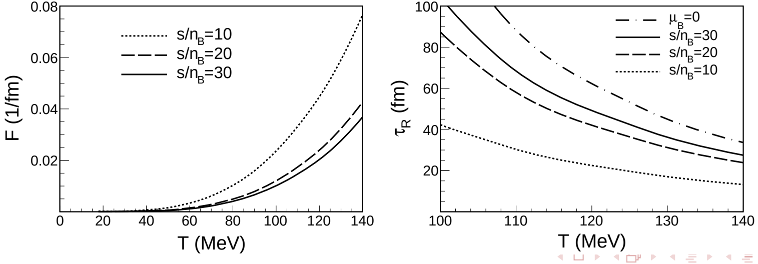
where $\overline{|T|^2}$ is the scattering amplitude squared, computed by the unitarized EFT

Results for D meson

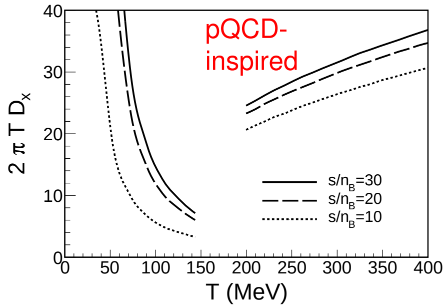
Drag Force and Diffusion coefficient at $p \to 0, \mu_B = 0$

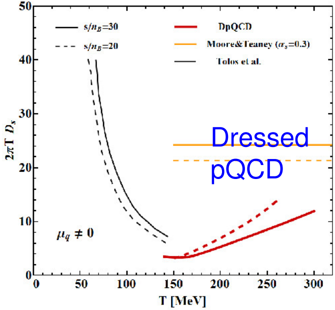


At isentropic evolution at FAIR energies, $p \to 0$



Spatial diffusion coefficient $D_x = T/M_H F(p \to 0)$

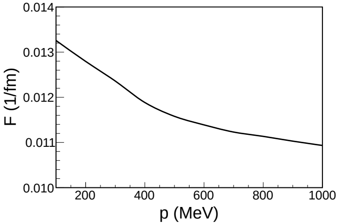


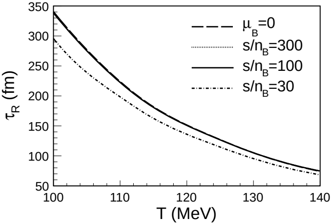


Minimum at the phase transition for finite chemical potential?

He et al. (2011), Abreu et al. (2011), Tolos and Torres-Rincon (2013), Berrehrah et al. (2014), Ozvenchuk et al. (2014)

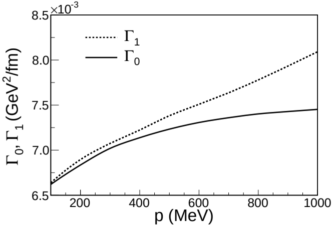
Results for $\bar{B}$ meson

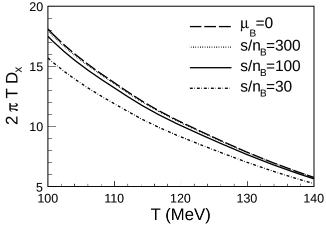




$\mu_B = 0$, $T = 140$ MeV

$p \to 0$ (adiabatic trajectories)





Das et al. (2012), Abreu et al. (2012), Torres-Rincon et al. (2014)

Conclusions

- We apply effective-field theories to describe interactions among heavy mesons and light hadrons at low energies. We exploit both chiral and heavy-quark spin-flavor symmetries in a systematic expansion
- We use a unitarization technique to impose exact unitarity for the scattering amplitudes. This extends the practical validity of the EFT to higher energies and leads to a potential generation of resonances and bound states ($D_0(2400), D^*_{s0}(2317), \Lambda_c(2595)$...)
- We obtain physical elastic cross sections for the heavy meson–light hadron scattering
- We compute the relevant transport coefficients for D and $\bar{B}$ mesons as a function of temperature, chemical potential and heavy-meson momentum

Thanks for your attention!

References

F.-K. Guo, C. Hanhart and U.G. Meissner *Eur.Phys.J. A40, 171-179 (2009)*
L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*
J.A.Oller and E. Oset *Nucl.Phys.A620 (1997) 438*
L. Roca, E. Oset and J. Singh *Phys.Rev.D72 (2005) 014002*
T.Mizutani, A.Ramos *Phys.Rev.C74, 065201 (2006)*
C.Garcia-Recio et al. *Phys.Rev.D79, 054004 (2012)*
O.Romanets et al. *Phys.Rev.D85 114032 (2012)*
M.He, R.J.Fries and R. Rapp *Phys.Lett. B701 (2011) 445-450*
L.Abreu, D.Cabrera, F. Llanes-Estrada and JMTR *Annals Phys. 326 (2011) 2737-2772*
L.Tolos and JMTR *Phys.Rev. D88 (2013) 074019*
H.Berrerah, P.B.Gossiaux, J.Aichelin, W.Cassing, JMTR, E.Bratkovskaya *Phys.Rev. C90 (2014) 5, 051901*
V.Ozvenchuk, JMTR, P.B.Gossiaux, L.Tolos, J.Aichelin *Phys.Rev. C90 (2014) 5, 054909*
S.K.Das, S.Ghosh, S.Sarkar and J.Alam *Phys.Rev. D85 (2012) 074017*
L.Abreu, D.Cabrera and JMTR *Phys.Rev. D87 (2013) 3, 034019*
JMTR, L.Tolos and O.Romanets and L.Tolos *Phys.Rev. D89 (2014) 7, 074042*

Back-up slides

Langevin equation

It is an alternative (but equivalent) description to the Fokker-Planck equation.

Langevin eq.

$$\frac{dx^i}{dt} = \frac{p^i}{m_H}$$
$$\frac{dp^i}{dt} = -F^i(p) + \xi^i(t)$$

with ξ^i a stochastic Gaussian force

$$\langle \xi^i(t) \rangle = 0$$
$$\langle \xi^i(t) \xi^j(t') \rangle = \Gamma^{ij}(p)\, \delta(t-t')$$

Relaxation time

Consider Newton's law (with $F^i = Fp^i$)

$$\frac{dp_i}{dt} = -F\, p^i$$

Assuming constant F one can solve the equation for $p(t)$

$$p(t) = p(0)\, e^{-t/F}$$

The inverse of F plays the role of a relaxation time τ_R

Relaxation time

$$\tau_R \; = \; 1/F$$

Fluctuation-Dissipation Theorem

$F(p)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.

The fluctuation-dissipation theorem relates the 3 coefficients:

Fluctuation-Dissipation Theorem

$$F(p) + \frac{1}{p}\frac{\partial \Gamma_1(p)}{\partial p} + \frac{2}{p^2}\left[\Gamma_1(p) - \Gamma_0(p)\right] = \frac{\Gamma_1(p)}{m_H T}$$

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In the static limit, i.e. when $p \to 0$ the two diffusion coefficients become degenerate and the Einstein relation is recovered

Einstein Relation

$$F = \frac{\Gamma}{m_H T}$$

Non relativistic formulae

$$F \sim \sigma P \sqrt{\frac{m_l}{T}} \frac{1}{m_H}$$

$$\Gamma \sim \sigma P \sqrt{m_l T}$$

$$P = nT$$

$$n \propto m_l^{3/2} T^{3/2} e^{\frac{\mu - m_l}{T}}$$

$$D_x \sim \frac{T^{3/2}}{\sigma P \sqrt{m_l}}$$

$$F = \frac{\Gamma}{m_H T}$$

$$D_x = \frac{\Gamma}{m_H^2 F^2}$$

$D-\pi$, $D^*-\pi$ interaction

Effective Lagrangian: L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

Chiral symmetry (NLO) + Heavy Quark symmetry (LO)

$$\mathcal{L}^{(1)} = Tr[\nabla^\mu D \nabla_\mu D^\dagger] - M_D^2 Tr[DD^\dagger] - Tr[\nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger}] + M_{D^*}^2 Tr[D^{*\mu} D_\mu^{*\dagger}]$$

$$+ig Tr\left[\left(D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger}\right)\right] + \frac{g}{2M_D} Tr\left[\left(D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger}\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$\mathcal{L}^{(2)} = -h_0 Tr[DD^\dagger] Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger] Tr[u^\mu u_\mu] + h_3 Tr[D u^\mu u_\mu D^\dagger]$$

$$+h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger] Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger] + \{D \to D^\mu\}$$