# Transport Coefficients and quark–hadron phase transition(s) from PLSM in vanishing and finite magnetic field

## Strangeness in Quark Matter 2015

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# Previous Works

1. **SU(3) Polyakov linear-σ model in magnetic fields: Thermodynamics, higher-order moments, chiral phase structure, and meson masses**
   AT, N. Magdy, **Phys.Rev. C91 (2015) 015206, 1501.01124 [hep-ph]**

2. Polyakov SU(3) extended linear-σ model: Sixteen mesonic states in chiral phase structure
   AT, A. Diab, **Phys.Rev. C91 (2015) 1, 015204**, 1412.2395 **[hep-ph]**

3. Thermodynamics and higher order moments in SU(3) linear σ-model with gluonic quasi-particles
   AT, N. Magdy, **J.Phys. G42 (2015) 1, 015004**, 1411.1871 **[hep-ph]**

4. SU(3) Polyakov linear-σ model in an external magnetic field
   AT, N. Magdy, **Phys.Rev. C90 (2014) 1, 015204**, 1406.7488 **[hep-ph]**

5. Polyakov linear SU(3) σ model: Features of higher-order moments in a dense and thermal hadronic medium
   AT, N. Magdy, A. Diab, **Phys.Rev. C89 (2014) 5, 055210**, 1405.0577 **[hep-ph]**

# Outline

- Sigma model and symmetries
- SU(3) LσM with Polyakov-Loop Potential
- Hadron-Quark Phase Transition(s)
- Electrical and Heat Conductivity
- Bulk and Shear Viscosity

Origin of magnetic field in Heavy-Ion Collisions $\approx m^2_\pi$

# Sigma Models

## Sigma-Model is a Physical system with the Lagrangian

$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij}\, \mathrm{d}\phi_i \wedge *\mathrm{d}\phi_j$$

Exterior algebraic Wedge Product

The fields $\phi_i$ represent **map** from a **base manifold** spacetime (worldsheet) to a **target** (Riemannian) **manifold** of the scalars linked together by internal symmetries.

The scalars **g**ij determines linear and non-linear properties.

It was introduced by **Gell-Mann** and **Levy** in **1960**. The name **σ-model** comes from a field corresponding to the spinless meson ***σ***, scalar introduced earlier by **Schwinger**.

# Why LσM?



- It is one of the lattice QCD alternatives
- Various symmetry-breaking scenarios can be investigated in a more easy way
- Various properties of strongly interacting matter can be studied
- But, finite temperature LσM requires many-body resummation schemes, because the IR divergences cause perturbation theory to break down
- Limitations are given by Sigma-fields

# LSM Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma) - g_\pi\left[(i\bar{\psi}\gamma_5\bar{\tau}\psi)\bar{\pi} + (i\bar{\psi}\psi)\sigma\right] - \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

K. E Of nucleons

K. E Of Mesons

interaction term between nucleons and the mesons

Pion nucleon Potential

Nucleon mass term

## The chiral part of LσM-Lagrangian has $SU(3)_R \times SU(3)_L$ symmetry

where fermionic part $\mathcal{L}_q = \sum_f \overline{\psi}_f(i\gamma^\mu D_\mu - gT_a(\sigma_a + i\gamma_5\pi_a))\psi_f$

and mesonic part
$$\mathcal{L}_m = \mathrm{Tr}(\partial_\mu\Phi^\dagger\partial^\mu\Phi - m^2\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)],$$

- $m^2$ is tree-level mass of the fields in the absence of symmetry breaking
- $\lambda_1$ *and* $\lambda_2$ are the two possible quartic coupling constants,
- $c$ is the cubic coupling constant,
- $g$ flavor-blind Yukawa coupling of quarks to mesons and of quarks to background gauge field $A_\mu = \delta_{\mu 0}A_0$

$c = 4.80; g = 6.5; \lambda_1 = 5.90; \lambda_2 = 46.48; m^2 = (0.495)$^2;

# LSM involving Polyakov-Loop Potential

The coupling between Polyakov loop and quarks is given by the covariant derivative

$$D_\mu = \partial_\mu - iA_\mu \text{ in PLSM Lagrangian}$$

$$A_\mu = \delta_{\mu 0} A_0$$ **in the chiral limit**

$$\mathcal{L}_{PLSM} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0 - \mathcal{U}(\phi, \phi^*, T),$$

**invariant under chiral flavor group (like QCD Lagrangian)**

$$\mathcal{L}_{chiral} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0$$

In vanishing μ, then $\phi = \phi^*$ and the Polyakov loop is considered as an order parameter for the deconfinement phase-transition

# Partition Function

In thermal equilibrium, grand partition function can be defined by using a path integral over quark, antiquark and meson fields

$$\mathcal{Z} = \mathrm{Tr}\exp[-(\hat{\mathcal{H}} - \sum_{f=u,d,s} \mu_f \hat{\mathcal{N}}_f)/T]$$

$$= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right],$$

where $\int_x \equiv i \int_0^{1/T} dt \int_V d^3x$ and $\mu_f$ chemical potential

Thermodynamic potential density

$$\Omega(T,\mu) = \frac{-T\ln\mathcal{Z}}{V} = U(\sigma_x,\sigma_y) + \mathcal{U}(\phi,\phi^*,T) + \Omega_{\bar{\psi}\psi}.$$

$$\frac{\mathcal{U}(\phi,\phi^*,T)}{T^4} = -\frac{b_2(T)}{2}\phi\phi^* - \frac{b_3}{6}(\phi^3 + \phi^{*3}) + \frac{b_4}{4}(\phi\phi^*)^2$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

All other parameters: pure gauge QCD thermo.

# LSM involving Polyakov-Loop Potential

## Quarks and antiquarks Potential contribution

$$\Omega_{\bar{\psi}\psi} = -2TN \int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3} \{\ln\left[1+3(\phi+\phi^* e^{-(E-\mu)/T})\times e^{-(E-\mu)/T}+e^{-3(E-\mu)/T}\right] + \ln\left[1+3(\phi^*+\phi e^{-(E+\mu)/T})\times e^{-(E+\mu)/T}+e^{-3(E+\mu)/T}\right]\},$$

**where N gives the number of quark flavors,** $E=\sqrt{\vec{p}^2+m^2}$

$$m_q = g\frac{\sigma_x}{2},$$
$$m_s = g\frac{\sigma_y}{\sqrt{2}}.$$

**Mesonic potential** $U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4.$

## Vandermonde determinant is found negligibly small

Gives change of variables from vector potential to ϕ in path integral and guarantees a reasonable behavior of the mean field approximation

# Thermodynamic Potential

## Again the thermodynamic potential

$$\Omega(T,\mu) = \frac{-T \ln \mathcal{Z}}{V} = U(\sigma_x, \sigma_y) + \mathcal{U}(\phi, \phi^*, T) + \Omega_{\bar{\psi}\psi}.$$

It has various parameters:

$m^2, h_x, h_y, \lambda_1, \lambda_2, c$ and $g$

$\sigma_x$ and $\sigma_y$ condensates (chiral order parameters)

$\phi$ and $\phi^*$ (deconfinement order parameters

$m^2, h_x, h_y, \lambda_1, \lambda_2$ and $c$ can be fixed, experimentally

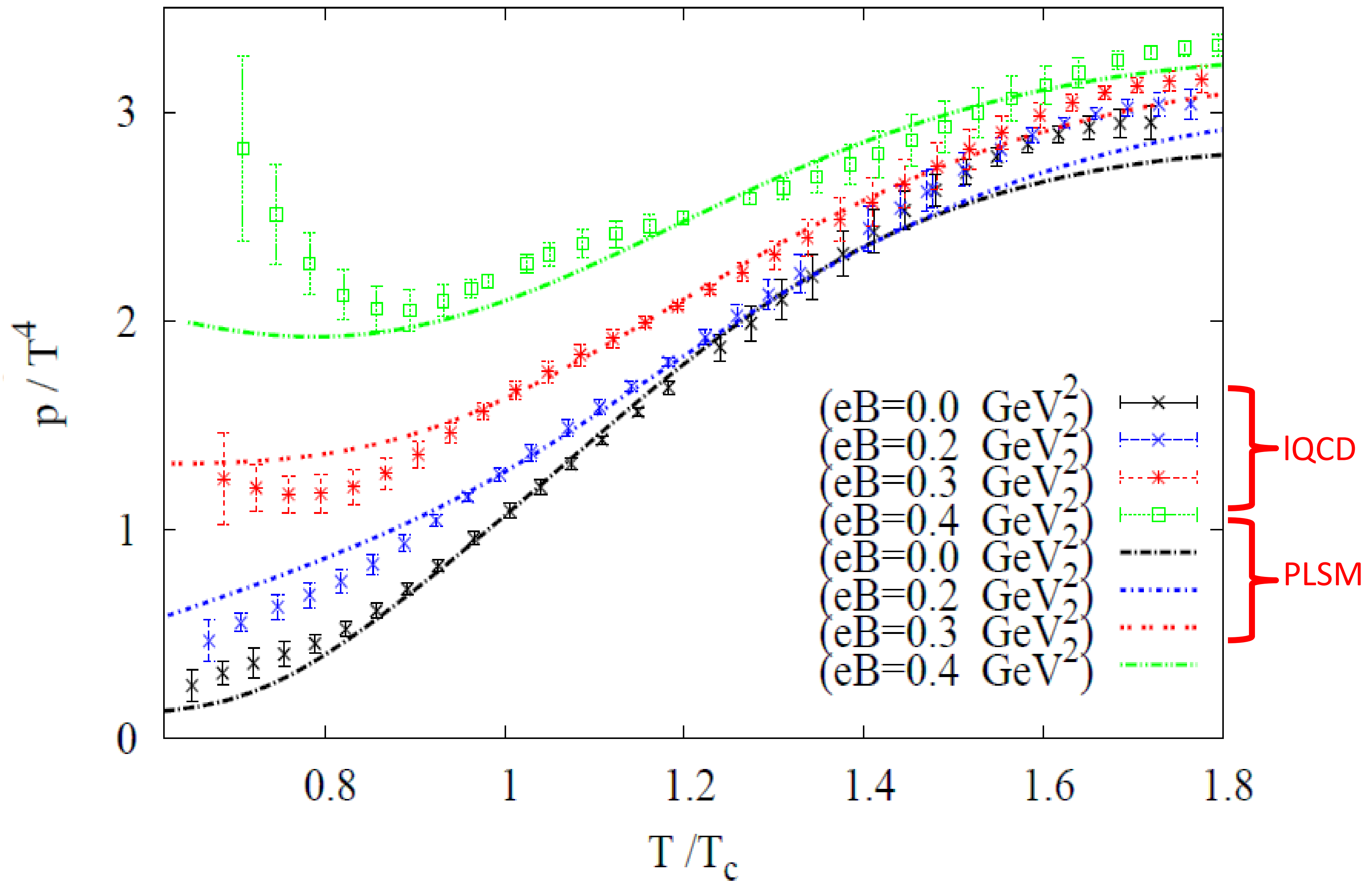
$\sigma_x, \sigma_y, \phi$ and $\phi^*$ minimizing the potential

$$\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\phi} = \frac{\partial\Omega}{\partial\phi^*}\bigg|_{min} = 0,$$

refined by lattice QCD,

$\sigma_x = \bar{\sigma_x}, \sigma_y = \bar{\sigma_y}, \phi = \bar{\phi}$ and $\phi^* = \bar{\phi}^*$ are the global minimum

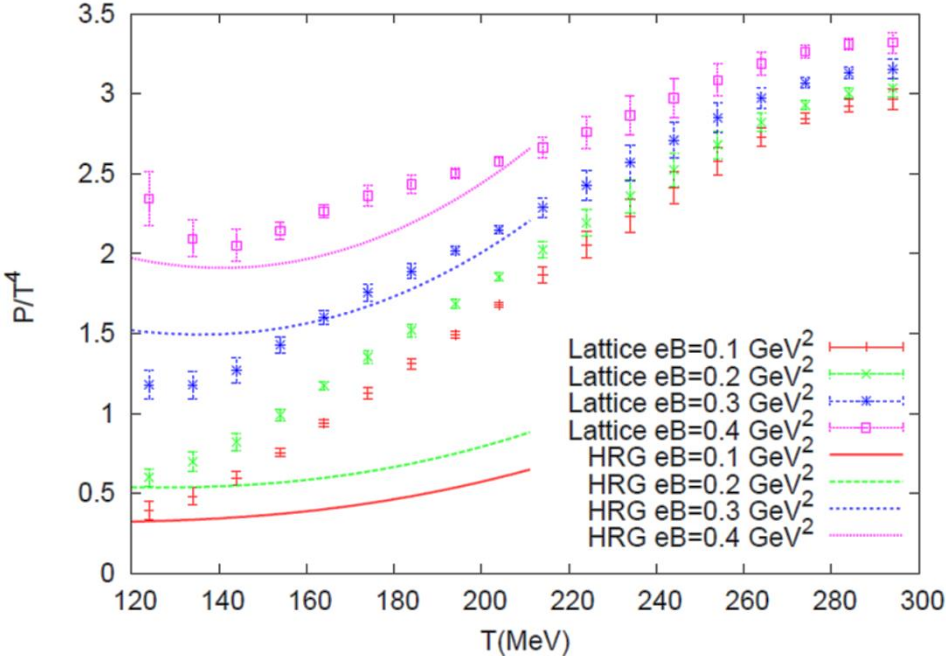
# Lattice QCD Thermodynamics: PLSM

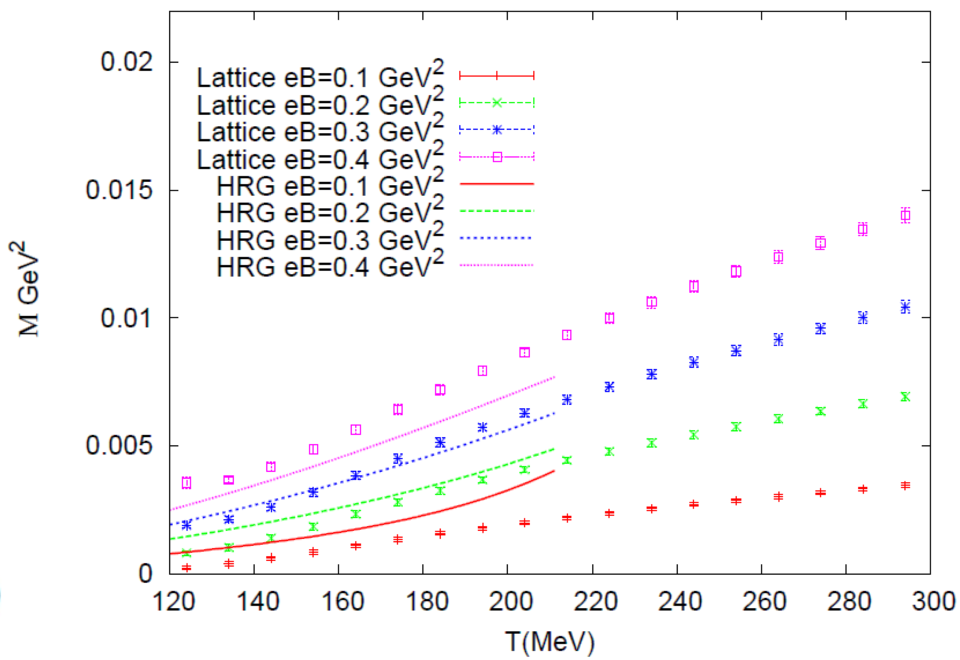



**Poster #26**

# Lattice QCD Thermodynamics: HRG






$$f_c(s) = \mp \sum_{s_z} \sum_{k=0}^{\infty} \frac{qB}{2\pi} \int \frac{dp_z}{2\pi} \left( \frac{E(p_z,k,s_z)}{2} + T \log(1 \pm e^{-E(p_z,k,s_z)/T}) \right),$$

$$E(p_z,k,s_z) = \sqrt{p_z^2 + m^2 + 2qB\,(k + 1/2 - s_z)},$$

$$\mathcal{M} = -\frac{\partial \mathcal{F}}{\partial (eB)}$$

**Non-charged hadrons** $f_n(s) = \mp \sum_{s_z} \int \frac{d^3p}{(2\pi)^3} \left( \frac{E_0(p)}{2} + T \log(1 \pm e^{-E_0(p)/T}) \right)$

**Why HRG fails to reproduce lattice data at finite eB? eB reduces hadron asses!?**

# (P)LSM in finite eB

## The quarks and antiquarks Potential contribution

$$\Omega_{\bar{q}q}(T,\mu_f) = -2T\sum_{f=l,s}\int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3}\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_f-\mu_f}{T}}\right)\times e^{-\frac{E_f-\mu_f}{T}}+e^{-3\frac{E_f-\mu_f}{T}}\right]\right.$$
$$\left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_f+\mu_f}{T}}\right)\times e^{-\frac{E_f+\mu_f}{T}}+e^{-3\frac{E_f+\mu_f}{T}}\right]\right\},$$

$$\Omega_{\bar{q}q}(T,\mu_f,B) = -\sum_{f=l,s}\frac{|q_f|BT}{(2\pi)^2}\sum_{\nu=0}^{\nu_{max\,f}}(2-\delta_{0\nu})\int_0^\infty dp_z$$
$$\left\{\ln\left[1+3\left(\phi+\phi^* e^{-\frac{E_{B,f}-\mu_f}{T}}\right)e^{-\frac{E_{B,f}-\mu_f}{T}}+e^{-3\frac{E_{B,f}-\mu_f}{T}}\right]\right.$$
$$\left.+\ln\left[1+3\left(\phi^*+\phi e^{-\frac{E_{B,f}+\mu_f}{T}}\right)e^{-\frac{E_{B,f}+\mu_f}{T}}+e^{-3\frac{E_{B,f}+\mu_f}{T}}\right]\right\}$$

$$E_{B,f}(B) = \sqrt{p_z^2+m_f^2+|q_f|(2n+1-\sigma)B}$$

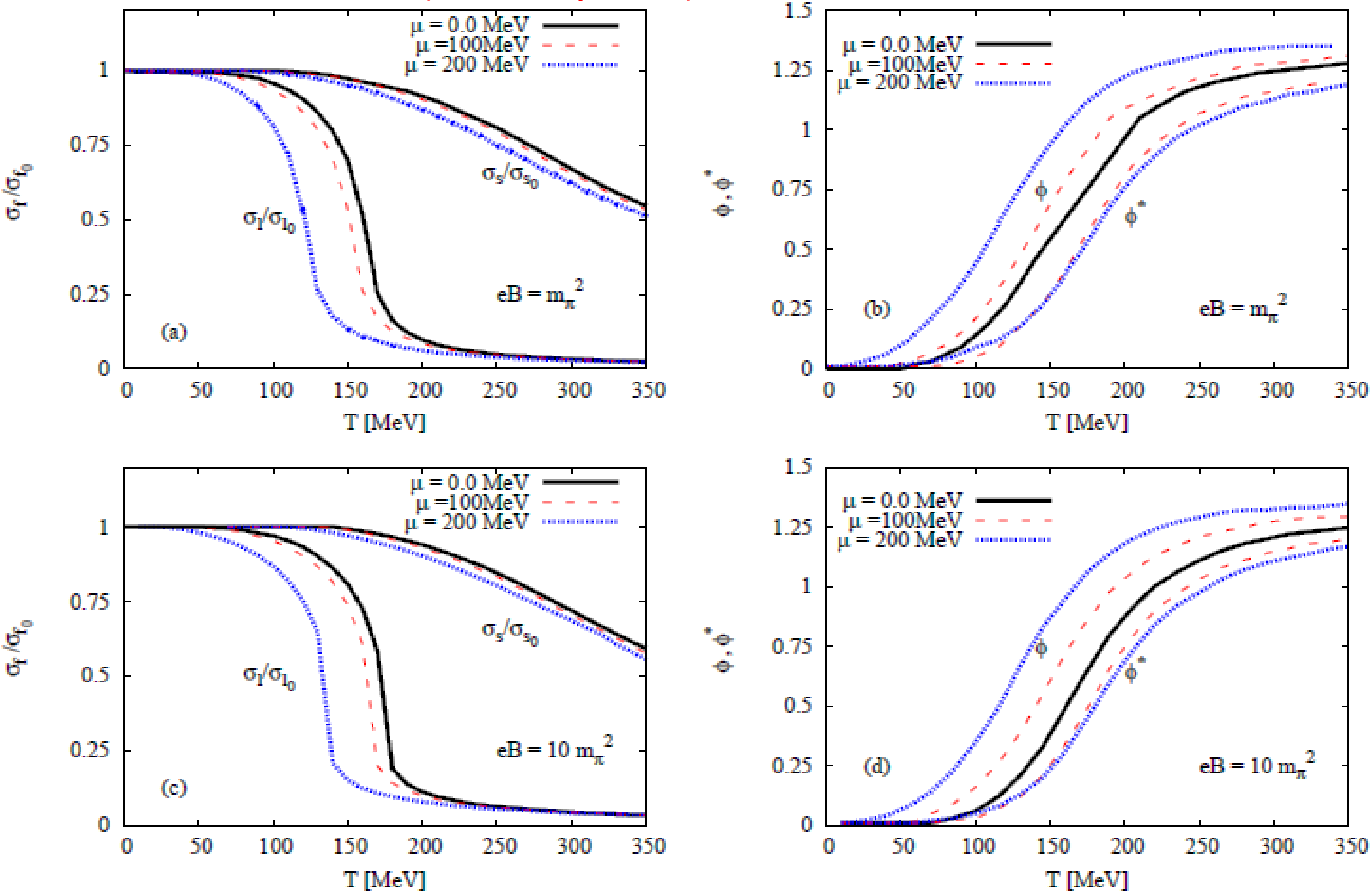
**Relations to spins** $\sigma = \pm S/2.$

**Landau levels** $\nu_{max\,f} = \left\lfloor \frac{\tau_f^2-\Lambda_{QCD}^2}{2|q_f|B} \right\rfloor$

$\tau$ is related to baryon chemical potentials at varying temperatures.

# Light and strange sigma/phi-condensates at eB≠0

**normalized to vacuum value ($\pi$&K decay width)**



# Light and strange sigma/phi-condensates at eB≠0

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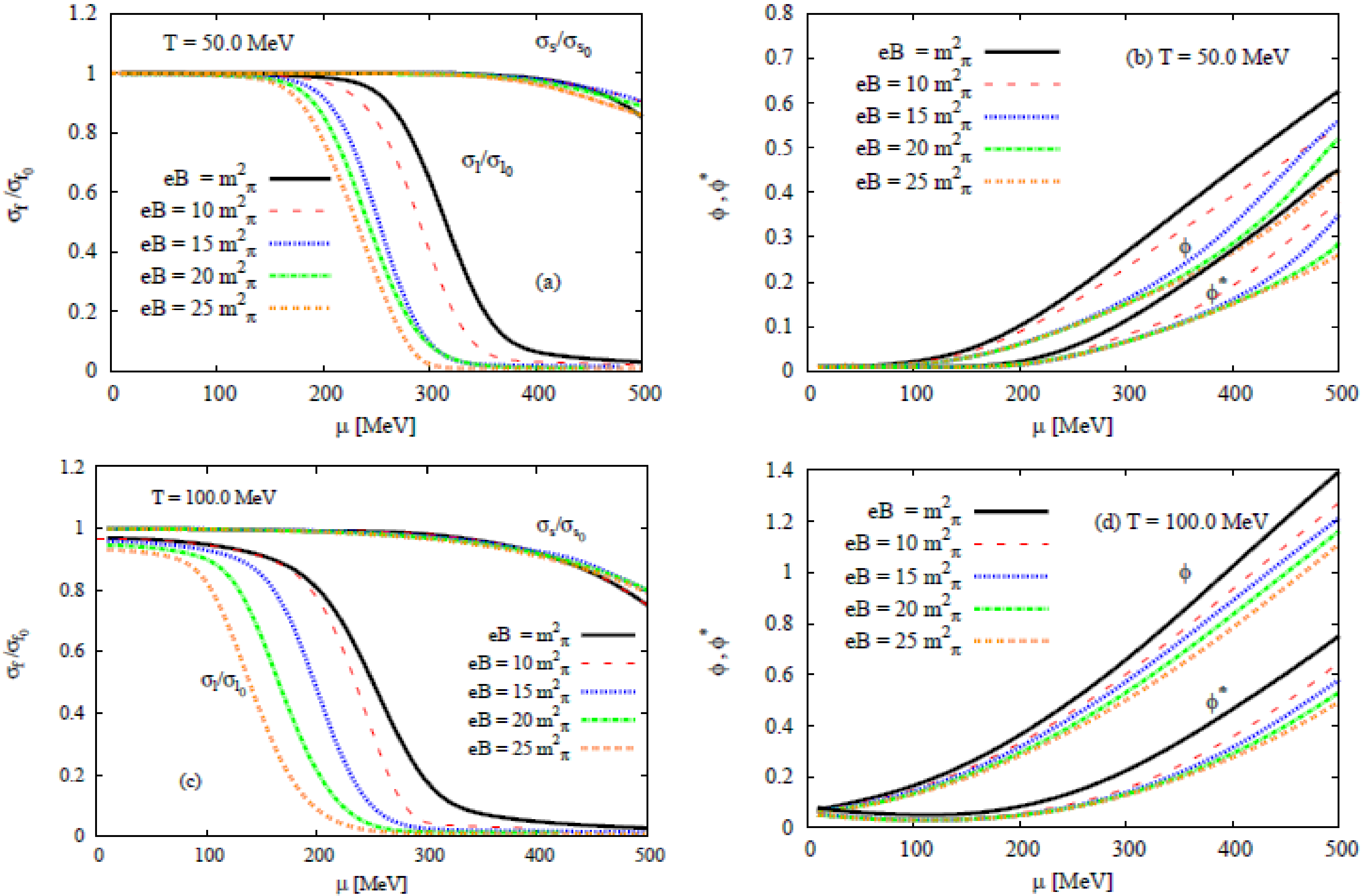


# Transport Coefficients from PLSM at eB=0



## Electrical and Heat Conductivity

Fine structure etc.

Number density

$$\sigma_e(T,\mu) = \sum_k \frac{4\pi}{137} q_k^2 \frac{n_k(T,\mu)\tau_k(T,\mu)}{m_k(T,\mu)}$$

Quark mass

Quarks flavors

Decay time

$$\kappa(T,\mu) = \frac{1}{3} v_{rel}\, c_V(T,\mu) \sum_k \tau_k(T,\mu)$$

Relative velocity

Specific heat

## Bulk and Shear Viscosity

$$\xi = \frac{1}{9T}\left[T^5 \frac{\partial}{\partial T}\left(\frac{\epsilon - 3p}{T^4}\right) + 16\,|\epsilon_v|\right] = \frac{1}{9T}\left[-16\,\epsilon + 9\,T\,S + T\,c_V + 16\,|\epsilon_v|\right]$$

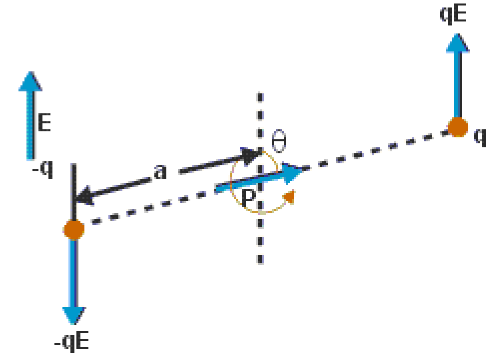
Vacuum energy density

$$\eta \sim \frac{\xi}{-0.45(c_s^2 - \frac{1}{3})}$$

Pressure, energy density, entropy, speed of sound

# Electrical Conductivity from PHSD at eB=0

**Based on parton-hadron-string dynamics transport approach**




$$\frac{d}{dt} p_z^j = q_j e E_z$$

**an additional force causes the propagation of charge.**

**The electrical current density**

$$j_z(t) = \frac{1}{V} \sum_j q_j e \frac{p_z^j(t)}{M_j(t)}$$

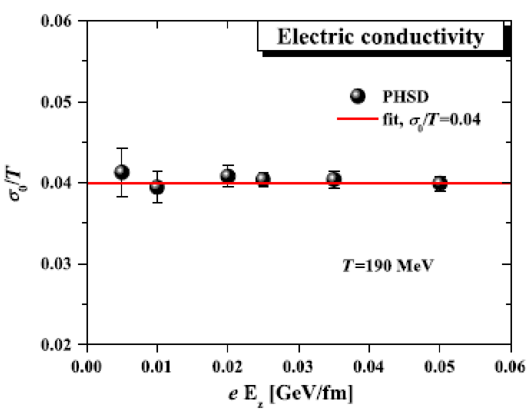
z-momentum of j-th particle at time t

Mass of j-th particle at time t

**In natural units, the ratio of current density and electric field strength → electric conductivity**

$$\frac{\sigma_0}{T} = \frac{j_{eq}}{E_z T}$$

**proportionality between e-current and e-field**




**F. Reif, Fundamentals of Statistical and Thermal Physics, (McGraw-Hill, New York, 1965).**
**W. Cassing, O. Linnyk, T. Steinert, and V. Ozvenchuk, Phys. Rev. Lett. 110, 182301 (2013).**

# Electrical Conductivity at eB=0

in relaxation time approximation, $\sigma$ is described in Gases, Liquids and Solid State,

$$\sigma_0 = \frac{e^2 n_e \tau}{m_e^*}$$

n density of nonlocalized charges

$\tau$ relaxation time of charge carriers

$m_e^*$ effective masses

for partonic degrees of freedom within the dynamical quasiparticle model (DQPM), the thermal dependence reads

$$\frac{\sigma_0(T)}{T} \approx \frac{2}{9} \frac{e^2 n_q(T)}{M_q(T)\Gamma_q(T) T}$$

$\Gamma_q$ width of quasiparticle spectral function

$M_q$ pole mass=spectral dist. of quark-mass

flavor averaged fractional quark charge squared

In PHSD: DQPM matches quasiparticles properties to lQCD results in equilibrium for EOS, electromagnetic correlator, among others.

# Electrical Conductivity at eB=0

## Durde-Lorentz conductivity

$$\sigma_e(T,\mu) = \sum_k \frac{4\pi}{137} q_k^2 \frac{n_k(T,\mu)\,\tau_k(T,\mu)}{m_k(T,\mu)}$$

Fine structure etc.

Quarks flavors

Number density

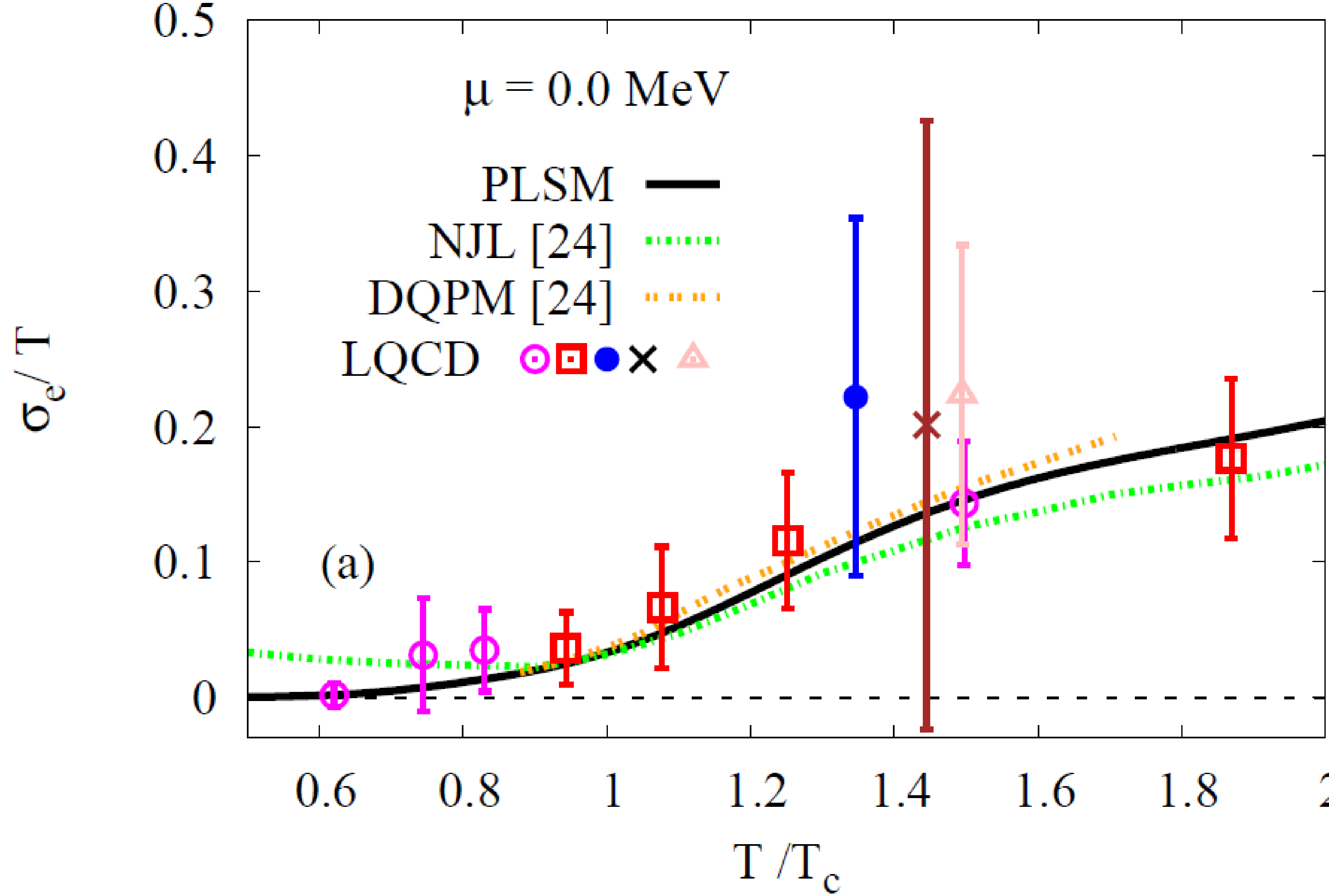
Quark mass

Decay time

**σ is related to flow of charges in presence of an electric field**
(decay constant & relaxation time)
**response of the strongly interacting system in equilibrium to an external e-field**

- external e-field is applied on flowing charges, the induced electric current J is related to the e-field. σ is the proportionally constant.
- self-interaction between quarks and gluons, Green-Kubo corrector

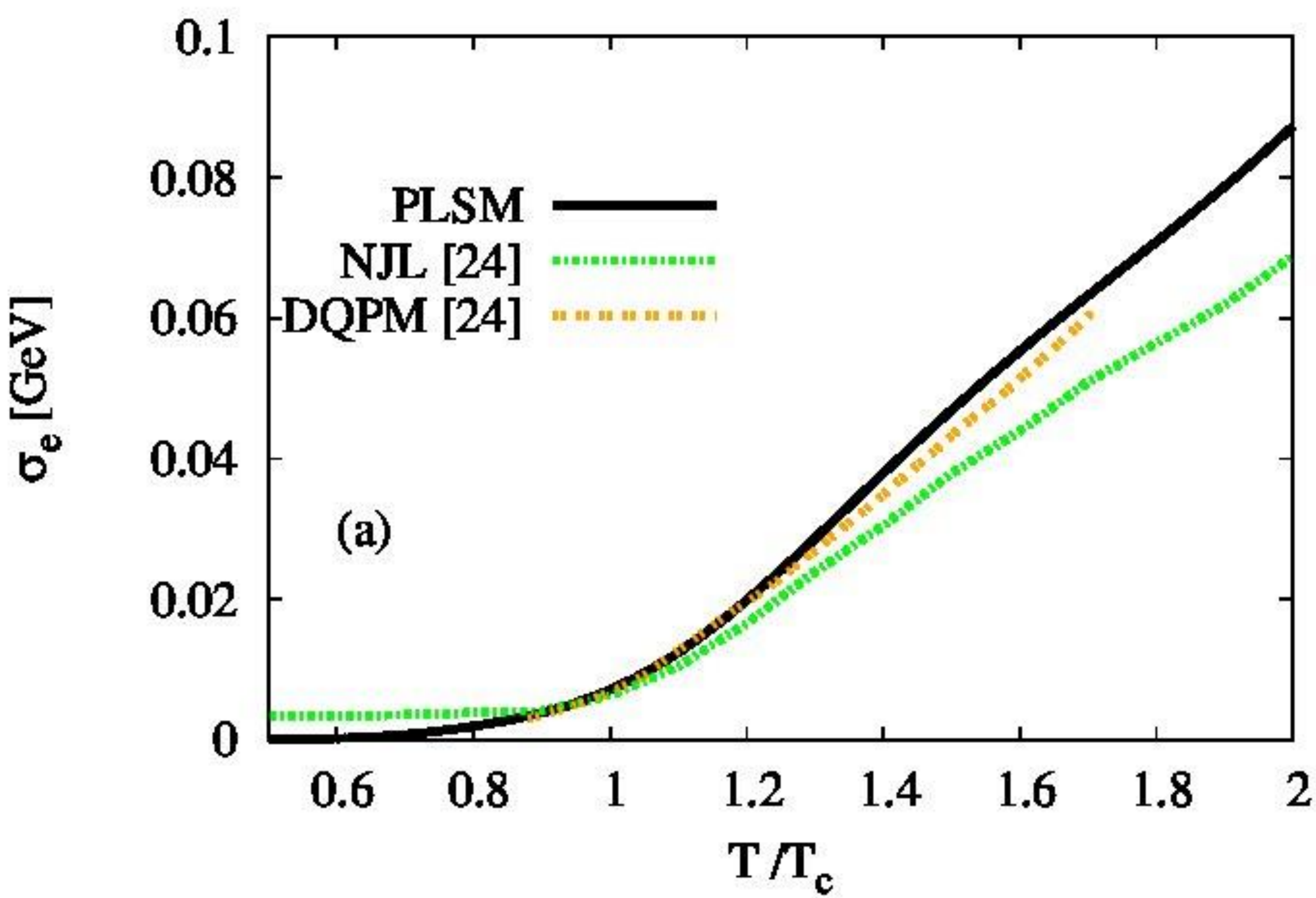
# Normalized Electrical Conductivity at eB=0




**NJL/DQPM:** PRC88, 045204 (2013)

**LQCD:** PRL111,172001 (2013), PRD83,034504 (2011), JHEP1303,100 (2013), 1412.6411, 1501.0018

# Non-Normalized Electrical Conductivity at eB=0




**NJL/DQPM:** PRC88, 045204 (2013)

# Heat Conductivity at eB=0

**From relativistic Navier-Stokes ansatz, heat flow is proportional to the gradient of thermal potential**

$$q^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \alpha = \kappa \left( \nabla^\mu T - \frac{T}{\epsilon + p} \nabla^\mu p \right)$$

PRE87, 033019 (2013)

$$q^x = \kappa (\nabla^x T) = -\kappa \partial_x T(x)$$

Temperature profile → Modeling

$$\kappa = q^x \frac{(ax+b)^2}{ap}$$

**Alternatively, linearizing Boltzmann Eq. →** PRD48, 2916 (1993)

$$f_i = f_i^{le} + \frac{\partial f_i^0}{\partial \varepsilon_i} \Phi_i \frac{\nabla T}{T}$$

Non-Equilibrium distribution function

$$f_i^{le} = \{\exp[(\varepsilon_i - \mu)/T(z)] + 1\}^{-1}$$

Equilibrium distribution function

**Then, the thermal current reads** $J_T = \nu_q \sum_{\mathbf{p}} (\varepsilon_p - \mu_q) v_z \frac{\partial f^0}{\partial \varepsilon_p} \Psi_{\mathbf{p}} = \frac{1}{3} \mu_q^2 T^2$

$$\frac{1}{\kappa} = \frac{24}{\pi^3} \alpha_s^2 T^{-2} I_\kappa(T/q_D)$$

$$I_\kappa(T/q_D) = \begin{cases} \frac{1}{3} \ln(T/q_D) + 0.30\,, & T \gg q_D\,, \\ 2\zeta(3) \left(\frac{T}{q_D}\right)^2\,, & T \ll q_D\,. \end{cases}$$

**$\alpha_s$ running strong coupling**

**$q_D$ Debye wave number** $g^2 N_q \mu^2 / (2\pi^2)$

# Heat Conductivity at eB=0

$$\kappa = \frac{1}{3} v_F^2 c_v \tau_\kappa$$

Fermi velocity

Specific heat

Relaxation time

**Relaxation time, specific heat are T- and mu-dependent**

**Relative velocity** $\nu_{rel} = \sqrt{(p_1 p_2)^2 - (m_1 m_2)^2} / E_1 E_2$

$$\kappa(T,\mu) = \frac{1}{3} \nu_{rel}\, c_V(T,\mu) \sum_k \tau_k(T,\mu)$$

Relative velocity

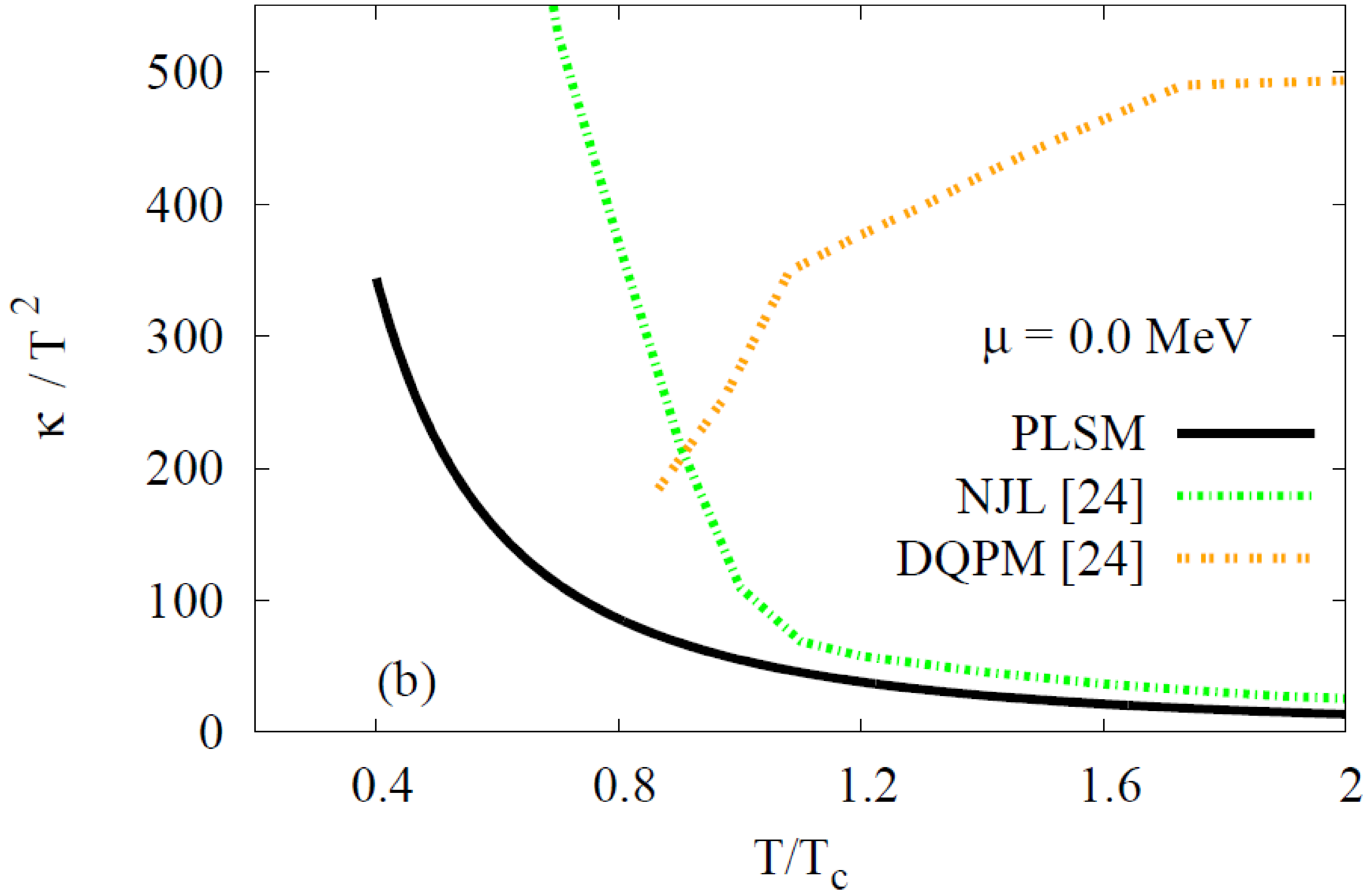
Specific heat

Decay time

**$\kappa$ is related to heat flow of relativistic fluid** (rate of energy change)

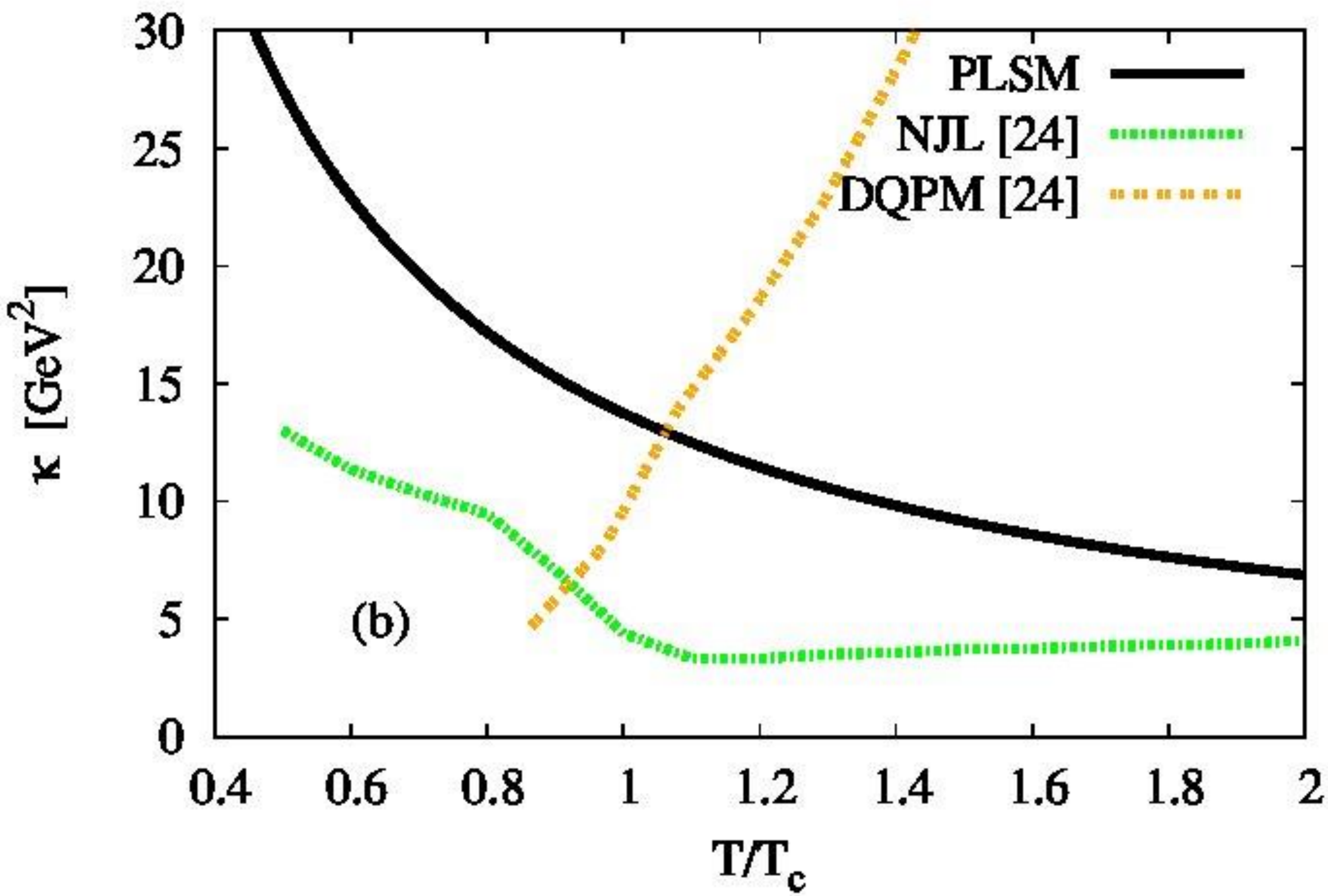
**$\kappa$ can be estimated through irradiation caused by energetic ions**

# Normalized Heat Conductivity




**NJL/DQPM:** PRC88, 045204 (2013)

# Non-Normalized Heat Conductivity at eB=0




**NJL/DQPM:** PRC88, 045204 (2013)

# Bulk Viscosity at eB=0



Kubo's formula: shear η and bulk ζ viscosities are related to the **correlation function of stress tensor**

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt \int d^3r\, e^{i\omega t}\, \langle[\theta_{ii}(x),\theta_{kk}(0)]\rangle$$

**PLB663, 217 (2008)**

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt \int d^3r\, e^{i\omega t}\, \langle[\theta_\mu^\mu(x),\theta_\mu^\mu(0)]\rangle$$

**LI- operators**

**In low energy theorems: bulk viscosity is a measure for violation of conformal invariance**

$$(\mathcal{E}-3P)^* = \langle m\bar{q}q\rangle^* + \langle m\bar{q}q\rangle_0 - 4|\epsilon_v|$$

$$\langle m\bar{q}q\rangle_0 = -M_\pi^2\, f_\pi^2 - M_K^2\, f_K^2$$

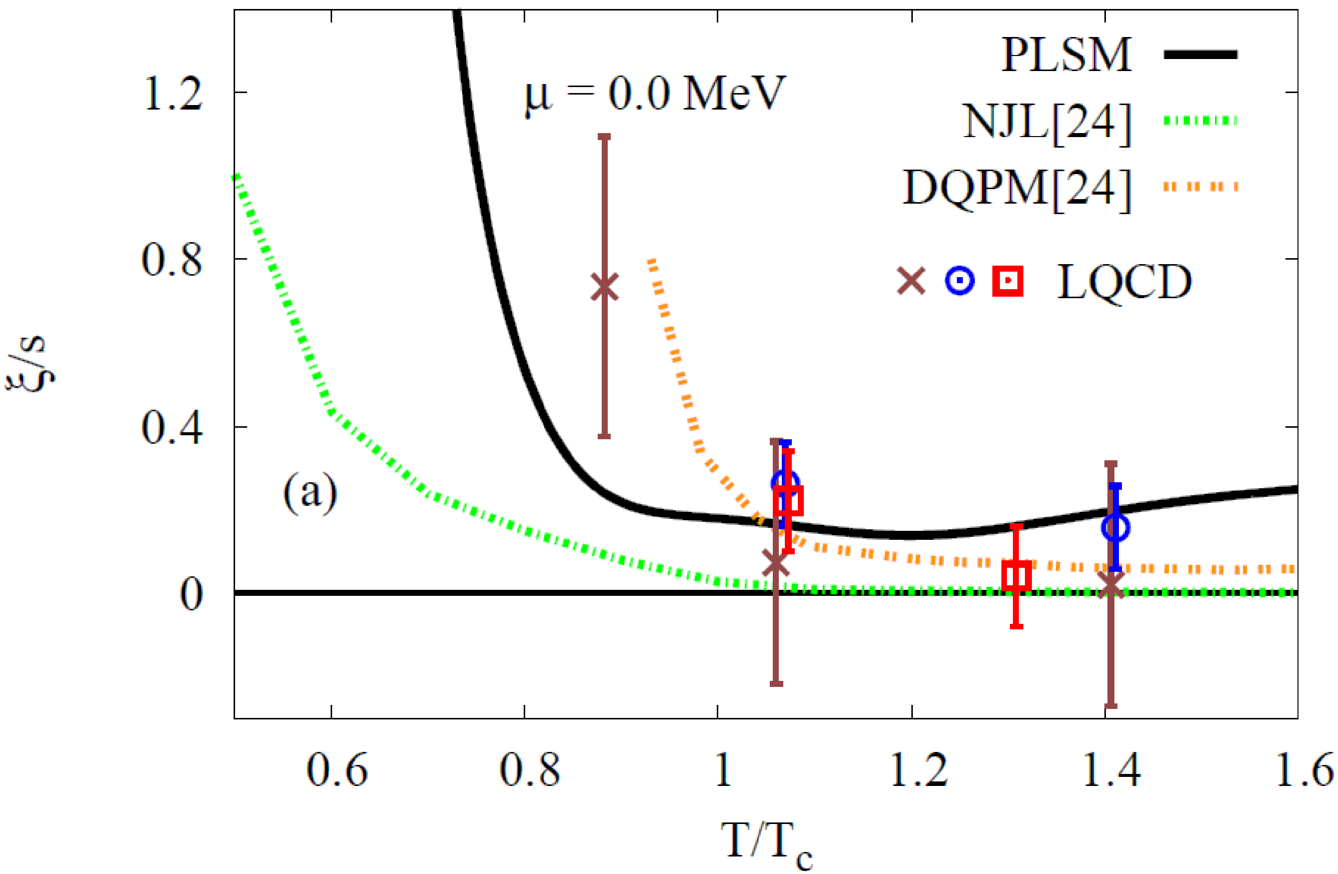
**PCAC relations**

$$9\,\omega_0\,\zeta = T\,s\left(\frac{1}{c_s^2}-3\right) - 4(\mathcal{E}-3P) + \left(T\frac{\partial}{\partial T}-2\right)\langle m\bar{q}q\rangle^* + 16|\epsilon_v| + 6(M_\pi^2\, f_\pi^2 + M_K^2\, f_K^2)$$

# Bulk Viscosity at eB=0

$$\xi = \frac{1}{9T}\left[T^5 \frac{\partial}{\partial T}\left(\frac{\epsilon - 3p}{T^4}\right) + 16|\epsilon_v|\right] = \frac{1}{9T}\left[-16\,\epsilon + 9\,T\,S + T\,c_V + 16|\epsilon_v|\right]$$

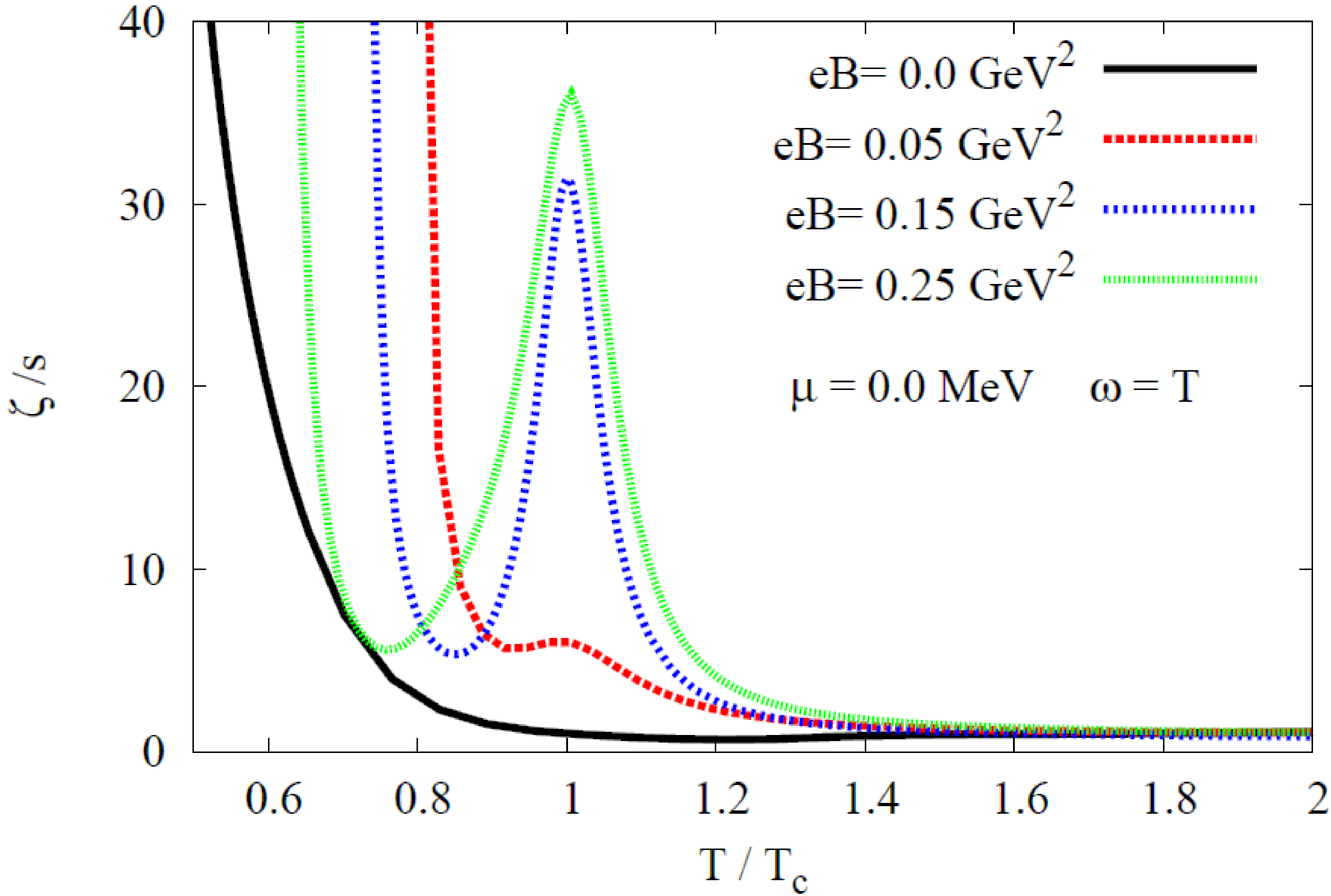
Vacuum energy density




**NJL/DQPM:** PRC88, 045204 (2013)

PRD76, 101701(2007); PRL100, 162001(2008); PoS LAT2007, 221(2007); PRL94, 072305(2005).

# PLSM Bulk Viscosity at eB≠0



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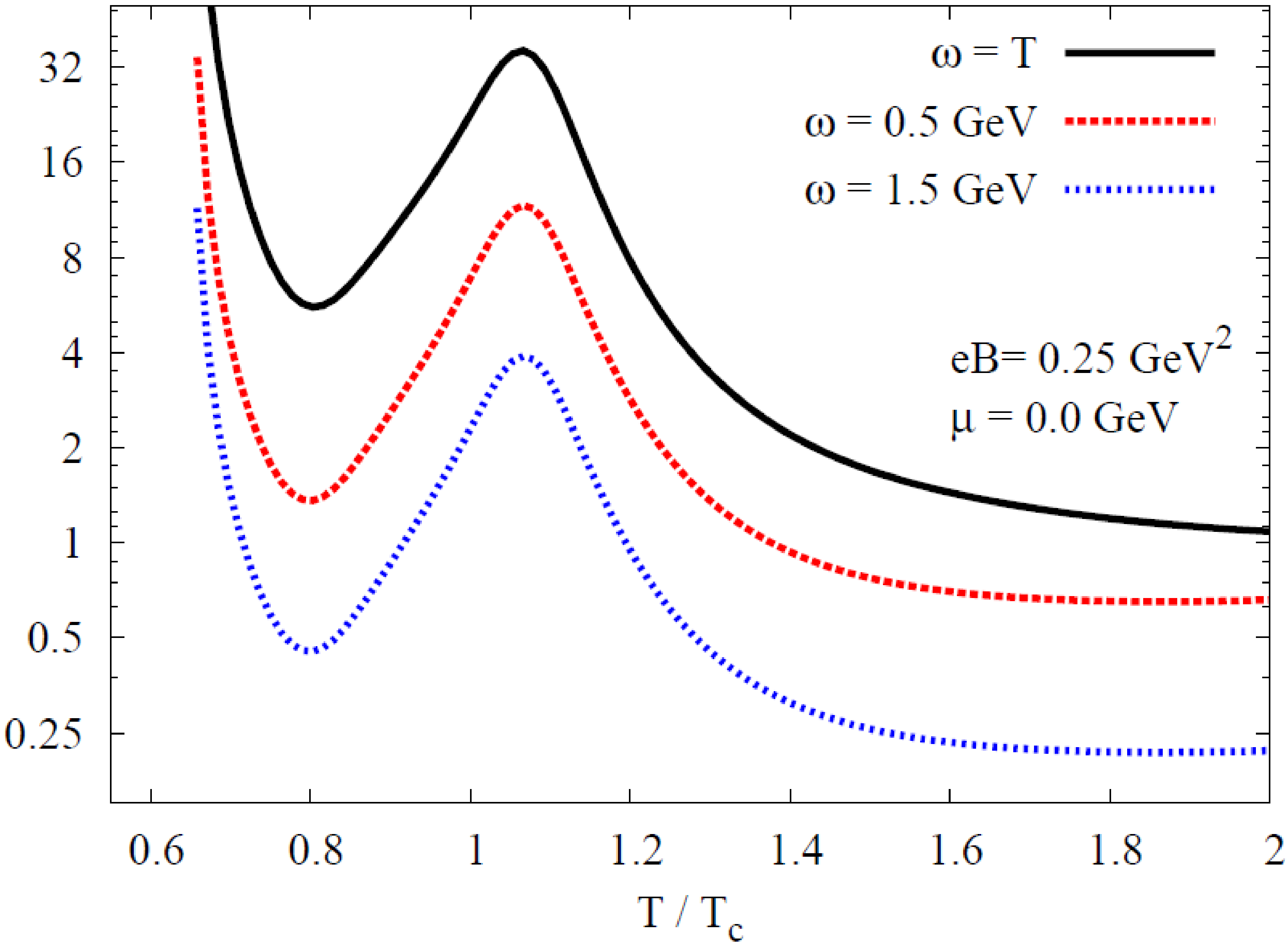
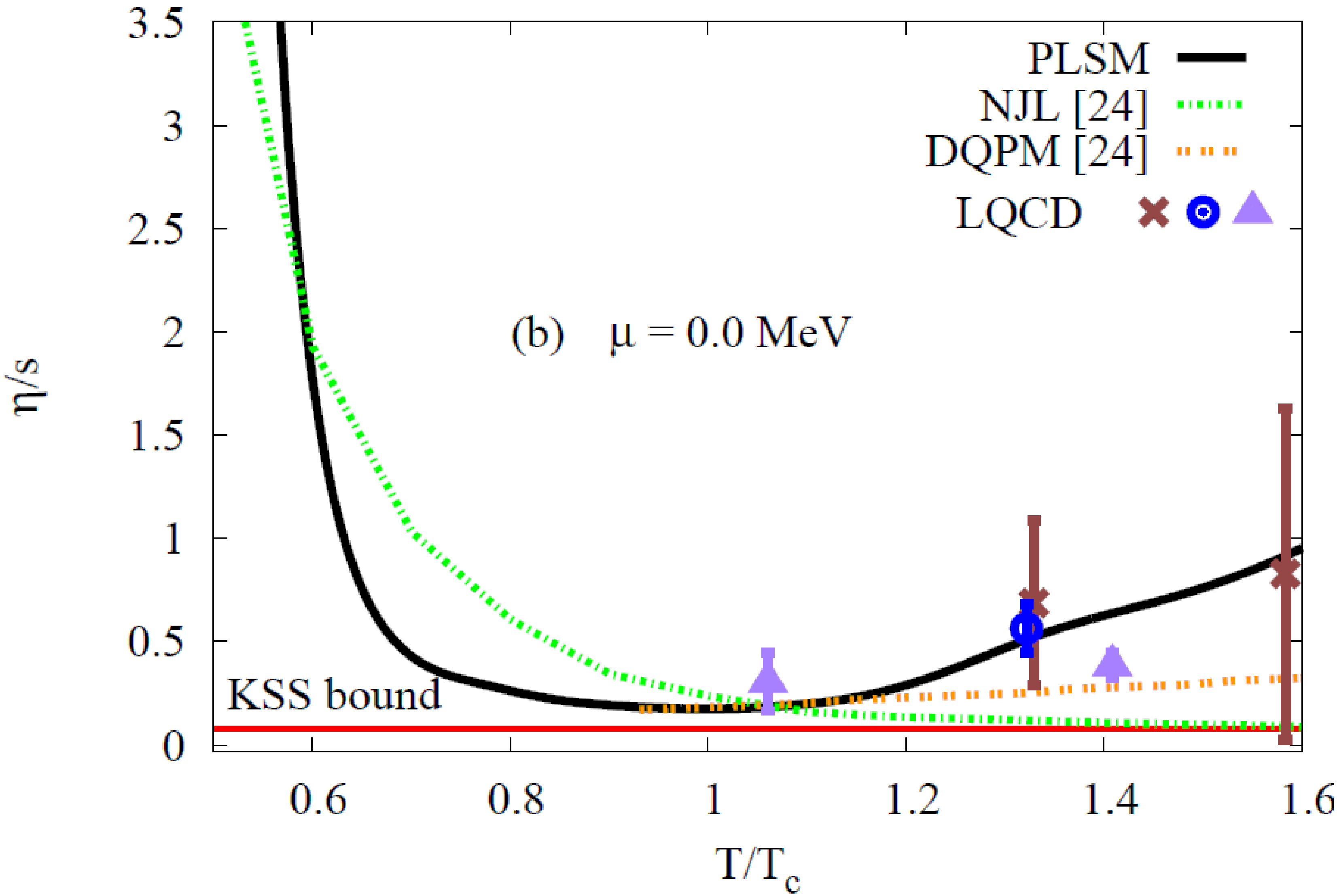
# PLSM Bulk Viscosity at eB≠0



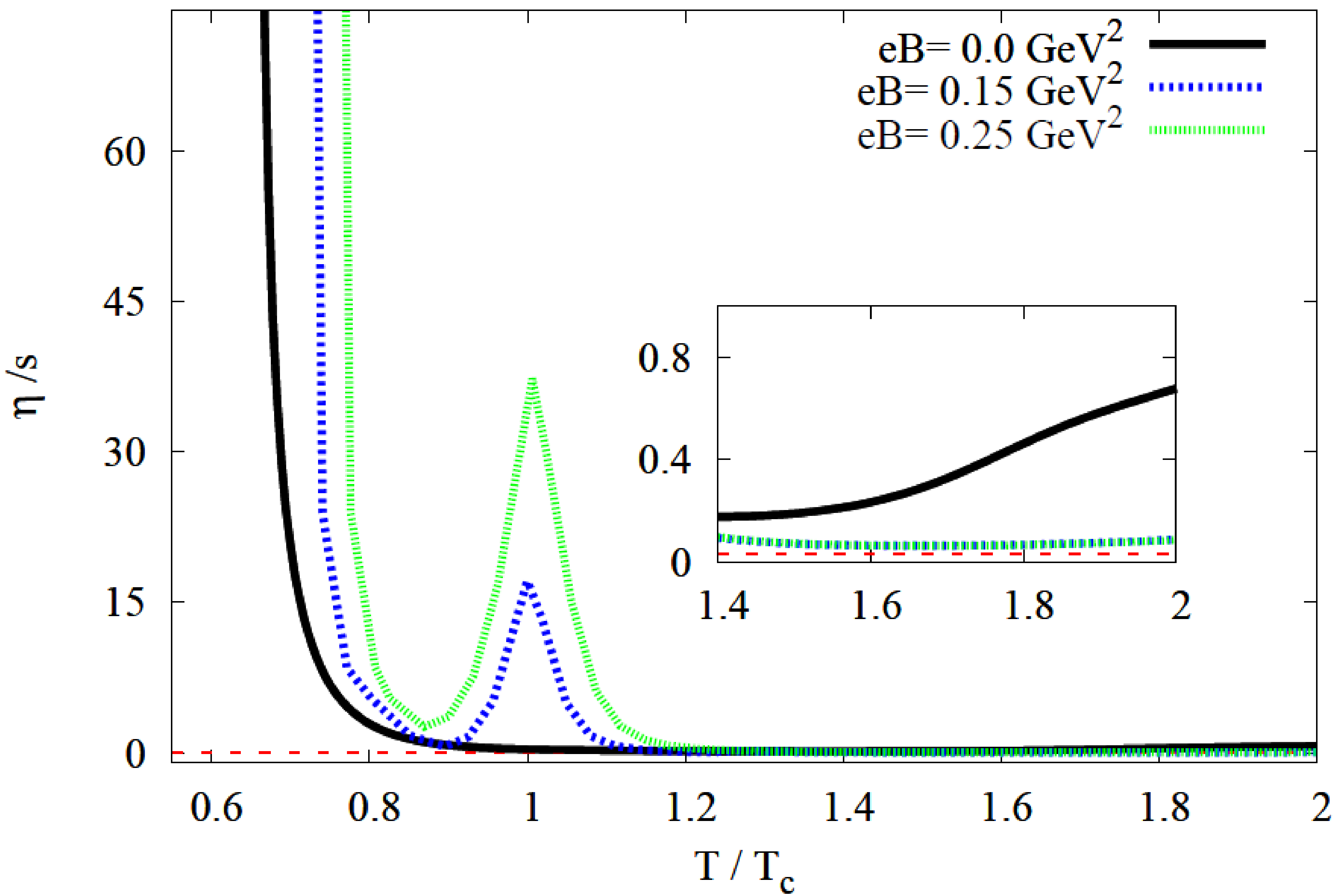
# Shear Viscosity at eB=0



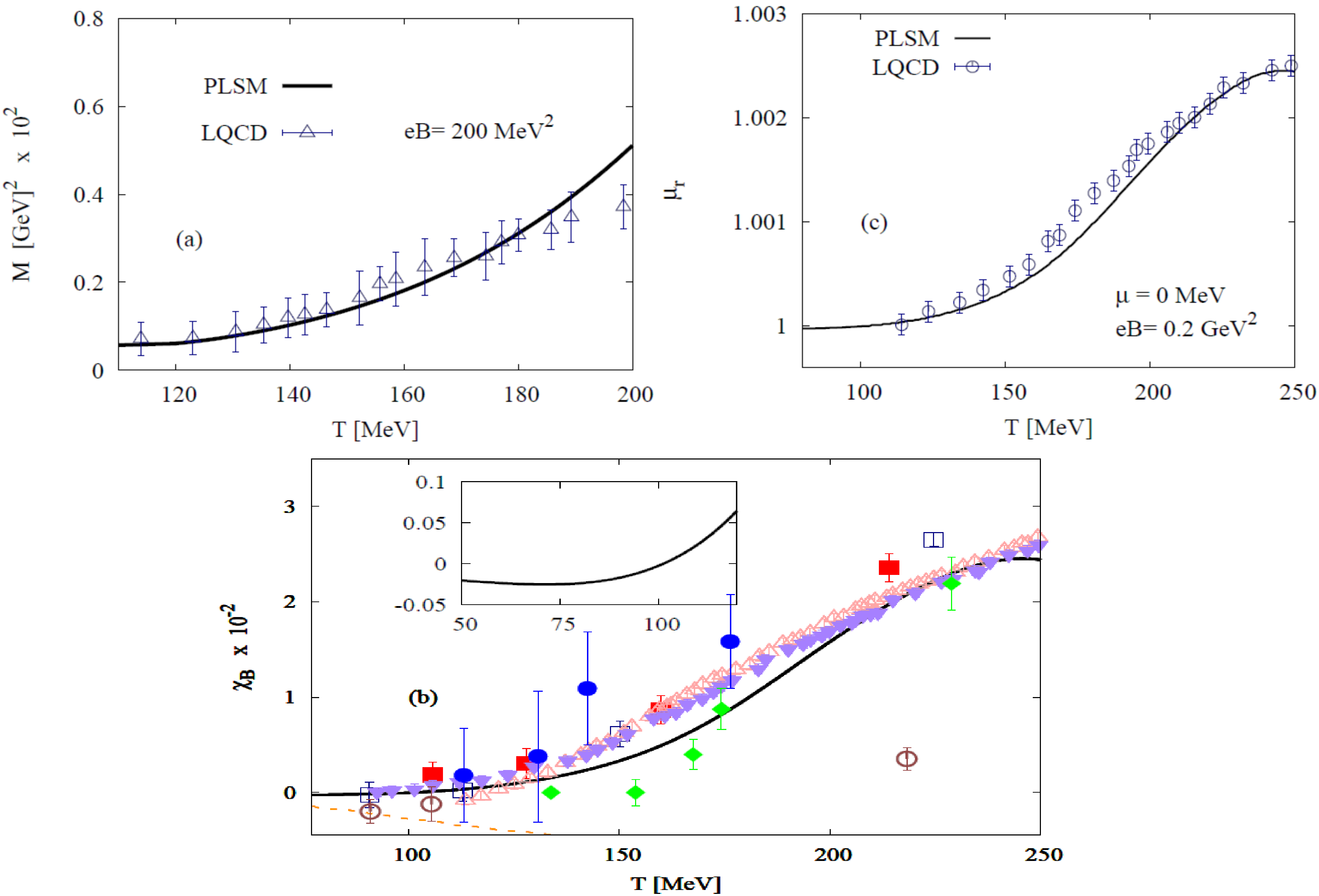

**NJL/DQPM:** PRC88, 045204 (2013)

**KSS:** Kovtun, Son, Starinets, PRL94, 111601 (2005).

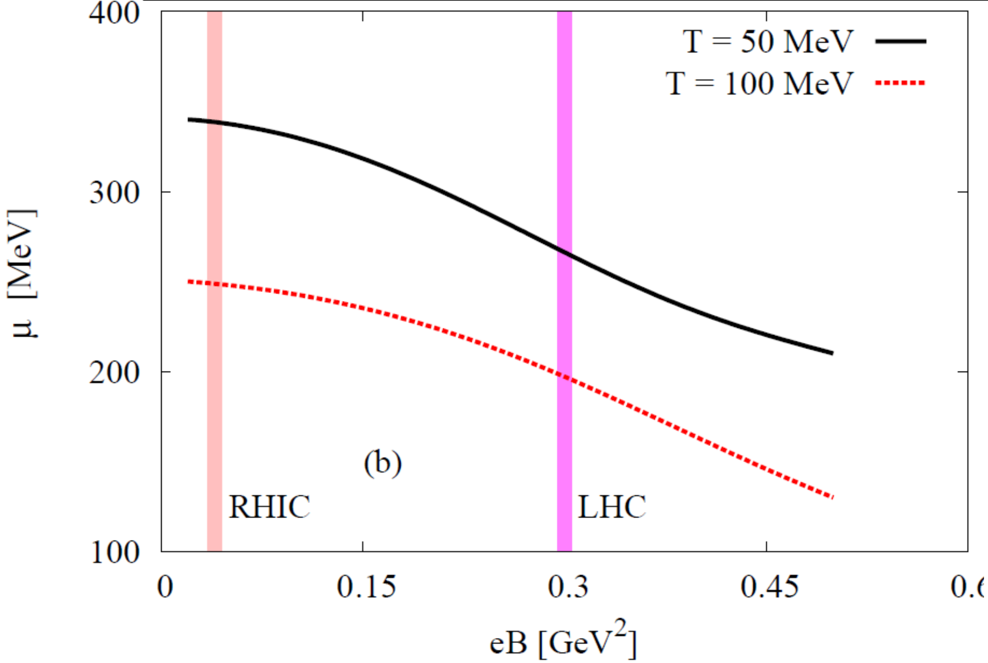
# PLSM Shear Viscosity at eB≠0

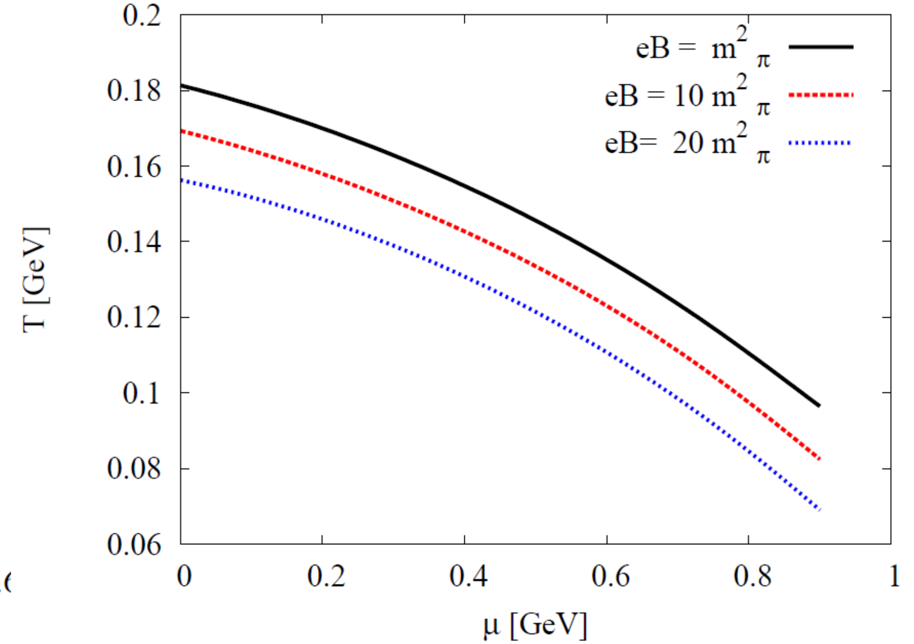


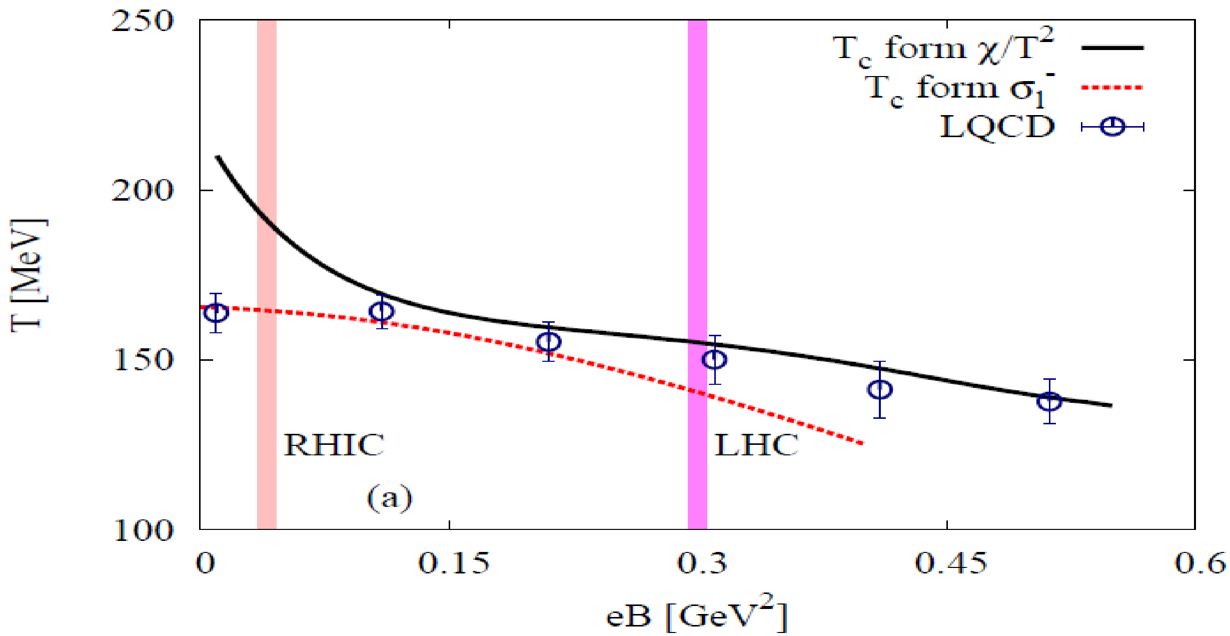
# PLSM and LQCD Thermodynamics at eB≠0



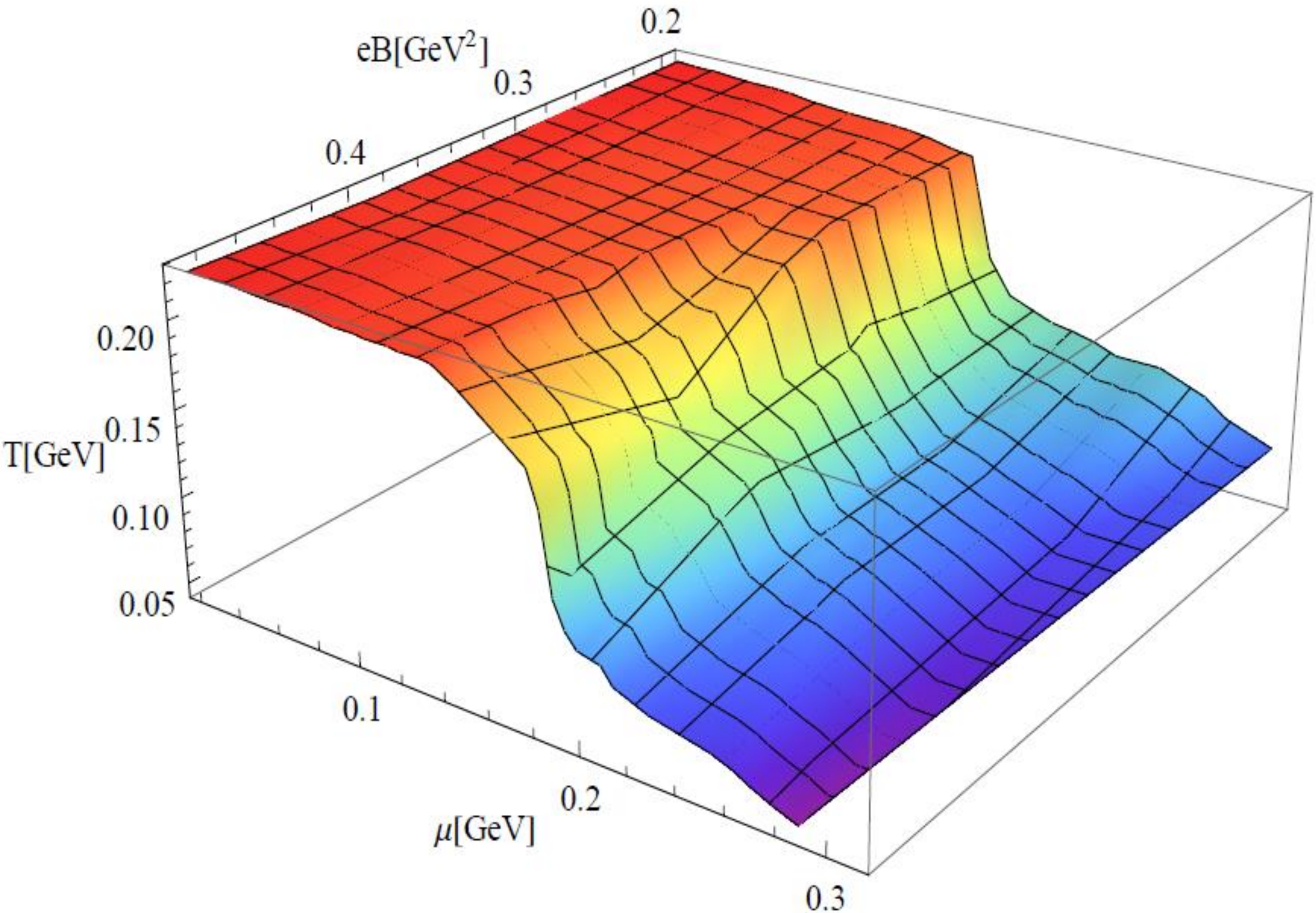
# T-mu diagram at eB≠0



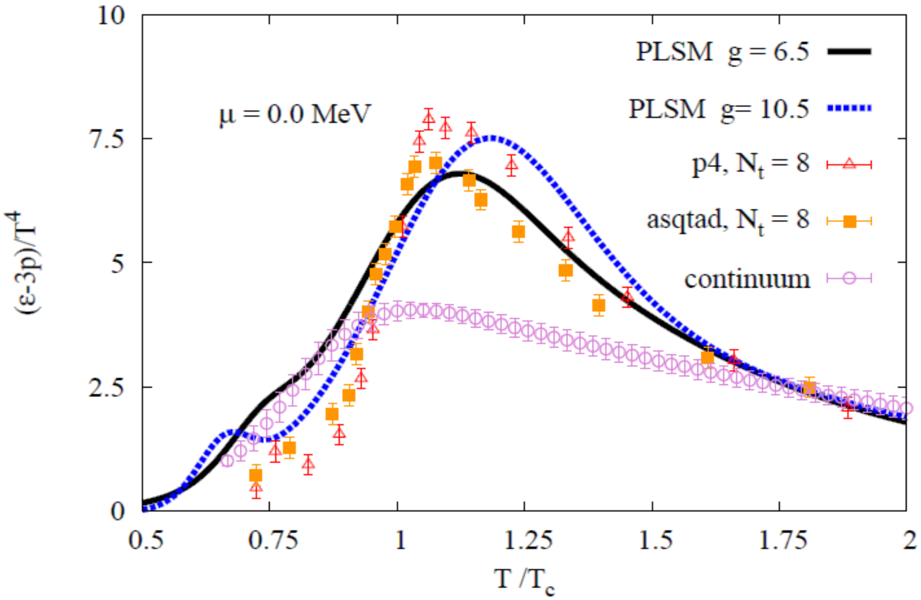



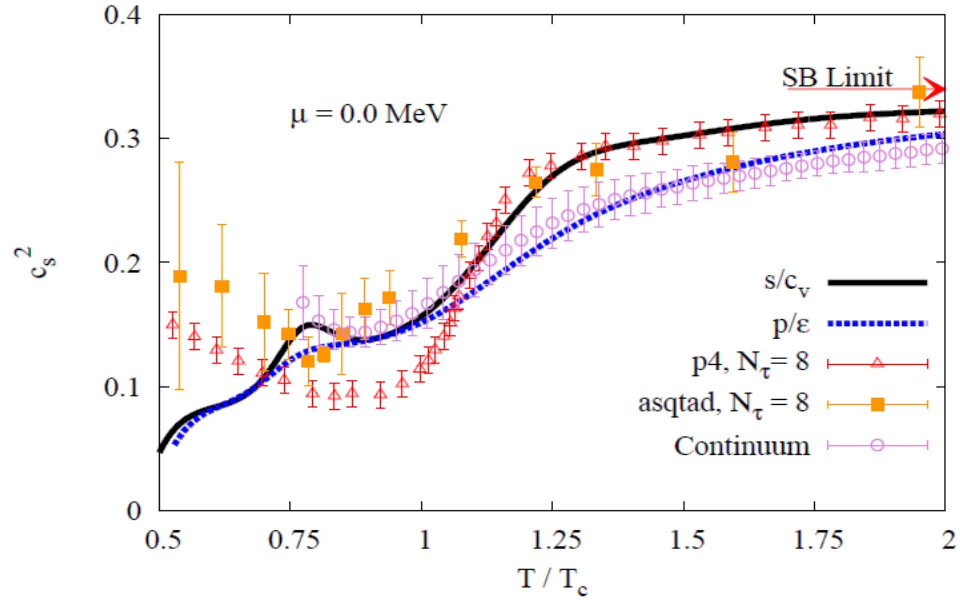



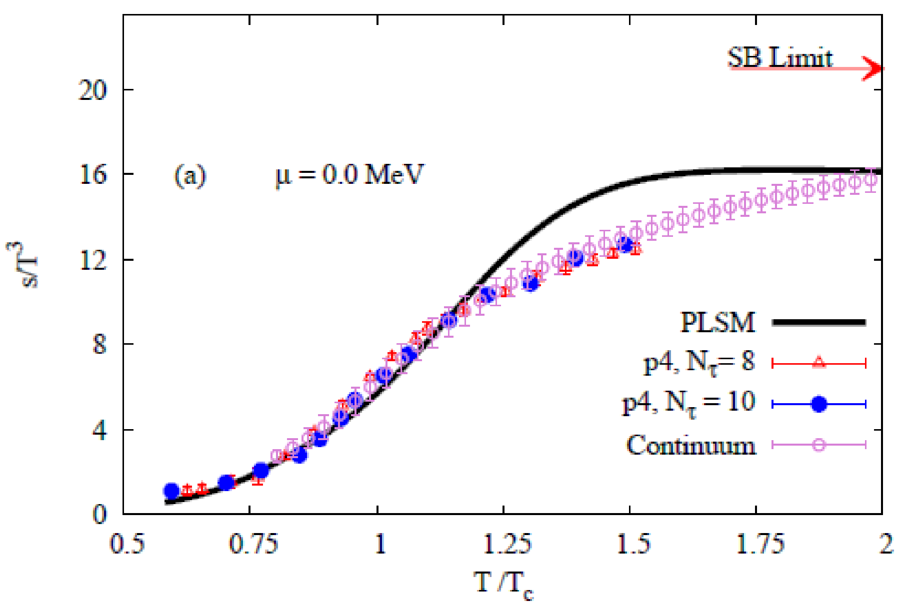
# Freezeout Diagram at eB≠0 and for s/T^3=7

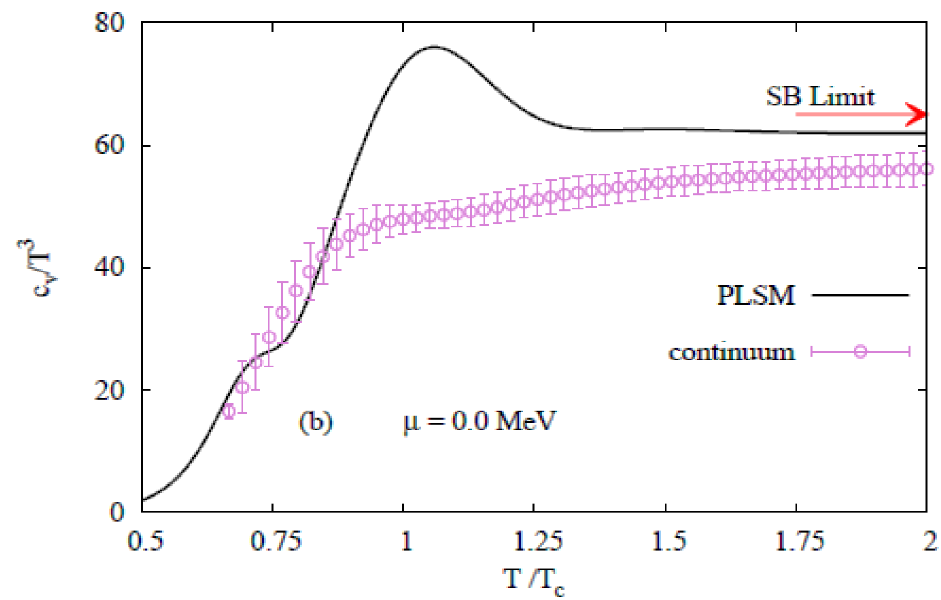


# PLSM and LQCD Thermodynamics at eB=0

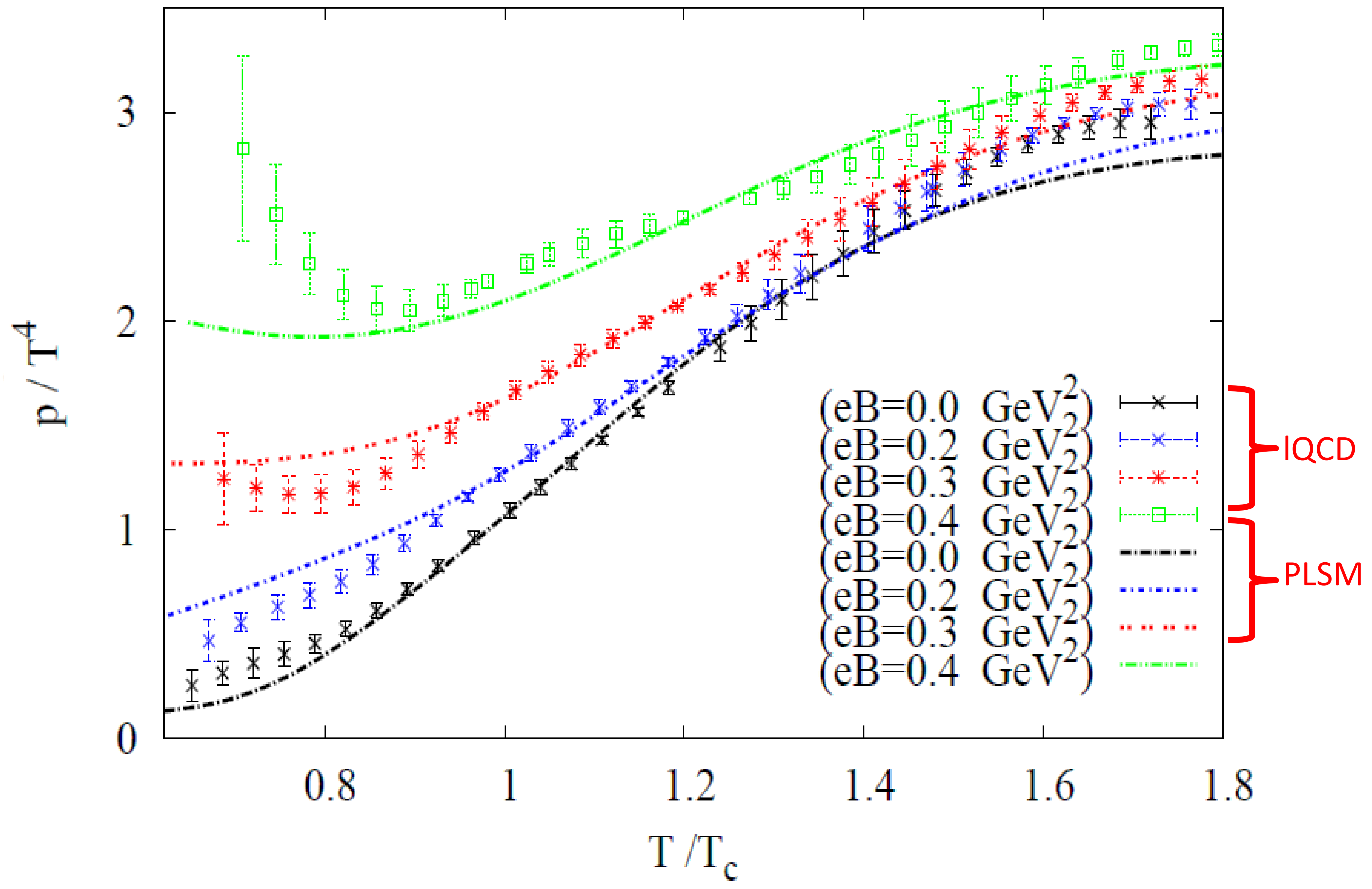


# PLSM and LQCD Thermodynamics at eB≠0




**Poster #26**

# Thank You!