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Analytical Results for Hard-Scattering Production of Heavy Quarks

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1. Introduction
2. $d\sigma(pp \to c\bar{c}X)/dy_c\, c_T dc_T\, dy_{\bar{c}}\, \bar{c}_T d\bar{c}_T$
3. $d\sigma(pp \to c\bar{c}X)/d\Delta\phi\, d\Delta y$
4. $d\sigma(pp \to cX)/dy_c\, c_T dc_T$
5. Conclusion

Why study analytical formulas for heavy quark production in pp collisions

- Knowledge of heavy quark production in pp collisions provide insight to guide our intuition
- Analytical formulas summarize important features and factors in the collision process
- Similar analysis in light hadron production lead to new insights and new results

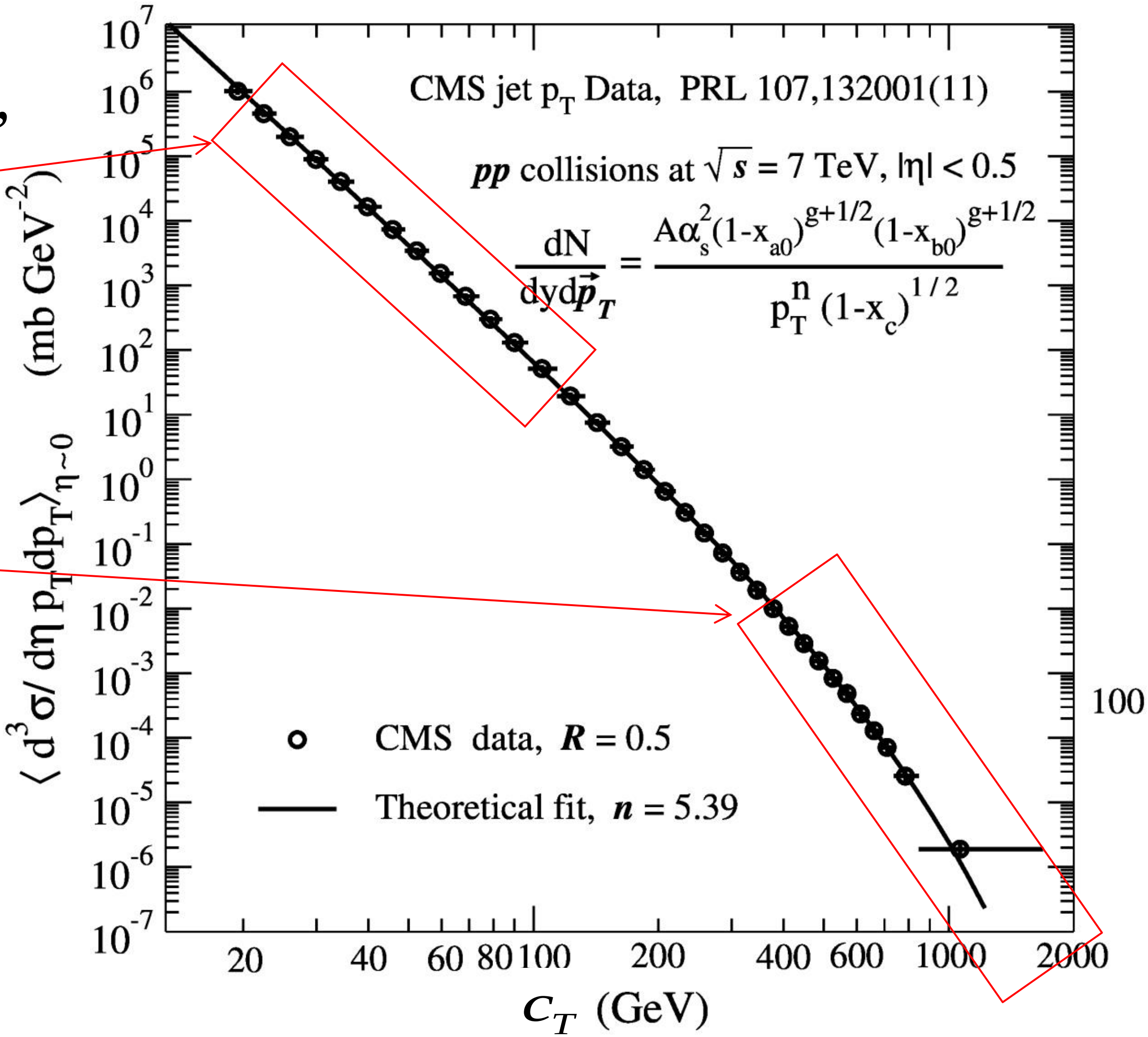
The log-log plot of σ_{inv} and c_T(jet)

(i) in the lower c_T region, the slope gives n,

$$\frac{d(\ln \sigma_{\text{inv}})}{d(\ln c_T)} = -n,$$

(ii) in the very high c_T region, the bending of the curve gives g,

$$\frac{d^2(\ln \sigma_{\text{inv}})}{d(\ln c_T)^2} \approx g.$$



Tsallis distribtuion can describe LHC p_T distributions

Tsallis distribution

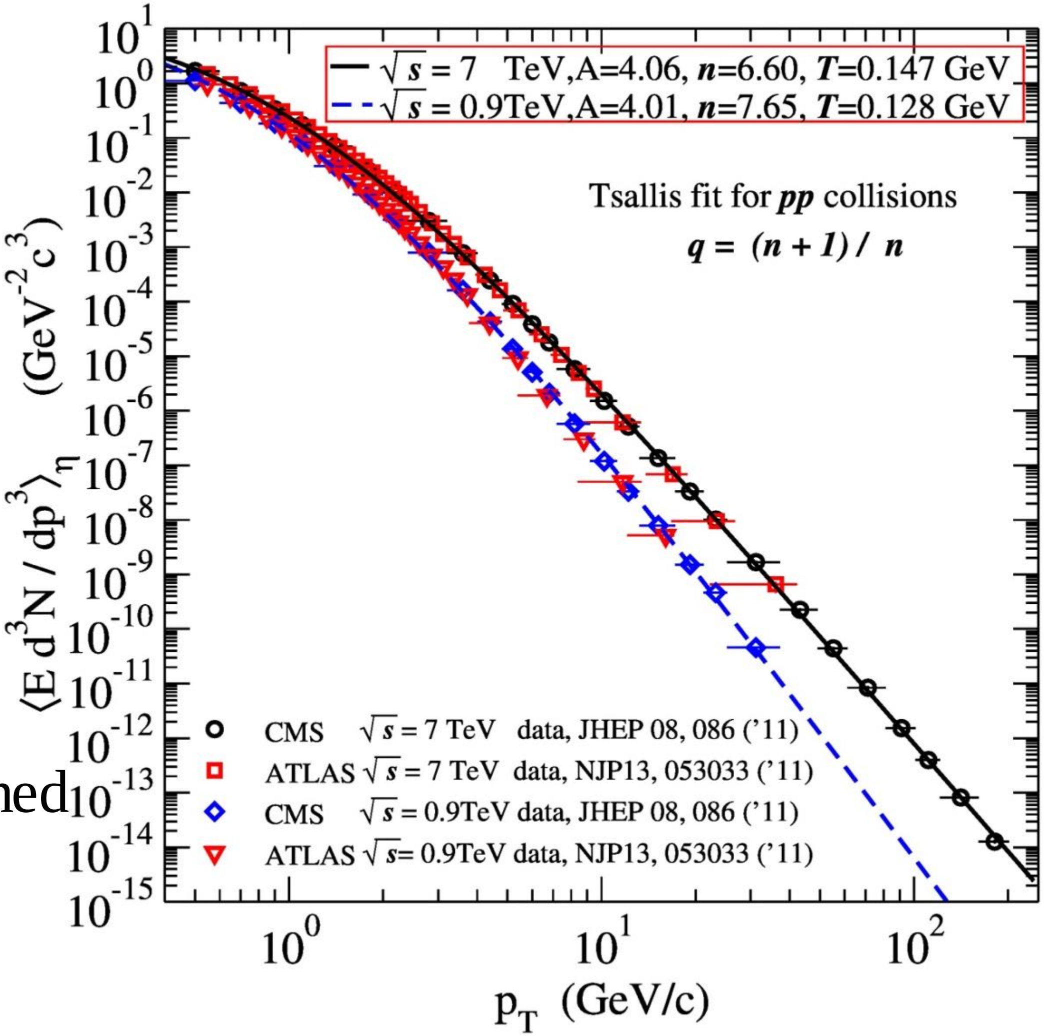
$$E\frac{d\sigma}{d^3p} = \frac{A}{\left(1+\frac{m_T - m}{nT}\right)^n}$$

$$m_T = \sqrt{m_\pi^2 + p_T^2}$$

can describe the hadron p_T spectra in pp collisions at LHC in the central rapdity region.

Good Tsallis fits have been obtained

$$\begin{cases} \text{for } \sqrt{s} = 7\text{ TeV}, \quad n = 6.60 \\ \text{for } \sqrt{s} = 0.9\text{ TeV}, n = 7.65 \end{cases}$$



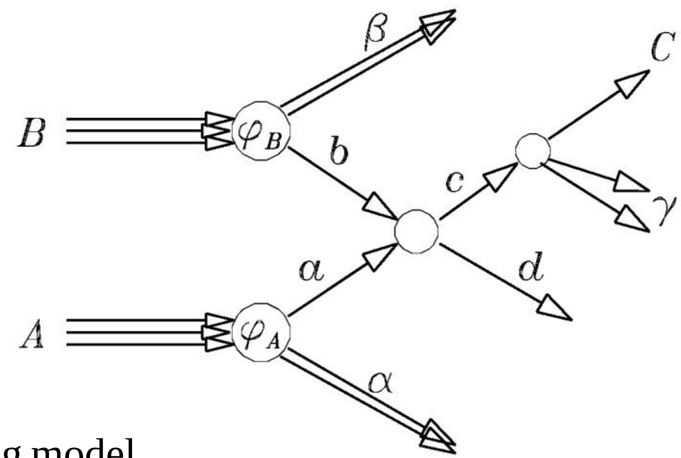
Wong and Wilk, ActaPhysPol.B43,2047(2012)

In terms of analytical expressions for the hard scattering integral, we provide evidences that

hadron production at $\eta \approx 0$ in high-energy pp and $\bar{p}p$ collisions is dominated by hard scattering over essentially the whole pT region

C. Y. Wong, G. Wilk, Acta Phys. Pol. B 43, 2047 (2012)
C. Y. Wong, G. Wilk, Phys. Rev. D87, 114007 (2013)
C. Y. Wong, G. Wilk, arXiv:1309.7330
C. Y. Wong, G. Wilk, L. J. L.Cirto, and C. Tsallis; EPJ Web of Conf.90, 04002 (2015)
L.J. L. Cirto, C. Tsallis, C.Y. Wong ,G. Wilk, arXiv:1409.3278
C. Y. Wong , G. Wilk, L.J.L.Cirto, C. Tsallis ,Phys. Rev. D91,114027 (2015)

Relativistic hard-scattering model



Cross section in the hard-scattering model

$$d\sigma(AB \to c\bar{c}X) = \sum_{ab} K_{ab} \int dx_a d\vec{a}_T dx_b d\vec{b}_T G_{a/A}(x_a, \vec{a}_T) G_{b/B}(x_b, \vec{b}_T) d\sigma(ab \to c\bar{c})$$

where K_{ab} is the K-factor.

$$d\sigma(ab \to c\bar{c}) = \frac{\hat{s}}{2\pi} \frac{d\sigma(ab \to c\bar{c})}{dt} \frac{d^3c}{E_c} \frac{d^3\bar{c}}{E_{\bar{c}}} \delta^4(a+b-c-\bar{c}),$$

Therefore,

$$\frac{E_c E_v d\sigma(AB \to c\bar{c}X)}{d^3c\, d^3\bar{c}} = \sum_{ab} K_{ab} \int dx_a d\vec{a}_T dx_b d\vec{b}_T G_{a/A}(x_a, \vec{a}_T) G_{b/B}(x_b, \vec{b}_T) \frac{\hat{s}}{2\pi} \frac{d\sigma(ab \to c\bar{c})}{dt} \delta^4(a+b-c-\bar{c}).$$

$$\frac{d\sigma(AB \to c\bar{c}X)}{dy_c d\vec{c}_T dy_v d\vec{v}_T} = \sum_{ab} K_{ab} \int dx_a d\vec{a}_T dx_b d\vec{b}_T G_{a/A}(x_a, \vec{a}_T) G_{b/B}(x_b, \vec{b}_T) \frac{\hat{s}}{2\pi} \frac{d\sigma(ab \to c\bar{c})}{dt} \delta^4(a+b-c-\bar{c}).$$

$$\hat{s} = (a+b)^2 = x_a x_b s, \qquad s = (A+B)^2.$$

Relativistic hard-scattering model

Assume $G_{a/A}(x_a,\vec{a}_T)=G_{a/A}(x_a)\dfrac{e^{-\vec{a}_T{}^2/2\sigma^2}}{2\pi\sigma^2}$ and weak dependence of $d\sigma(ab\to c\bar{c})/dt$ on $\vec{a}_T$ and $\vec{b}_T$, then

$$\int d\vec{a}_T d\vec{b}_T \frac{e^{-\frac{\vec{a}_T^2+\vec{b}_T^2}{2\sigma^2}}}{(2\pi\sigma^2)^2}\delta^2(\vec{a}_T+\vec{b}_T-\vec{c}_T-(\vec{\bar{c}})_T)=\frac{e^{-\frac{(\vec{c}_T+(\vec{\bar{c}})_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_v d\vec{\bar{c}}_T}=\sum_{ab}K_{ab}\int dx_a dx_b G_{a/A}(x_a)G_{b/B}(x_b)\frac{\hat{s}}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\frac{e^{-\frac{(\vec{c}_T+\vec{\bar{c}}_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}\delta(a_0+b_0-c_0-\bar{c}_0)\delta(a_z+b_z-c_z-\bar{c}_z)$$

$$\delta(a_0+b_0-c_0-\bar{c}_0)\delta(a_z+b_z-c_z-\bar{c}_z)=\frac{1}{s}\delta(x_a-x_{a0})\delta(x_b-x_{b0}),$$

$$s=(A+B)^2\,,\qquad \hat{s}=(a+b)^2,\qquad \hat{s}=x_a x_b s$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_{\bar{c}} d\vec{\bar{c}}_T}=\sum_{ab}K_{ab}\ [x_{a0}G_{a/A}(x_{a0})]\ [\,x_{b0}G_{b/B}(x_{b0})]\frac{1}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\frac{e^{-\frac{(\vec{c}_T+(\vec{\bar{c}})_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

$$x_{a0}G_{a/A}(x_{a0})\propto(1-x_{a0})^{g_a}\,,\qquad x_{b0}G_{b/B}(x_{b0})\propto(1-x_{b0})^{g_b}\,,$$

$$x_{a0}=\frac{m_{cT}}{\sqrt{s}}2\ \cosh\bar{y}\ \exp\{\Delta y/2\},\qquad x_{b0}=\frac{m_{cT}}{\sqrt{s}}2\ \cosh\bar{y}\ \exp\{-\Delta y/2\}$$

Relativistic hard-scattering model

$$\delta(a_0+b_0-c_0-\bar{c}_0)\delta(a_z+b_z-c_z-\bar{c}_z)=\frac{1}{\hat{s}}\delta(x_a-x_{a0})\delta(x_b-x_{b0})$$

$$x_{a0}=\frac{m_{cT}}{\sqrt{s}}2(\cosh Y)\,e^{\Delta y/2},\qquad x_{b0}=\frac{m_{cT}}{\sqrt{s}}2(\cosh Y)\,e^{-\Delta y/2},$$

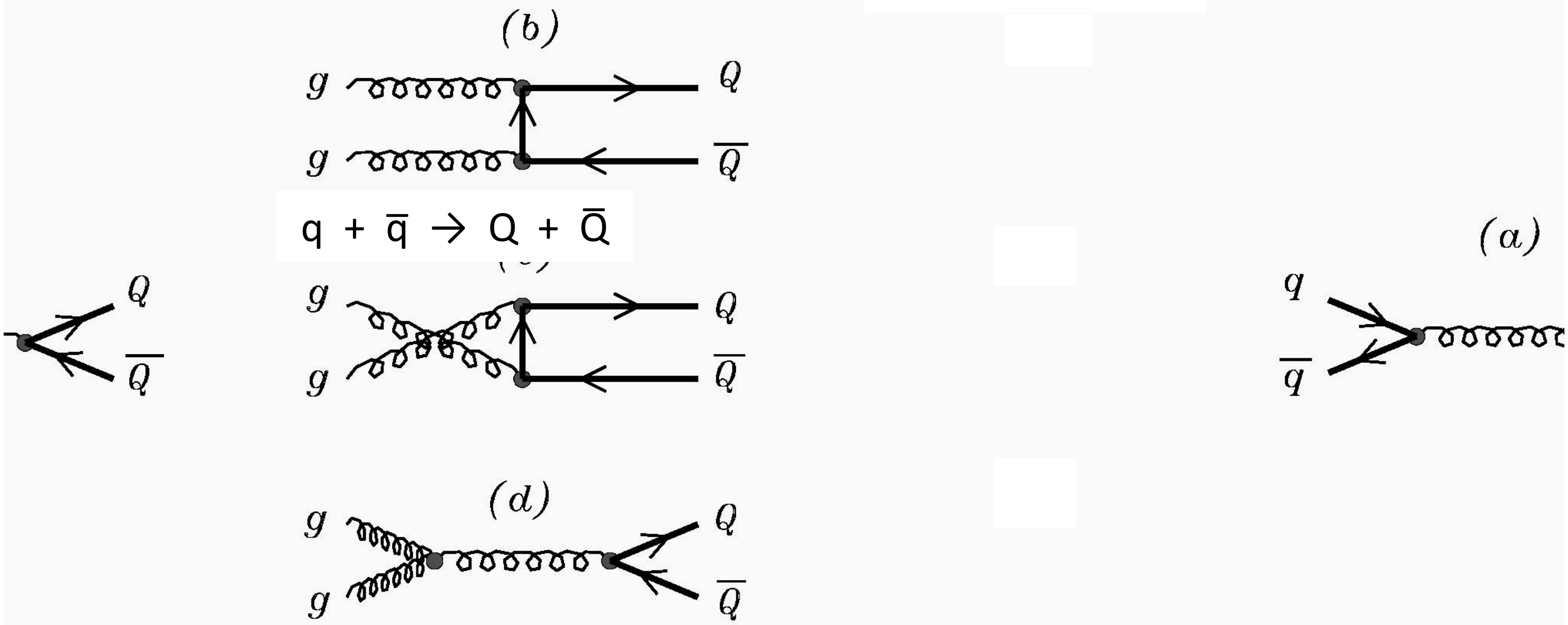
$$Y=\frac{y_c+y_{\bar{c}}}{2},\qquad \Delta y=y_c-y_{\bar{c}}$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_{\bar{c}} d\bar{c}_T}=\sum_{ab}K_{ab}x_{a0}G_{a/A}(x_{a0})x_{b0}G_{b/B}(x_{b0})\frac{1}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\bigg|_{x_{a0},x_{b0}}\frac{e^{-\frac{(\vec{c}_T+(\bar{c})_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_{\bar{c}} d\bar{c}_T}\propto K_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b}\frac{1}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\bigg|_{x_{a0},x_{b0}}\frac{e^{-\frac{(\vec{c}_T+(\bar{c})_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

Diagrams for heavy quark production

$g + \bar{g} \rightarrow Q + \bar{Q}$



$q + \bar{q} \rightarrow Q + \bar{Q}$

The dominant process is
$g + \bar{g} \rightarrow Q + \bar{Q}$

What is $d\sigma(gg \to c\bar{c})/dt$?

Gluck, Owen, Reya gives (PRD17,2324(1978))

$$\frac{d\sigma(gg \to Q\overline{Q})}{dt} = \frac{\pi\alpha^2}{64s^2}\left[12M_{ss} + \frac{16}{3}M_{tt} + \frac{16}{3}M_{uu} + 6M_{st} + 6M_{su} - \frac{2}{3}M_{tu}\right]$$

$$M_{ss} = \frac{4}{s^2}(t-M^2)(u-M^2)$$

$$M_{tt} = \frac{-2}{(t-M^2)^2}\left[4M^4 - (t-M^2)(u-M^2) + 2M^2(t-M^2)\right]$$

$$M_{uu} = \frac{-2}{(u-M^2)^2}\left[4M^4 - (u-M^2)(t-M^2) + 2M^2(u-M^2)\right]$$

$$M_{st} = \frac{4}{s(t-M^2)}\left[M^4 - t(s+t)\right]$$

$$M_{su} = \frac{4}{s(u-M^2)}\left[M^4 - u(s+u)\right]$$

$$M_{tu} = \frac{-4M^2}{(t-M^2)(u-M^2)}\left[4M^2 + (t-M^2) + (u-M^2)\right]$$

What is $\dfrac{d\sigma(gg \to c\bar{c})}{dt}$?

$$\frac{d\sigma(gg \to c\bar{c})}{dt} = \frac{\pi\alpha_s^2}{256\, m_{cT}^4 \cosh^4 \bar{y}} \left\{ \frac{12}{\cosh^2 \bar{y}} + \frac{64}{3}\cosh 2\bar{y} - 24 + \frac{m_c^2}{m_{cT}^2}\left(\frac{64}{3} + \frac{24\sinh^2 \bar{y}}{\cosh^2 \bar{y}} - \frac{8}{3} \right) + \frac{m_c^4}{m_{cT}^4}\left(-\frac{64}{3}\frac{\cosh 2\bar{y}}{\cosh^2 \bar{y}} + \frac{8}{3}\frac{1}{\cosh^2 \bar{y}} \right) \right\}$$

$\dfrac{d\sigma(gg \to c\bar{c})}{dt}$ depends on $m_c, m_{cT} = \sqrt{m_c^2 + c_T^2}$, and $\bar{y}$.

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_v d\bar{c}_T} = \sum_{ab} K_{ab} x_{a0} G_{a/A}(x_{a0}) x_{b0} G_{b/B}(x_{b0}) \frac{1}{2\pi} \frac{d\sigma(ab\to c\bar{c})}{dt}\bigg|_{x_{a0},x_{b0}} \frac{e^{-\frac{(\vec{c}_T+\bar{c}_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

Two particle correlation for the production of a $c\bar{c}$ pair :

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_v d\bar{c}_T} \sim AK_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b} e^{-\frac{(\vec{c}_T+(\bar{c})_T)^2}{4\sigma^2}}$$

$$\times \frac{\pi\alpha_s^2}{256\, m_{cT}^4 \cosh^4\bar{y}} \left\{ \frac{12}{\cosh^2\bar{y}} + \frac{64}{3}\cosh 2\bar{y} - 24 + \frac{m_c^2}{m_{cT}^2}\left(\frac{64}{3} + \frac{24\sinh^2\bar{y}}{\cosh^2\bar{y}} - \frac{8}{3} \right) + \frac{m_c^4}{m_{cT}^4}\left(-\frac{64}{3}\frac{\cosh 2\bar{y}}{\cosh^2\bar{y}} + \frac{8}{3}\frac{1}{\cosh^2\bar{y}} \right) \right\}$$

$$x_{a0} = \frac{m_{cT}}{\sqrt{s}} 2(\cosh\bar{y})\, e^{\Delta y/2}, \qquad x_{b0} = \frac{m_{cT}}{\sqrt{s}} 2(\cosh\bar{y})\, e^{-\Delta y/2},$$

$$\bar{y} = \frac{y_c + y_v}{2}, \qquad \Delta y = y_c - y_d$$

This is the formula for the away-side ridge for the production of a c-c̄ pair.

C-C̄ Angular correlations

$$\frac{d\sigma(AB\to c\bar{c}X)}{dy_c d\vec{c}_T dy_v d\vec{\bar{c}}_T}=\sum_{ab}K_{ab}x_{a0}G_{a/A}(x_{a0})x_{b0}G_{b/B}(x_{b0})\frac{1}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\bigg|_{x_{a0},x_{b0}}\frac{e^{-\frac{(\vec{c}_T+\vec{\bar{c}}_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}$$

Change variables: $(y_c, y_d)\to(\bar{y},\Delta y)$

$$\bar{y}=\frac{y_c+y_d}{2},\qquad \Delta y=y_c-y_d$$

$$y_c=\bar{y}+\frac{\Delta y}{2},\qquad y_d=\bar{y}-\frac{\Delta y}{2},$$

$$d\vec{c}_T=c_T dc_T d\phi_c,\qquad d\vec{\bar{c}}_T=\bar{c}_T d\bar{c}_T d\phi_{\bar{c}}$$

$$d\phi_c d\phi_{\bar{c}}=d\phi_c d\Delta\phi,\qquad \Delta\phi=\phi_{\bar{c}}-\phi_c$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{d\bar{y}\,d\Delta y\,c_T d\vec{c}_T\,\bar{c}_T d\vec{\bar{c}}_T}\sim AK_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b}\frac{1}{2\pi}\frac{d\sigma(ab\to c\bar{c})}{dt}\bigg|_{x_{a0},x_{b0}} e^{-\frac{(\vec{c}_T+(\vec{\bar{c}})_T)^2}{4\sigma^2}}$$

$$\frac{d\sigma(AB\to c\bar{c}X)}{d\Delta\phi\,d\Delta y}\sim A\int d\bar{y}\,c_T dc_T d\phi_c\,\bar{c}_T d\bar{c}_T K_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b}e^{-\frac{(\vec{c}_T+(\vec{\bar{c}})_T)^2}{4\sigma^2}}$$

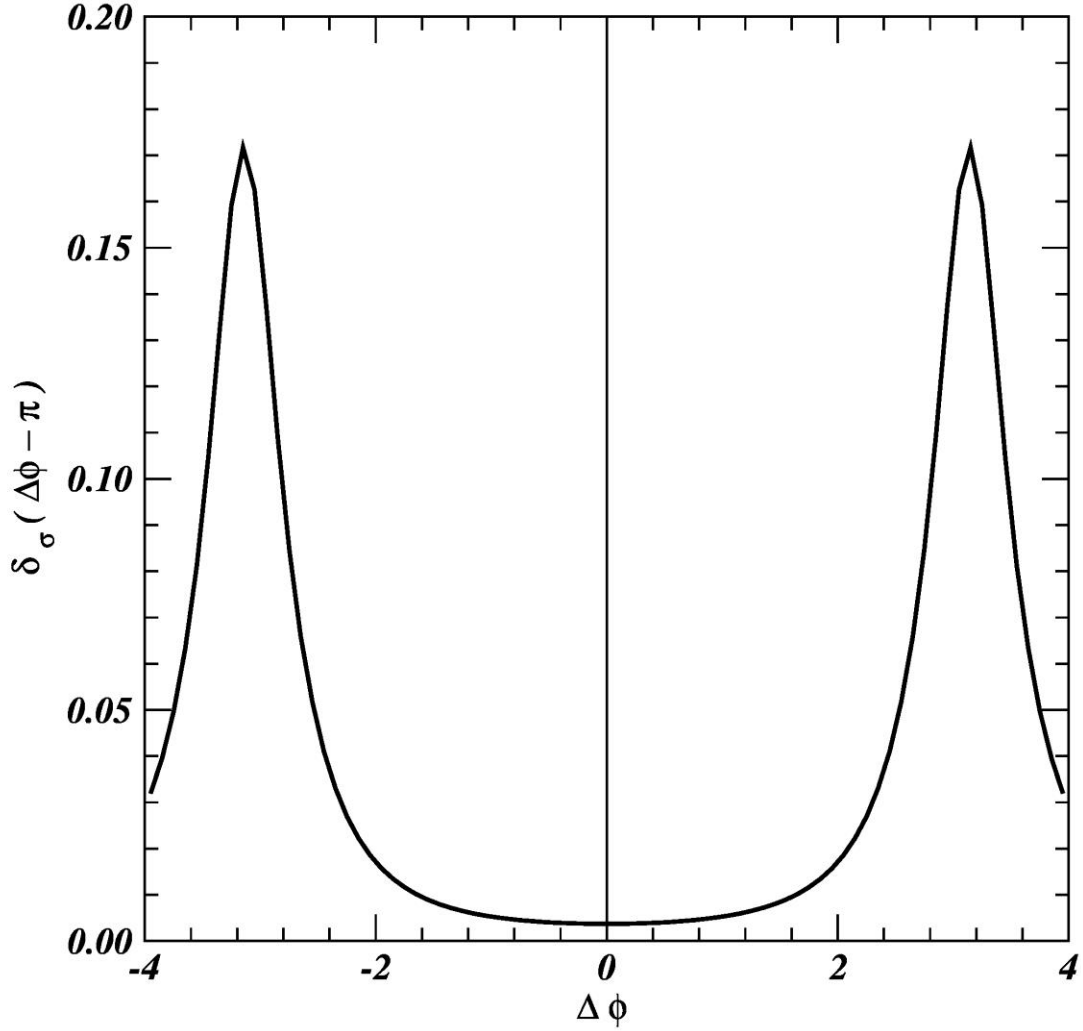
$$\times\frac{\pi\alpha^2}{256\,m_{cT}^4\cosh^4\bar{y}}\left\{\frac{12}{\cosh^2\bar{y}}+\frac{64}{3}\cosh 2\bar{y}-24+\frac{m_c^2}{m_{cT}^2}\left(\frac{64}{3}+\frac{24\sinh^2\bar{y}}{\cosh^2\bar{y}}-\frac{8}{3}\right)+\frac{m_c^4}{m_{cT}^4}\left(-\frac{64}{3}\frac{\cosh 2\bar{y}}{\cosh^2\bar{y}}+\frac{8}{3}\frac{1}{\cosh^2\bar{y}}\right)\right\}$$

$$\sim A\int d\bar{y}\,c_T dc_T d\phi_c\,\bar{c}_T d\bar{c}_T K_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b}e^{-\frac{(\vec{c}_T+(\vec{\bar{c}})_T)^2}{4\sigma^2}}\frac{\pi\alpha^2}{256\,m_{cT}^4\cosh^4\bar{y}}\left\{\frac{12}{\cosh^2\bar{y}}+\frac{64}{3}\cosh 2\bar{y}-24+O\left(\frac{m_c^2}{m_{cT}^2}\right)\right\}$$

After the integration over $c_T, \phi_c, \bar{c}_T$

$$\frac{d\sigma(AB\to c\bar{c}X)}{d\Delta\phi\,d\Delta y}\sim A\,K_{ab}(1-x_{a0})^{g_a}(1-x_{b0})^{g_b}\delta_\sigma(\Delta\phi-\pi)\,d\bar{y}\,\frac{\pi\alpha^2}{256\cosh^4\bar{y}}\left\{\frac{12}{\cosh^2\bar{y}}+\frac{64}{3}\cosh 2\bar{y}-24+O\left(\frac{m_c^2}{m_{cT}^2}\right)\right\}$$

Back-to-back correlation with a Δy ridge on the away side at Δϕ ~ π



0.20
0.15
0.10
0.05
0.00
-4
-2
0
2
4
Δ φ
δ σ (Δφ − π)

Heavy-quark production cross section

$$\frac{E_c E_{\bar c} d\sigma(AB\to c\bar c X)}{d^3c\, d^3\bar c} = \sum_{ab} K_{ab}\int dx_a d\vec a_T dx_b d\vec b_T G_{a/A}(x_a,\vec a_T) G_{b/B}(x_b,\vec b_T)\frac{\hat s}{2\pi}\frac{d\sigma(ab\to c\bar c)}{dt}\delta^4(a+b-c-\bar c).$$

$$\frac{E_c d\sigma(AB\to cX)}{d^3c} = \sum_{ab} K_{ab}\int \frac{d^4\bar c}{E_{\bar c}} dx_a d\vec a_T dx_b d\vec b_T G_{a/A}(x_a,\vec a_T) G_{b/B}(x_b,\vec b_T)\frac{\hat s}{2\pi}\frac{d\sigma(ab\to c\bar c)}{dt}\delta^4(a+b-c-\bar c).$$

$$= \sum_{ab} K_{ab}\int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b)\frac{e^{-\frac{(\vec c_T+\vec{\bar c}_T)^2}{4\sigma^2}}}{2(2\pi\sigma^2)^2}\frac{\hat s}{\pi}\frac{d\sigma(ab\to c\bar c)}{dt}\delta(\hat s+\hat t+\hat u-m_a^2-m_b^2-m_c^2-m_{\bar c}^2)$$

The delta function can be tranformed into

$$\delta(\hat s+\hat t+\hat u-m_a^2-m_b^2-m_c^2-m_{\bar c}^2) = \frac{\delta(x_a-x_a(x_b))}{s\left(x_b-\dfrac{m_{cT}^2}{x_c s}\right)}$$

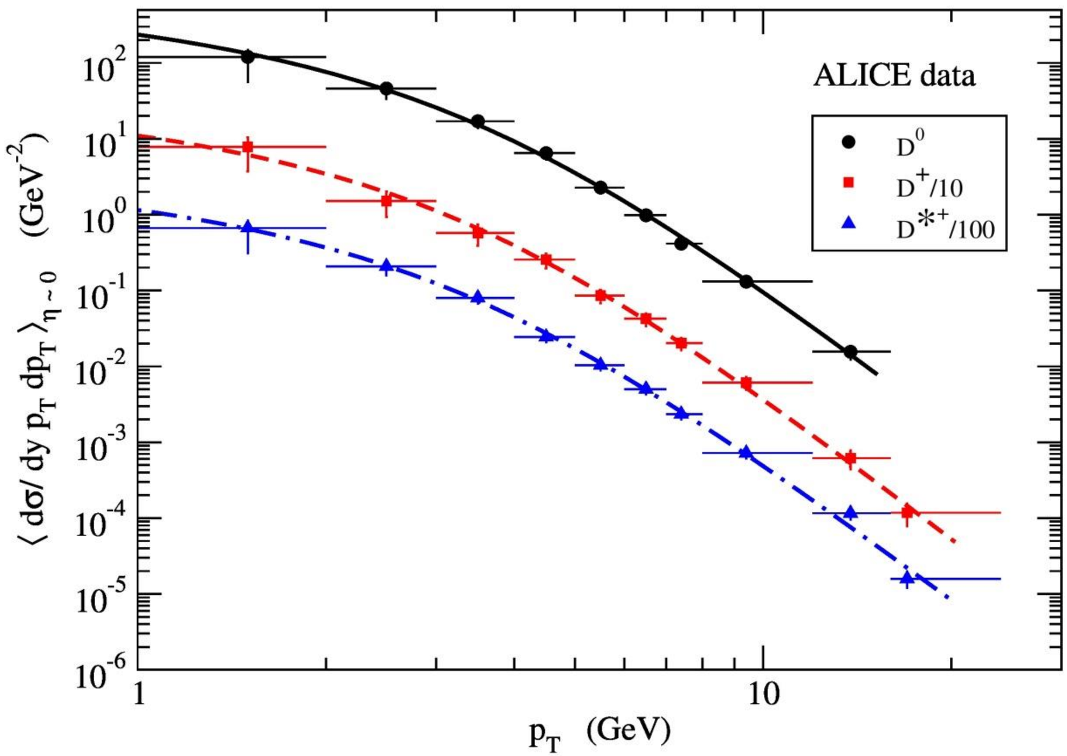
The other integral can be carried out by the saddle point method. We obtain at $y_c \approx 0$

$$\frac{E_c d\sigma(AB\to cX)}{d^3c} = AK_{ab}(1-x_{a0})^{g_a+1/2}(1-x_{b0})^{g_b+1/2}\frac{1}{\sqrt{x_c}}\frac{d\sigma(ab\to c\bar c)}{dt}$$

where $\quad x_{a0}=x_{b0}=2x_c, \qquad x_c = \dfrac{(m_c^2+c_T^2)^{1/2}}{\sqrt s}$

$$\frac{E_c d\sigma(AB\to cX)}{d^3c} \propto \frac{K_{ab}\alpha_s^2}{m_{cT}^4\cosh^4\bar y}\left\{\frac{12}{\cosh^2\bar y}+\frac{64}{3}\cosh 2\bar y-24+\frac{m_c^2}{m_{cT}^2}\left(\frac{64}{3}+\frac{24\sinh^2\bar y}{\cosh^2\bar y}-\frac{8}{3}\right)+\frac{m_c^4}{m_{cT}^4}\left(-\frac{64}{3}\frac{\cosh 2\bar y}{\cosh^2\bar y}+\frac{8}{3}\frac{1}{\cosh^2\bar y}\right)\right\}$$

The cross section is a mixture of $1/m_{cT}^4$, $1/m_{cT}^6$, $1/m_{cT}^8$.



$d\sigma/dy\, p_T dp_T = A/(1+p_T^2/m_0^2)^{n/2}$

	A	n	m_0(GeV)
D0	1600	5.8	3.5
D+	780	5.9	3.5
D*+	808	5.7	3.5

K-factor

$$K = \frac{2\pi\ f(v)}{1-\exp\{-2\pi\ f(v)\}}(1+(C_F\alpha_s)^2),$$

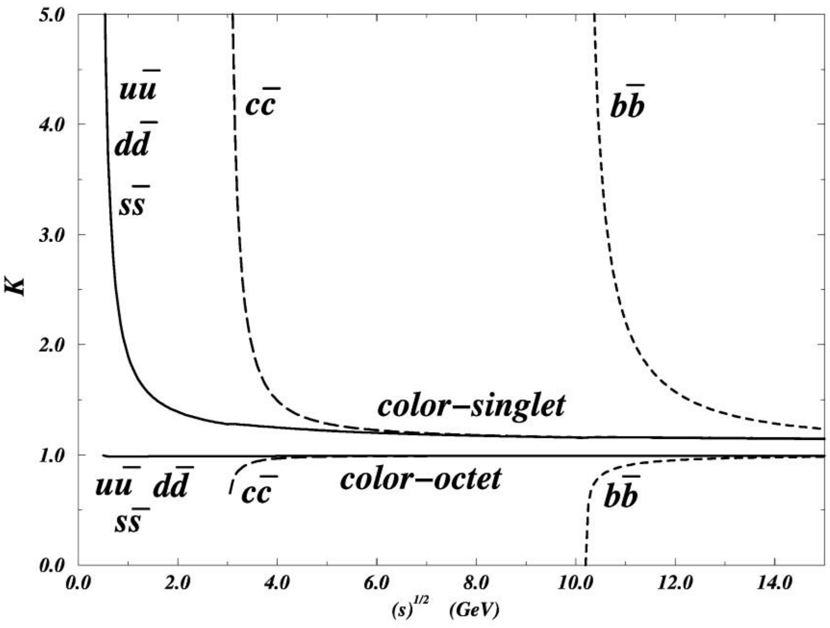
$$f(v) = C_F\alpha_s\left[\frac{1}{v} - v(1-\frac{4\pi}{3})\right]$$

$$C_F = \begin{cases} 4/3 & \text{for color singlet} \\ \text{-}1/6 & \text{for color octet} \end{cases}$$

For $c\bar{c}$ final-state interaction by gluon fusion

$$\frac{(\text{color}-\text{octet})}{(\text{color}-\text{singlet})} = \frac{(d^{abc})^2}{(\delta^{ab})^2} = \frac{5}{2}$$

$$K_{gg} = [5K(\text{octet}) + 2K(\text{singlet})]/2$$



Chatterjee+Wong, PRC51,2125(1995)

Conclusion

Analytical formulas have been obtained for various heavy-quark production cross sections. They will facilitate future comparisons and physical understanding of the production process.