Non-Extensive Statistical Approach for Hadronization and its Application

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Approach: Eur. Phys. J. A49 (2013) 110, Physica A 392 (2013) 3132

Application: J.Phys.CS 612 (2015) 012048 arXiv:1405.3963, 1501.02352, 1501.05959

SQM DUBNA 2015

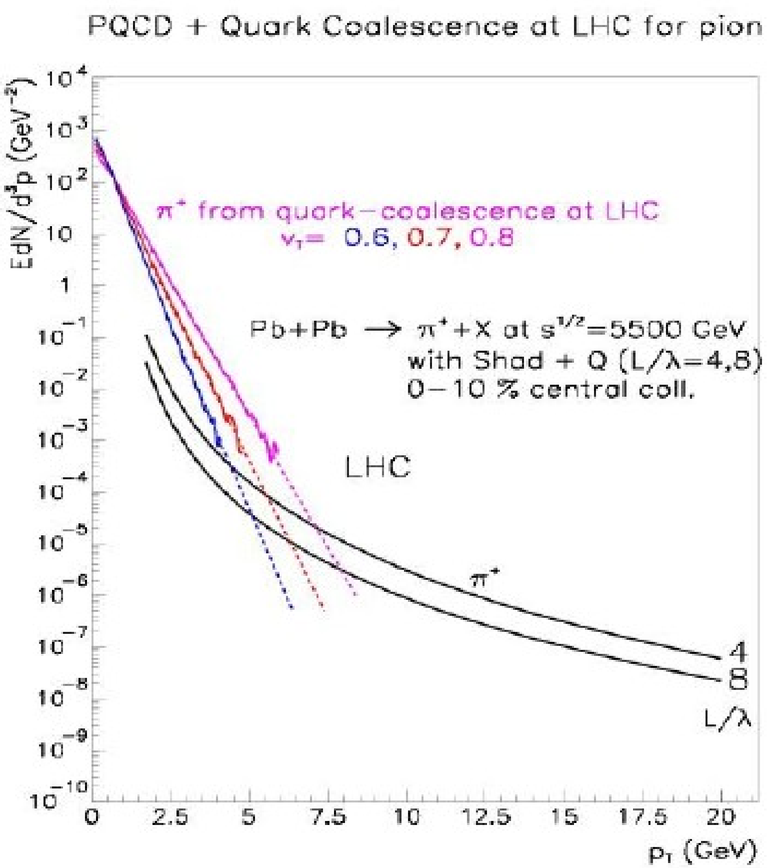
Strangeness in Quark Matter 2015, Dubna, Russia, 10th July 2015

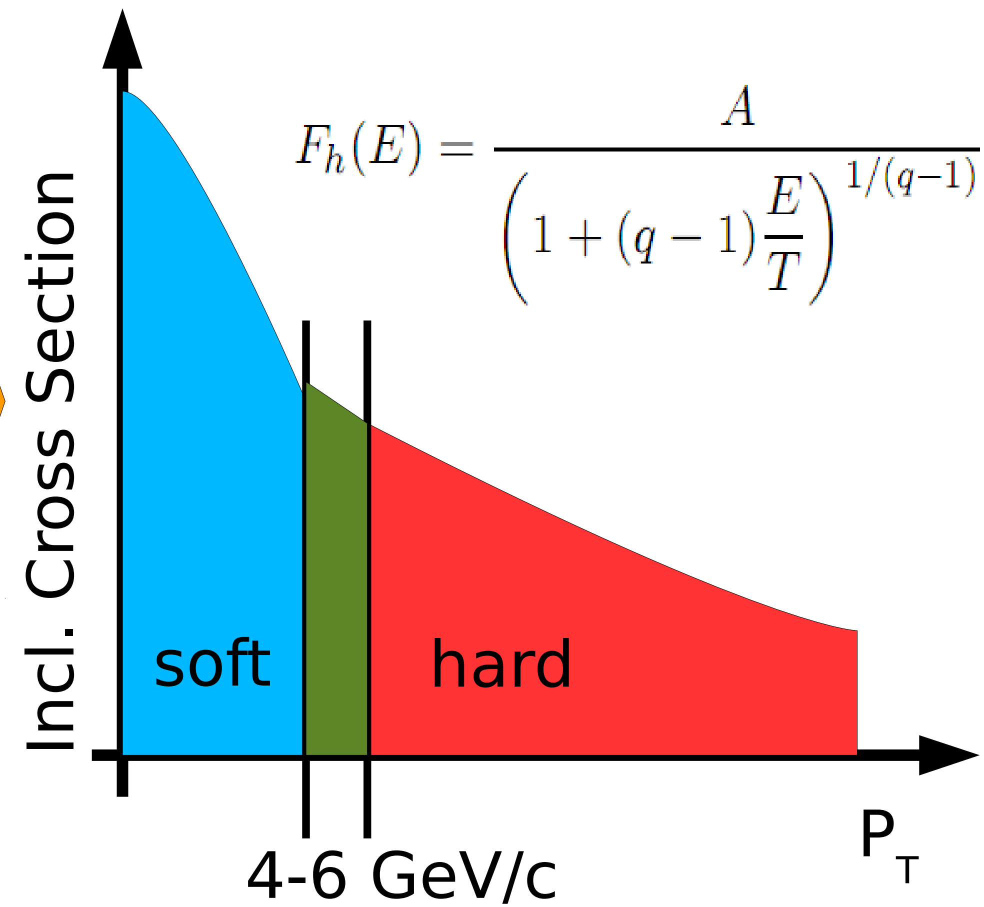
O U T L I N E

- Motivation...
  - by a student exercise
- Non-extensive statistical approach
  - Fits of experimental spectra from e^+e^-, pp
  - Non-extensive statistical approach
- Can Tsallis – Pareto fit spectra of HIC?
  - The soft+hard model and its applications
  - Spectra fit and extraction of q and T
  - Asimuthal anisotropy from the model

MOTIVATION

- Simplest and best fit to hadron spectra at low-p_T & high-p_T





P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

The student exercise...

- Why use Tsallis–Pareto distribution?
  - Is it true Boltzmann-Gibbs fits better at low momenta?
  - Is it true Power-law distribution is better at high momenta?
  - Is it true Tsallis – Pareto fits the whole mumentum range?
  - Can we apply this for any system: ee, pp, pA, AA?

- Let's see first a 'known' case:
  - PYTHIA6.4: π, K and p production in proton-proton @ 14 TeV
  - Fits of Boltzmann-Gibbs, Power law, and Tsallis–Pareto distributions
  - Low momenta: [1.2 GeV/c : 2.0 GeV/c] or [1.2GeV/c : 5.0 GeV/c]
  - High momenta: [5.0 GeV/c : 15.0 GeV/c]
  - Full range: [1.2 GeV/c : 15.0 GeV/c]

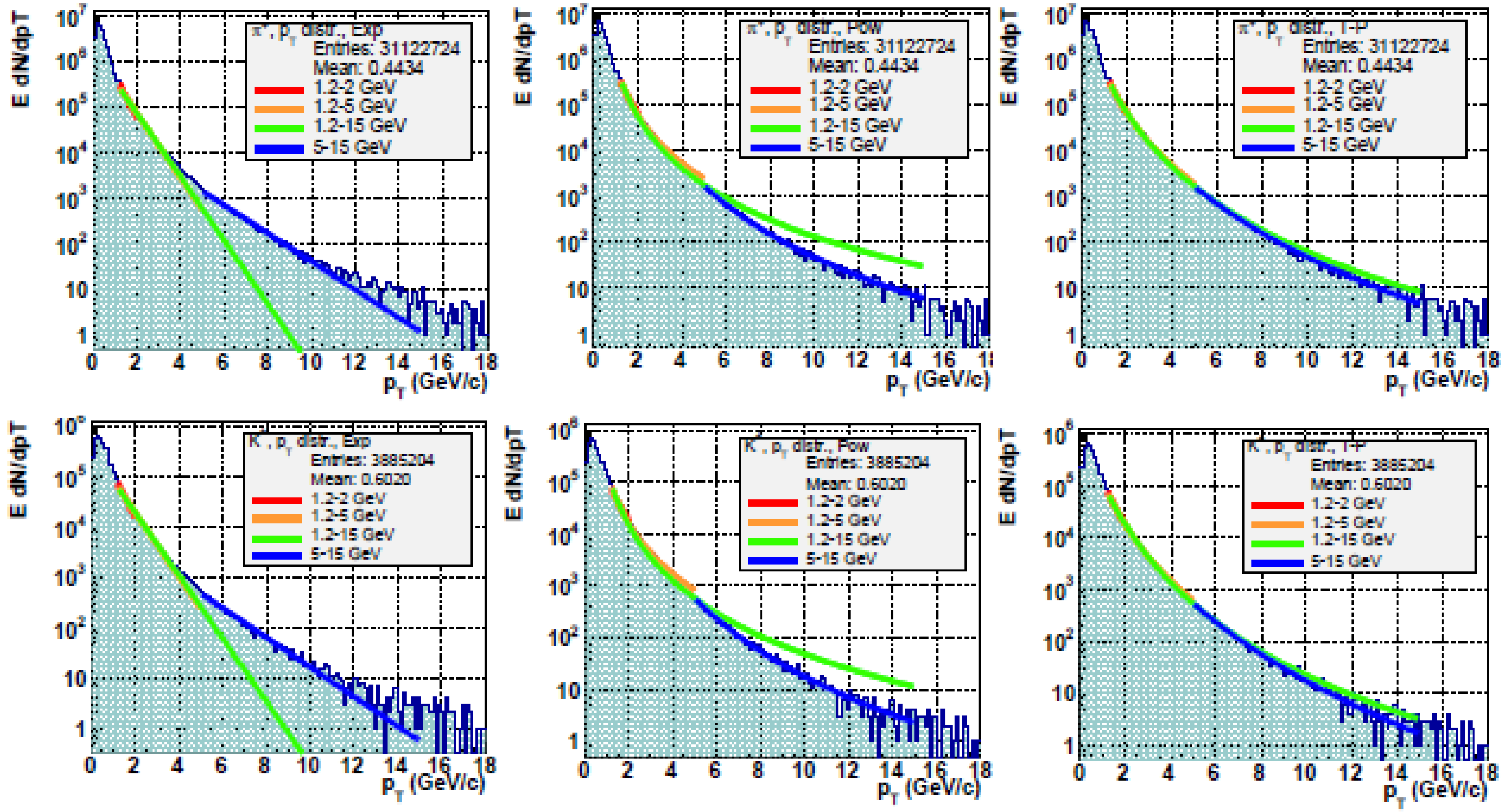
What can we learn form a simply exercise?

The student exercise...

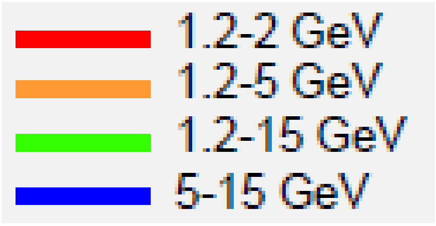
Boltzmann–Gibbs **Power Law** **Tsallis–Pareto**

Pions

Kaons



The fitted momentum regions:

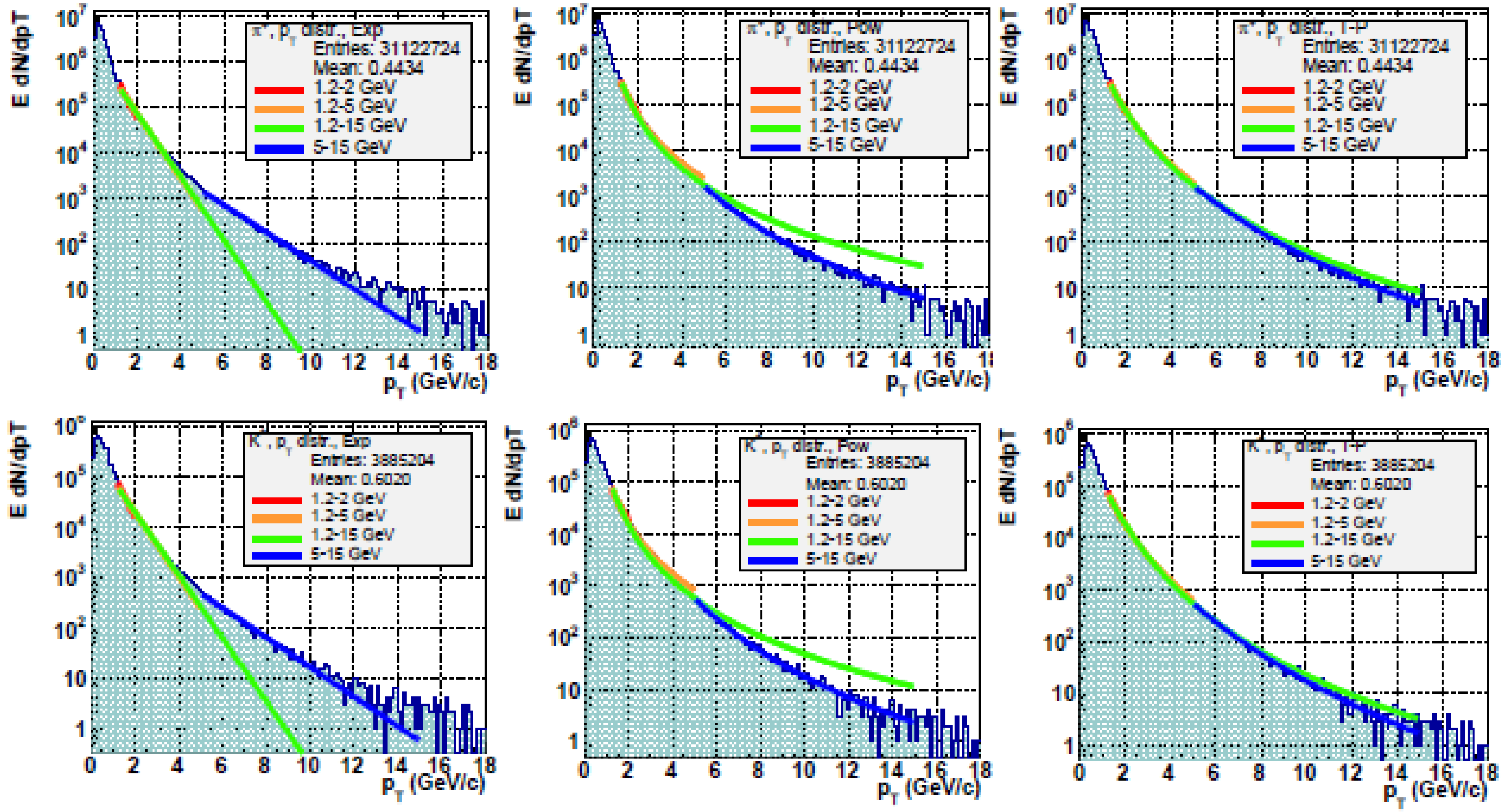


The student exercise...

Boltzmann–Gibbs **Power Law** **Tsallis–Pareto**

Pions

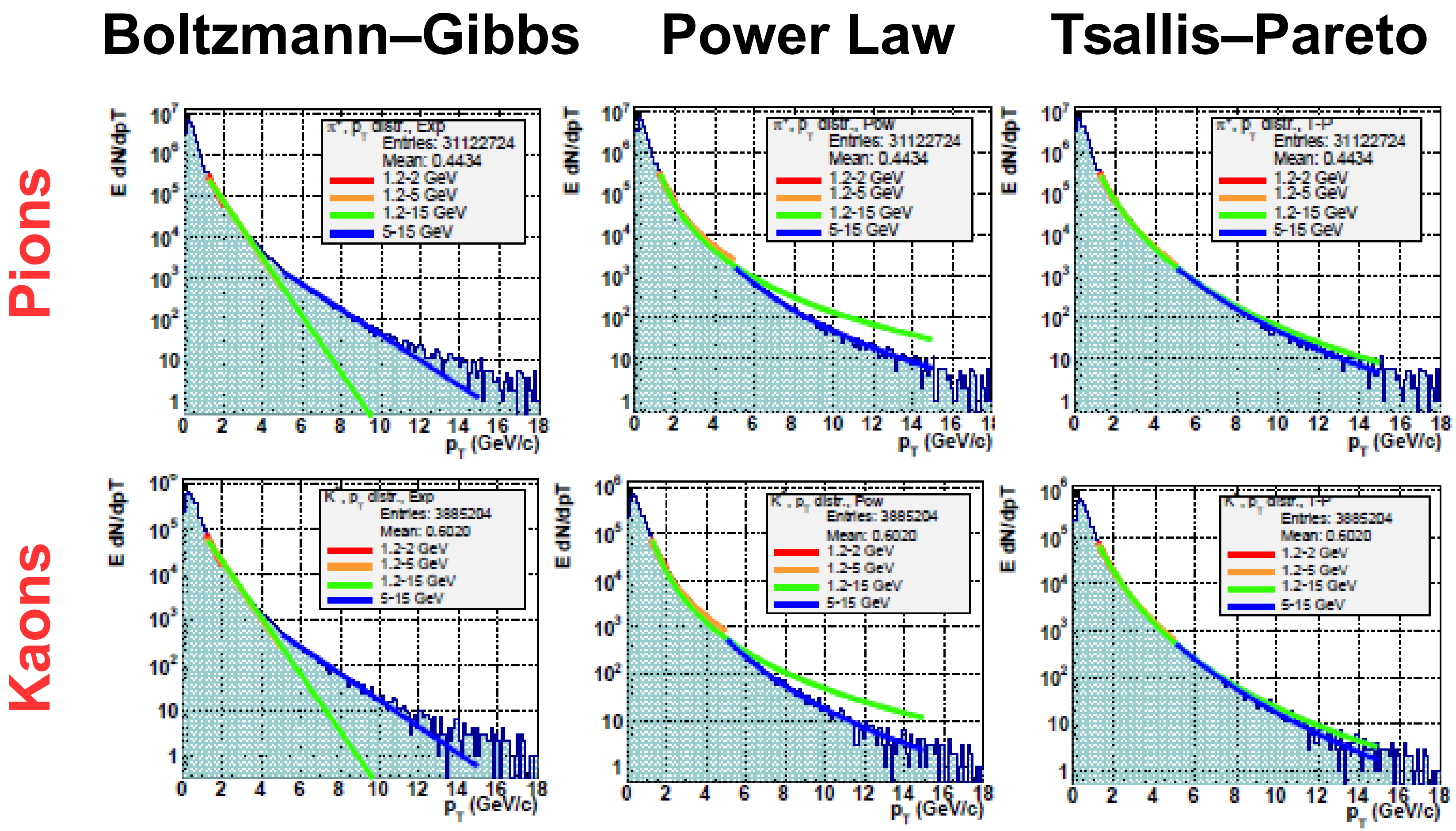
Kaons



χ2 values:

	[1,2:2] GeV/c	[1,2:5] GeV/c	[1,2:15] GeV/c	[5:15] GeV/c
Exp	112,37/29,81/27,34	623,89/130,48/109,26	254,12/61,71/48,13	3,01/1,44/1,45
Pow	1,71/0,98/0,47	161,27/55,68/56,08	214,12/76,92/77,26	1,37/1,144/0,91
TP	0,45/1,19/0,56	12,21/5,55/11,06	10,39/4,37/7,77	1,14/0,97/0,91

The student exercise...



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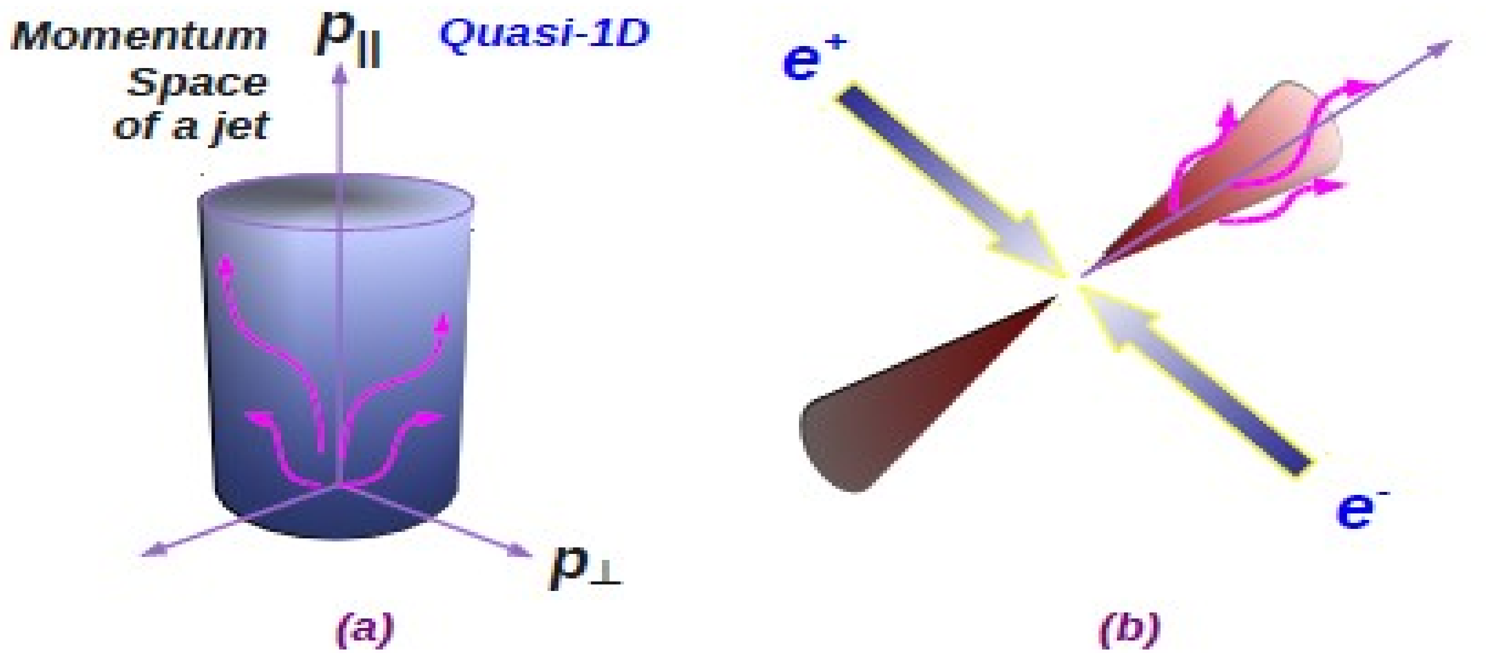
The student exercise...

- Why fit Tsallis–Pareto distribution?
  - Yes, it is true Boltzmann-Gibbs fits better at low momenta.
  - Yes, it is true Power-law distribution is better at high momenta.
  - Yes, it is true Tsallis – Pareto fits the whole mumentum range.
  - Can we apply this for any system: ee, pp, pA, AA?

- But carefully
  - BODY vs. TAIL (dependence on the momentom regions)
  - Need to find the proper variable E_{jet}, p_T, m_T, m_T^*
  - Need for
    - High-p_T PID hadron data
    - High statistic data
    - Spectra in several multiplicity bins
    - Dream: all of these on track-by-track basis

Application of the non-extensive statistical approach on small systems using experimental data.

The 'Thermodynamics of Jets'



K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

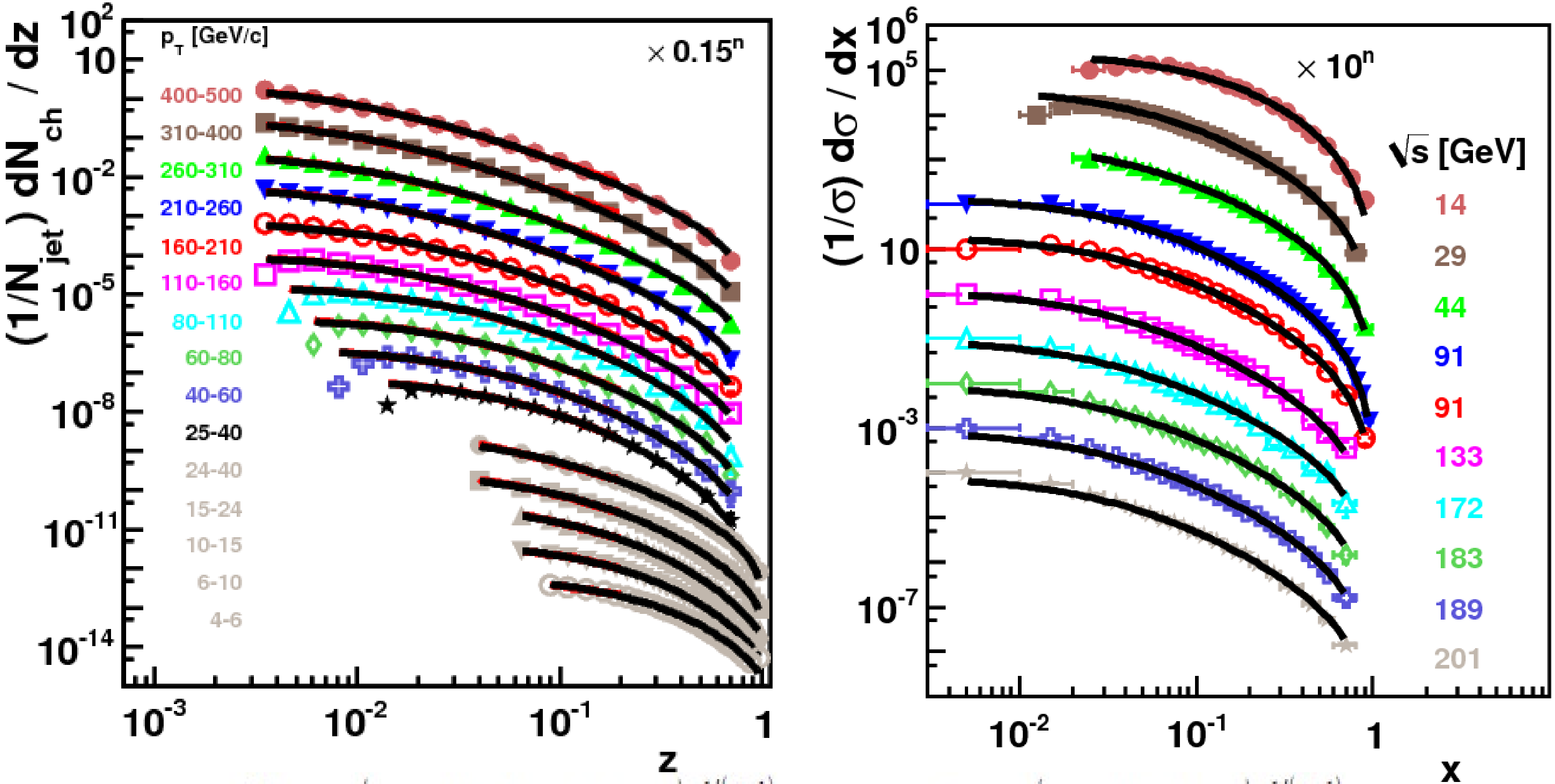
- Microcanonical Jet-Fragmentation in pp at LHC energies:

  Phys. Lett. B701 (2011) 111

- Generalized Tsallis distribution in e^+e^- collisons

  Phys. Lett. B718 (2012) 125

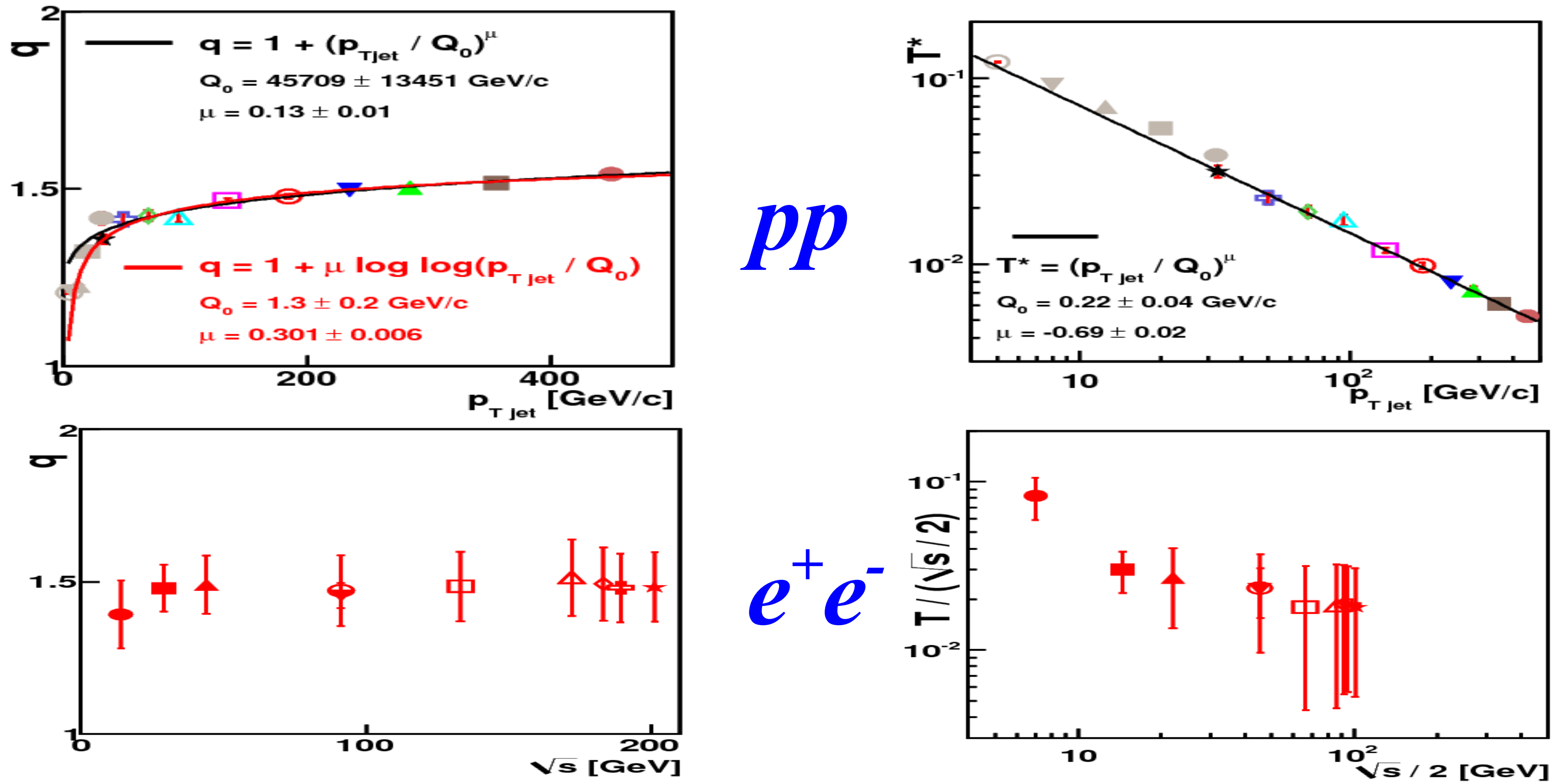
Fits for jet spectra in pp (left) and e^+e^- (right)



$$\frac{d\sigma}{dx} \propto \left(1 - \frac{q-1}{T/(\sqrt{s}/2)} \ln(1-x)\right)^{-1/(q-1)} \quad \Rightarrow \quad \left(1 + \frac{q-1}{T/(\sqrt{s}/2)} x\right)^{-1/(q-1)}$$

Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

The evolution of q and T parameters



K Ürmössy, GGB, TS Biró,
PLB 710 (2011) 111, PLB 718 (2012) 125.

- Energy dependence (hard)
  - Parameters *q* seem to saturate at high energies *q>1.1*
  - Parameter *T* is decreading with increasing energy

What is the physical meaning of these '*q*' and '*T*' parameters?

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The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2$$
$$E_{12} = E_2 + E_2$$

$$\Rightarrow \quad S_B = -\sum_i p_i \ln p_i$$

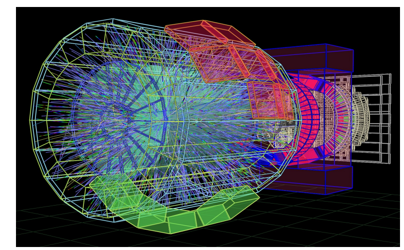
The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$\begin{aligned} S_{12} &= S_1 + S_2 \\ E_{12} &= E_2 + E_2 \end{aligned} \quad \Rightarrow \quad S_B = -\sum_i p_i \ln p_i$$

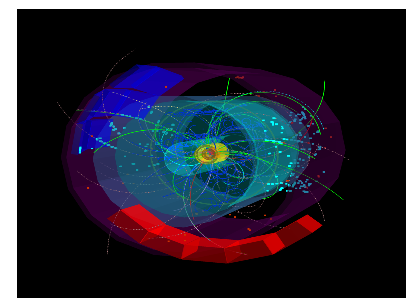
- Non-extensivity → generalized entropy

$$\begin{aligned} \hat{L}_{12}(S_{12}) &= \hat{L}_1(S_1) + \hat{L}_2(S_2), \\ L_{12}(E_{12}) &= L_1(E_1) + L_2(E_2) \end{aligned} \quad \Rightarrow \quad S_T = \frac{1}{1-q} \sum_i (p_i^q - p_i)$$



- Tsallis entropy

$$S_{12} = S_1 + S_2 + (q-1) S_1 S_2 \quad \Rightarrow \quad \hat{L}(S) = \frac{1}{q-1} \ln\left(1 + (q-1)S\right)$$



from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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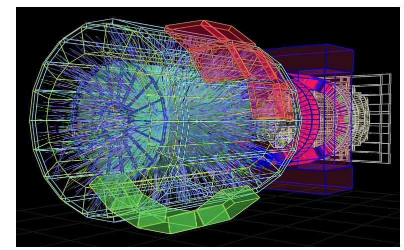
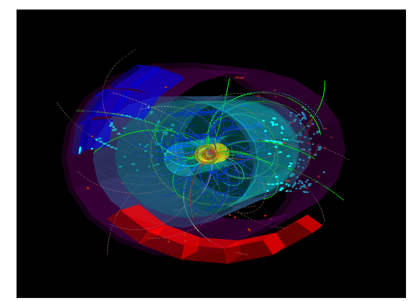
$$q = \frac{\langle S'(E)^2 + S''(E) \rangle}{\langle S'(E) \rangle^2}$$

$$\frac{1}{T} = \langle S'(E) \rangle$$





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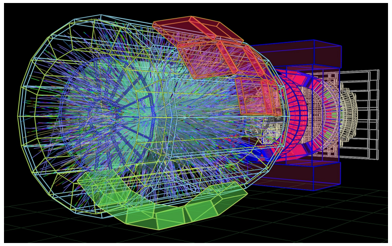
$$\frac{1}{T} = \langle S'(E)\rangle$$

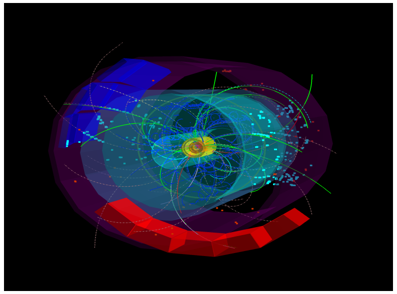
$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}.$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = \cdot \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$





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The non-extensive statistical approach

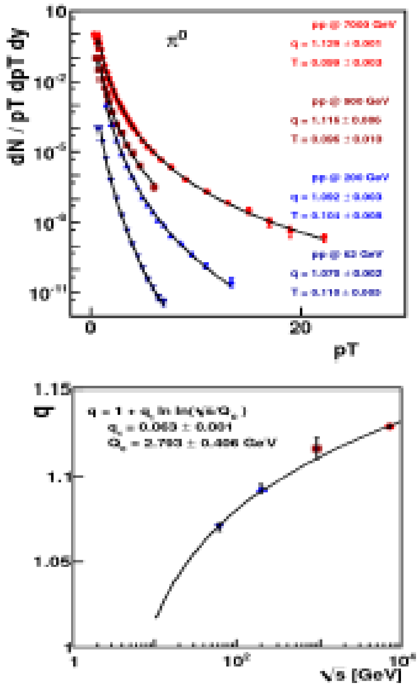
Hadron **spectra** in *pp* collisions can be described by the *Tsallis distribution*:

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

π **spectra** in *pp* collisions depends *similarly on √s* and on the multiplicity *N*

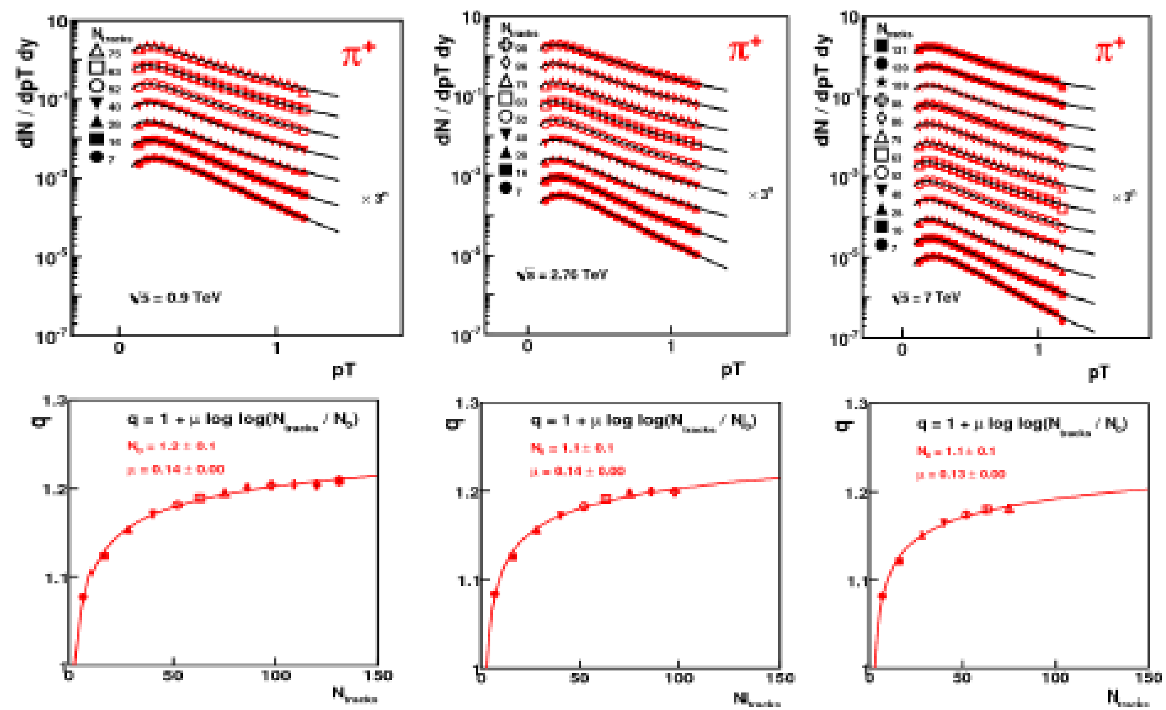
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

√s = fix



N = fix

arXiv:1405.3963, 1501.02352, 1501.05959



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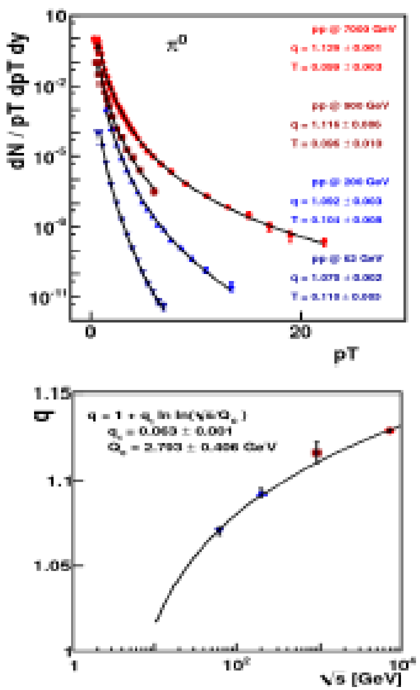

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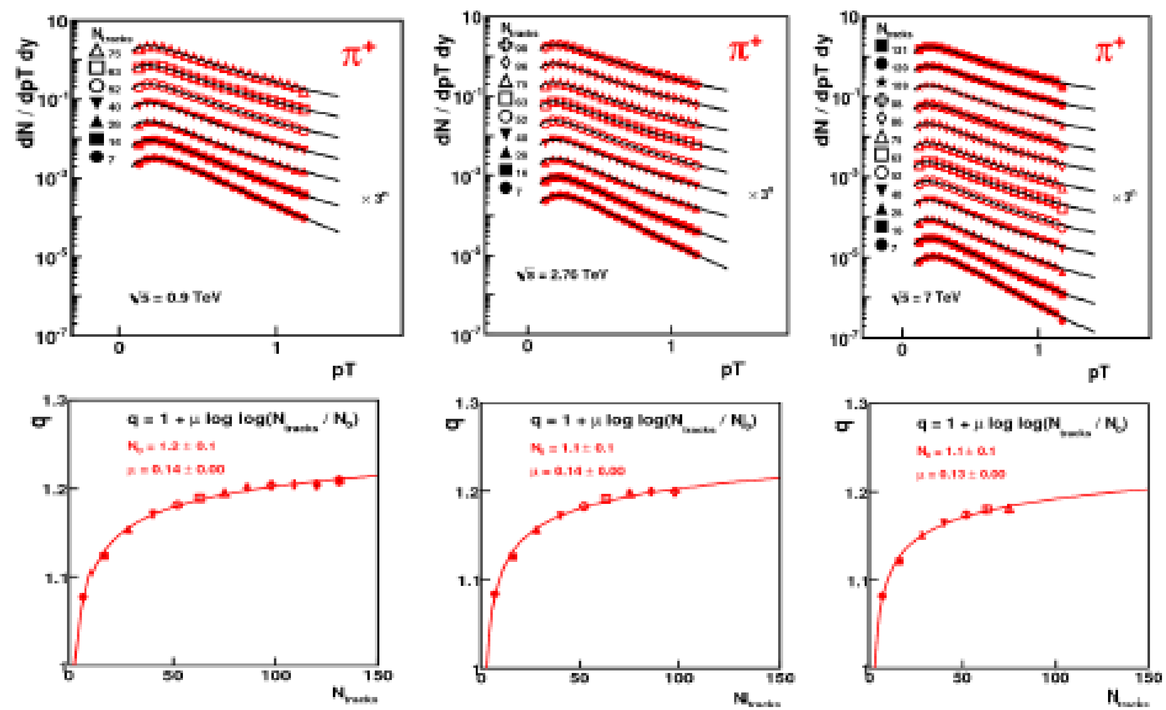
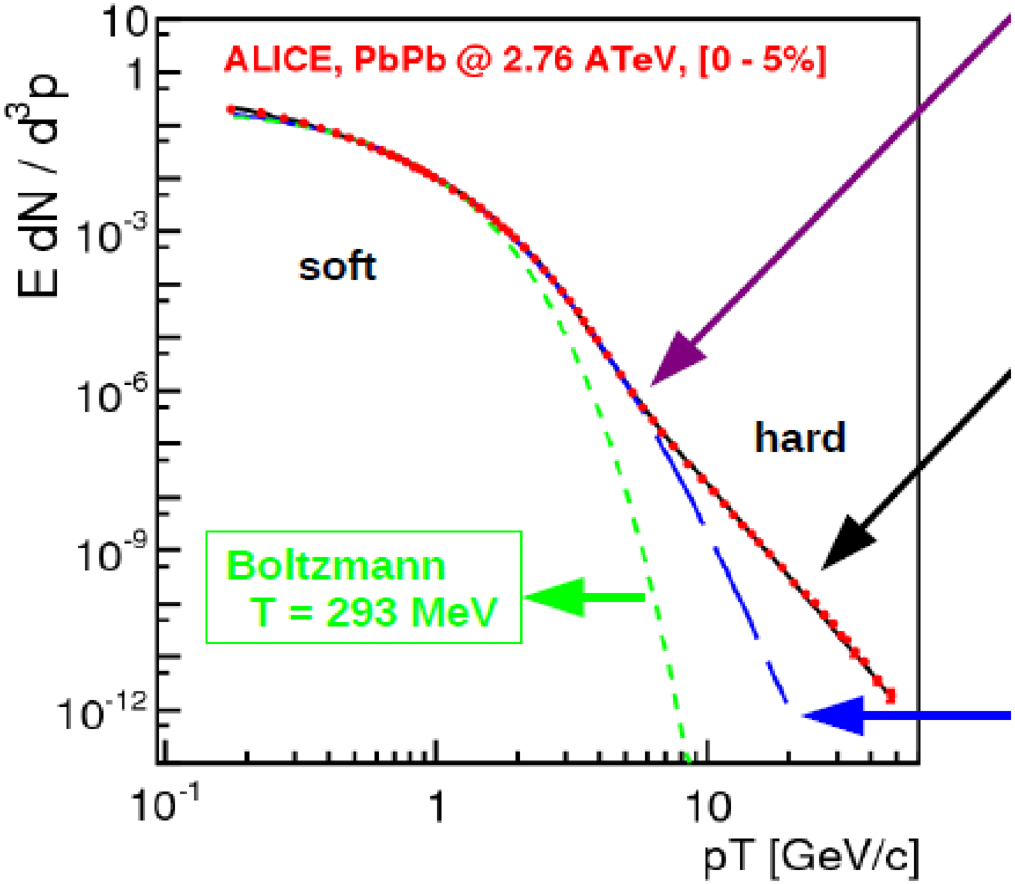


What if, we would apply this for
a bigger system (AA)
where
Boltzmann–Gibbs
use to work?

Test with real data in PbPb

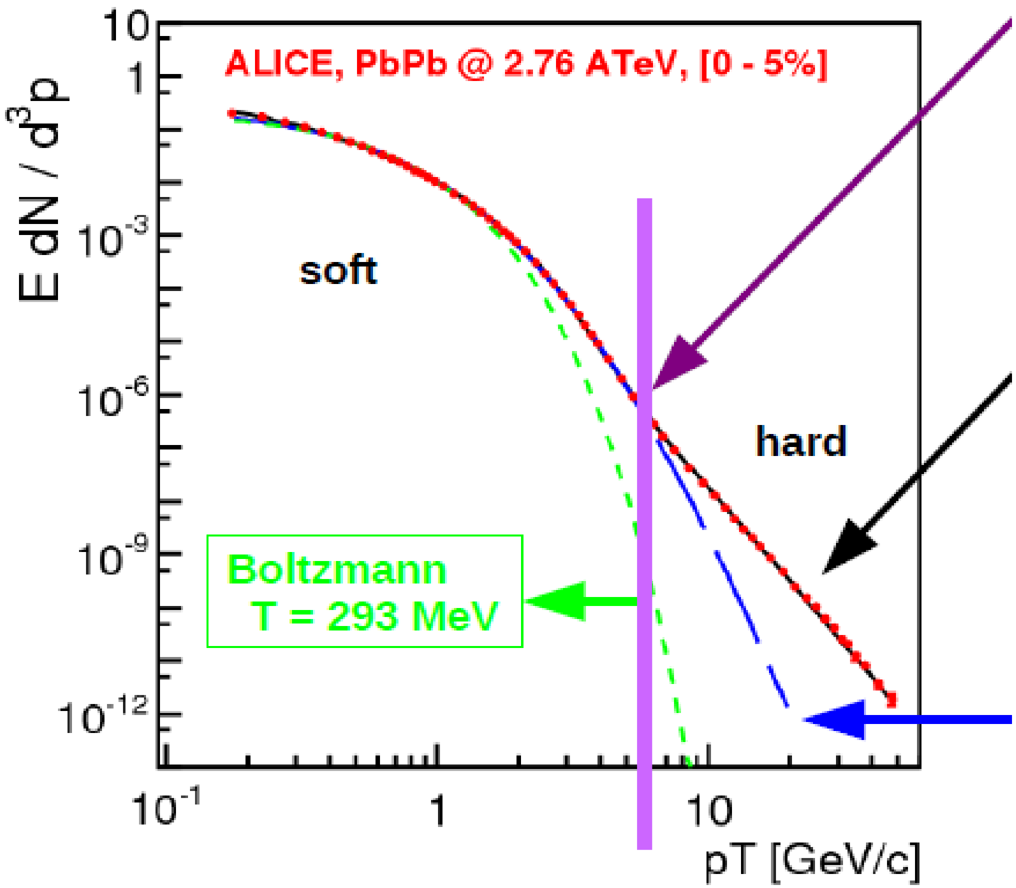


The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

Test with real data in PbPb



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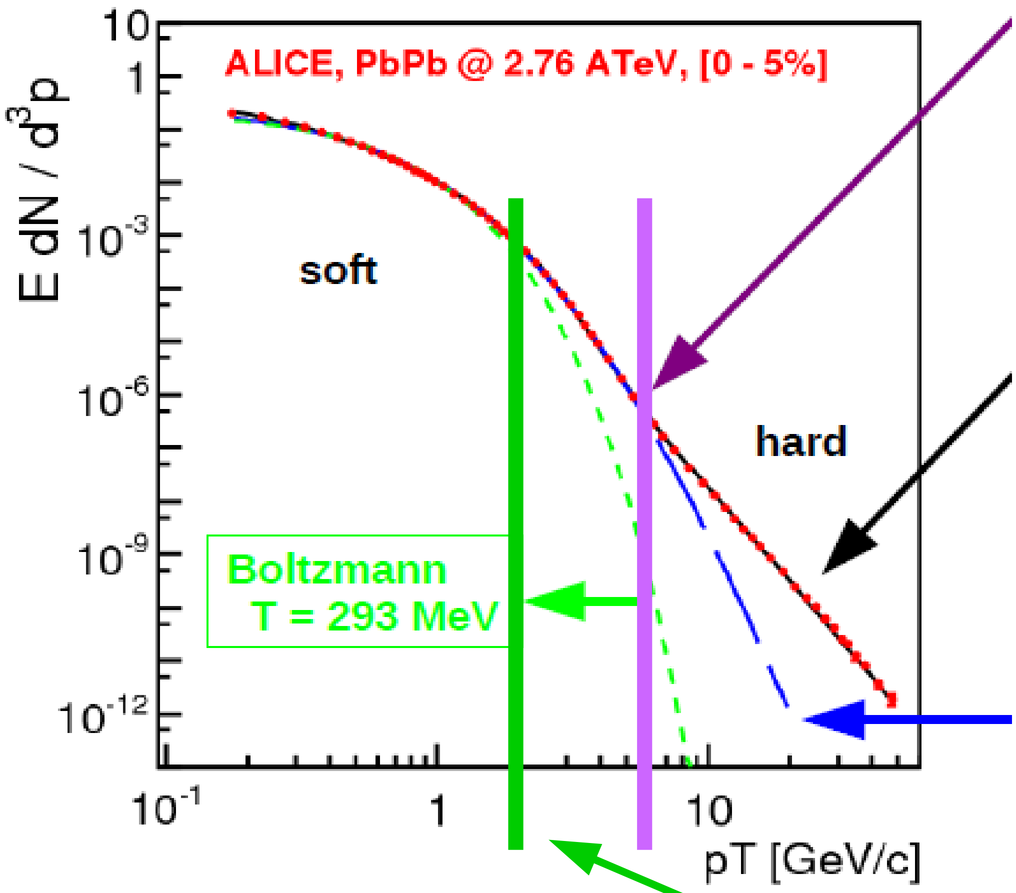

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Handling soft/hard regime with a new approach, using not only the temperature, T

The soft + hard model

- Simplest approximation: soft ('bulk') + hard ('jet') contribution

$$p^0 \frac{dN}{d^3\mathbf{p}} = p^0 \frac{dN}{d^3\mathbf{p}}^{\text{hard}} + p^0 \frac{dN}{d^3\mathbf{p}}^{\text{soft}}$$

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- Identified hadron spectra is given by double Tsallis–Pareto:

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft} \qquad f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i}[\gamma_i(m_T - v_i p_T) - m]\right]^{-1/(q_i-1)}$$

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in where parameters are given by

- Lorentz factor $\gamma_i = 1/\sqrt{1 - v_i^2}$
- Transverse mass $m_T = \sqrt{p_T^2 + m^2}$
- Doppler temperature $T_i^{Dopp} = T_i \sqrt{\frac{1 + v_i}{1 - v_i}}$

- Finally we assume N_{part} scaling for the parameters

$$q_i = q_{2,i} + \mu_i \ln(N_{part}/2)$$
$$T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part}) \ .$$

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Fit of pp and PbPb (centra/peripheral) data

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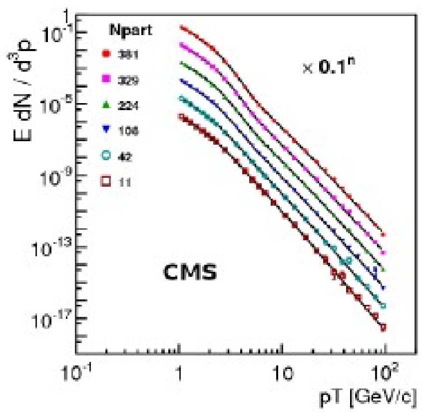

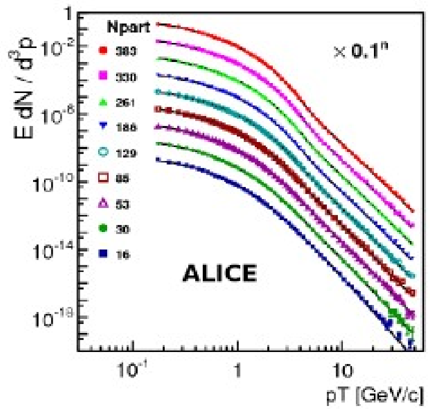

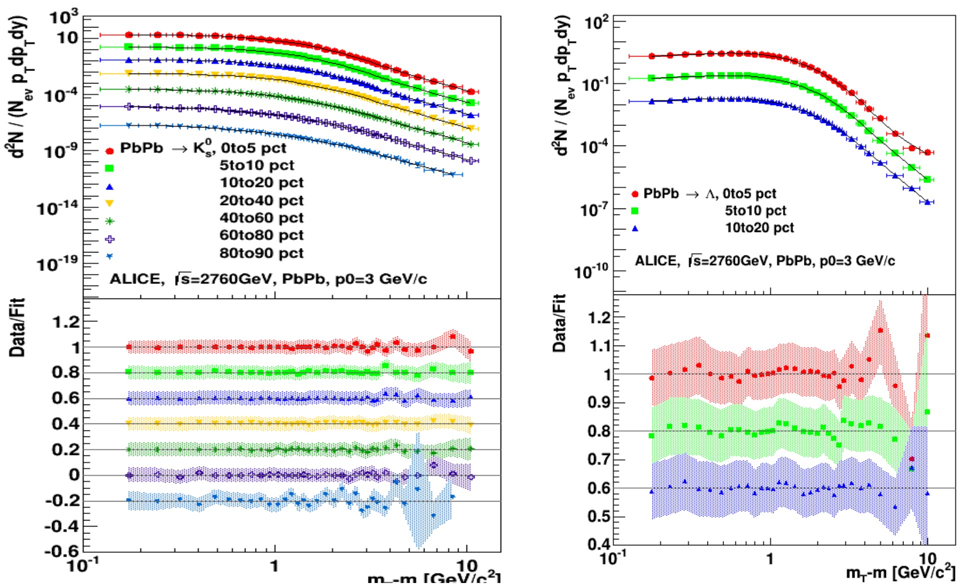

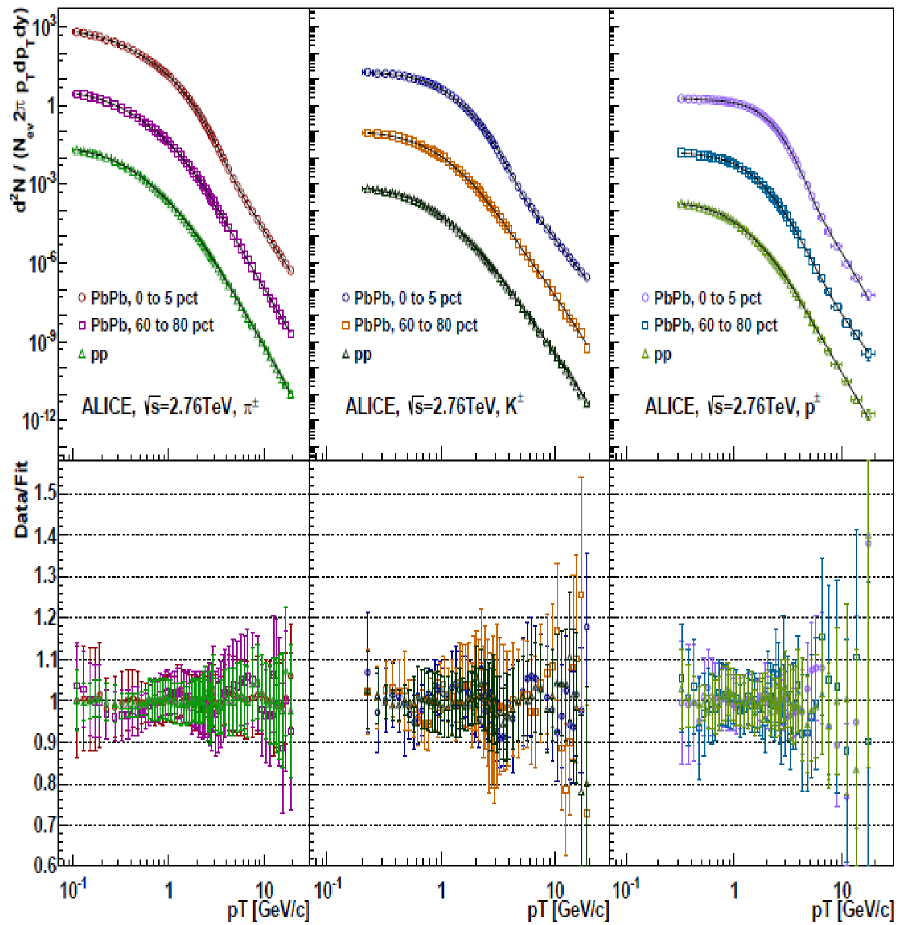

Parameters of the soft+hard model

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Parameters of the soft+hard model

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$$T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})$$

	$q_{2,soft}$	$q_{2,hard}$	μ_{soft}	μ_{hard}
CMS	1.058 ± 0.025	1.136 ± 0.001	-0.008 ± 0.005	0.005 ± 0.0003
ALICE	1.074 ± 0.018	1.131 ± 0.002	-0.009 ± 0.004	0.006 ± 0.0006
PHENIX	1.073 ± 0.016	1.100 ± 0.002	-0.005 ± 0.004	0.000 ± 0.0006

	T_1^{soft} [MeV]	T_1^{hard} [MeV]	τ_{soft} [MeV]	τ_{hard} [MeV]
CMS	310 ± 20	126 ± 5	9.9 ± 3.7	5.3 ± 0.8
ALICE	266 ± 16	194 ± 2	11.5 ± 2.9	-12.5 ± 0.5
PHENIX	165 ± 26	192 ± 20	9.3 ± 5.5	18.7 ± 4.6

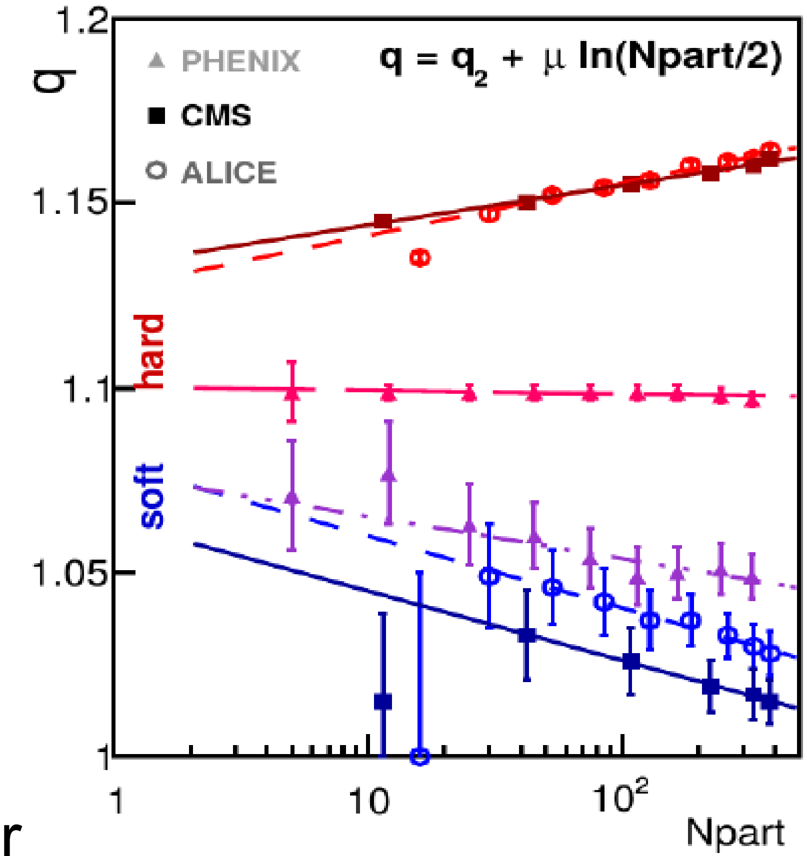
The N_{part} scaling of the q & T parameters

- Scaling of the $q_i = q_{2,i} + \mu_i \ln(N_{part}/2)$
  - Soft component, $q \to 1$
    - LHC: dereasing
    - RHIC: decreasing

    Higher N_{part} result BG statistics

  - Hard component, $q > 1.1$
    - LHC: slight increasing
    - RHIC: constant

    Without the soft part result clearer non-extensive behaviour, like e^+e^-



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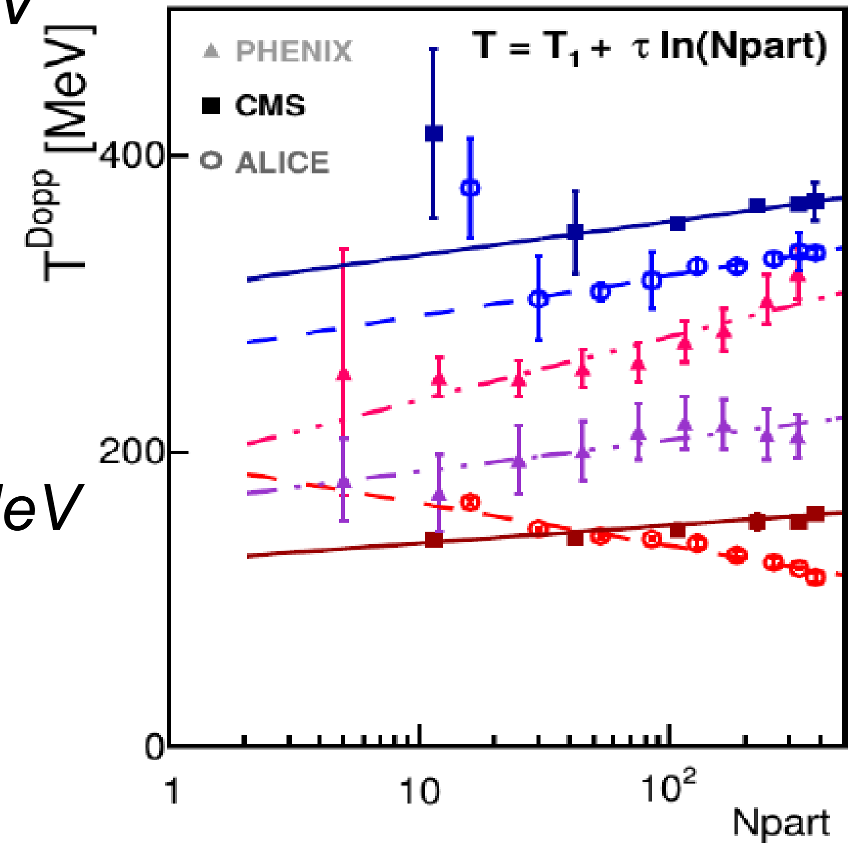
The N_{part} scaling of the *q* & *T* parameters

- Scaling of the $T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})$
  - Soft component, *T~200-400 MeV*
    - LHC: constant/increasing
    - RHIC: slightly increasing

    higher N_{part} results bit higher T
  - Hard component, *T ~100-300 MeV*
    - LHC: decreasing
    - RHIC: increasing

    N_{part} scalimg seems sensitive...



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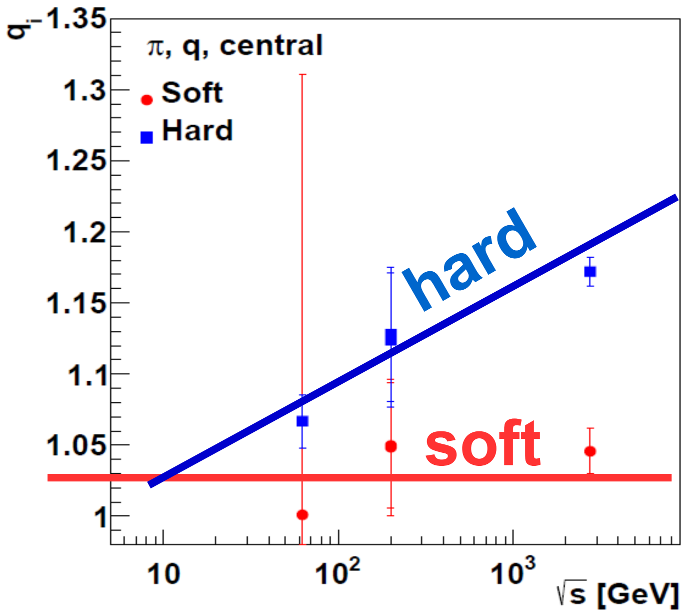
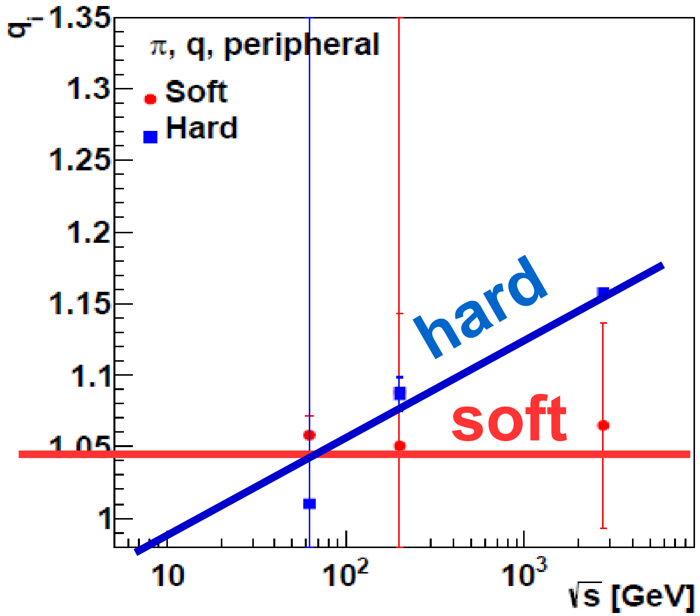
The c.m. energy dependence of *q* & *T*

q **measures non-extensivity**

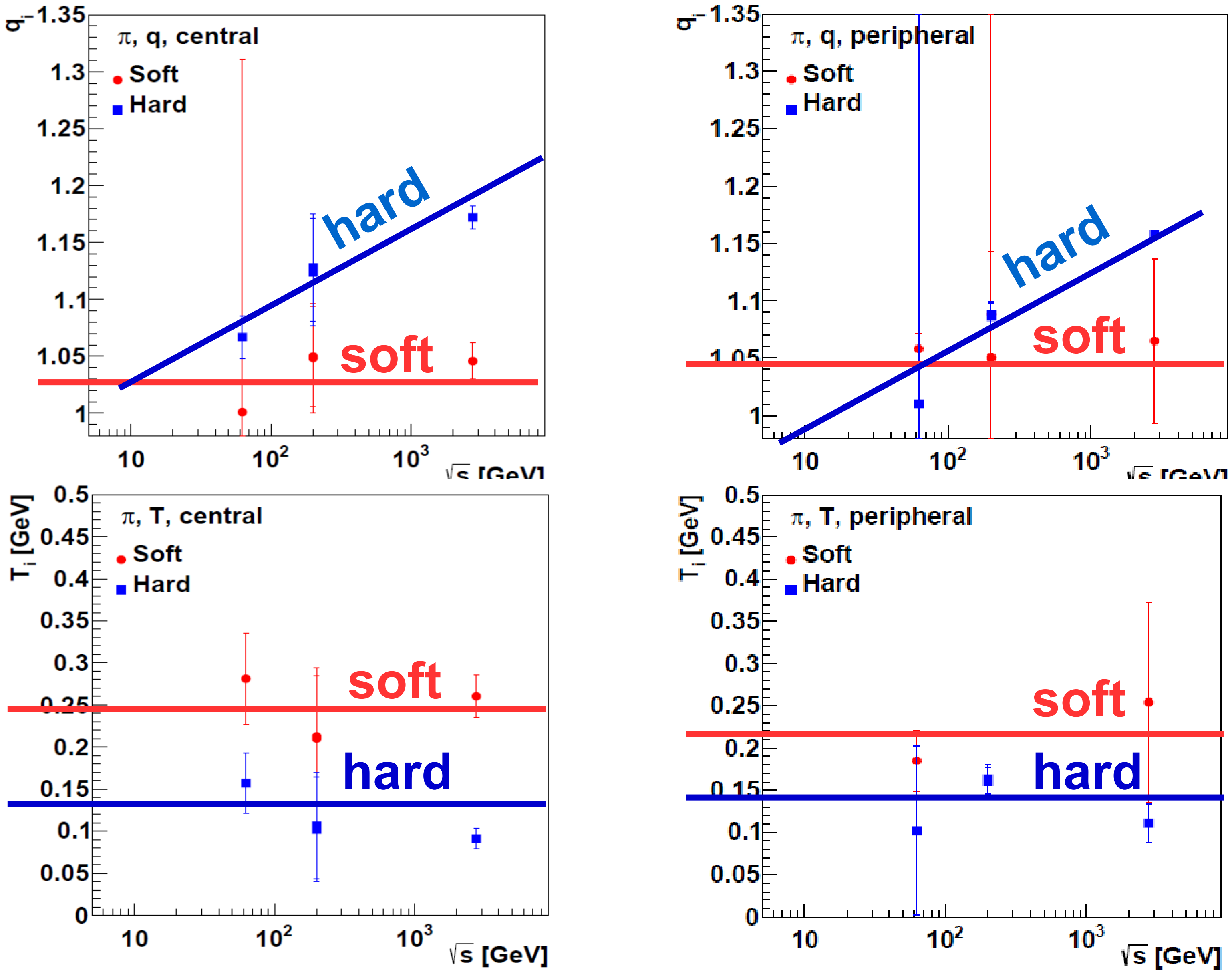




The c.m. energy dependence of *q* & *T*

q **measures non-extensivity**

T **measures average *E* per multiplicity**

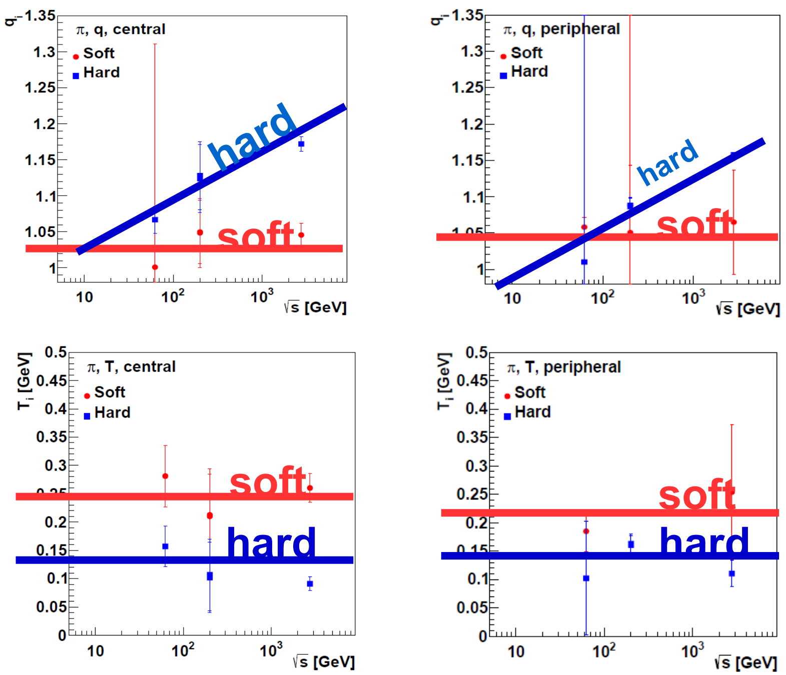


The c.m. energy dependence of *q* & *T*

- Energy dependence
  - Parameter q
    - HARD: clearly increasing
    - SOFT: no relevant change
  - Parameter *T*
    - HARD: central decreasing peripheral const?

      $T_{centr} = T_{periph}$
    - SOFT: similar trend

      T_{centr} ~100 MeV higher



The c.m. energy dependence of *q* & *T*

- Energy dependence
  - Parameter q
    - HARD: clearly increasing
    - SOFT: no relevant change
  - Parameter *T*
    - HARD: central decreasing peripheral const?

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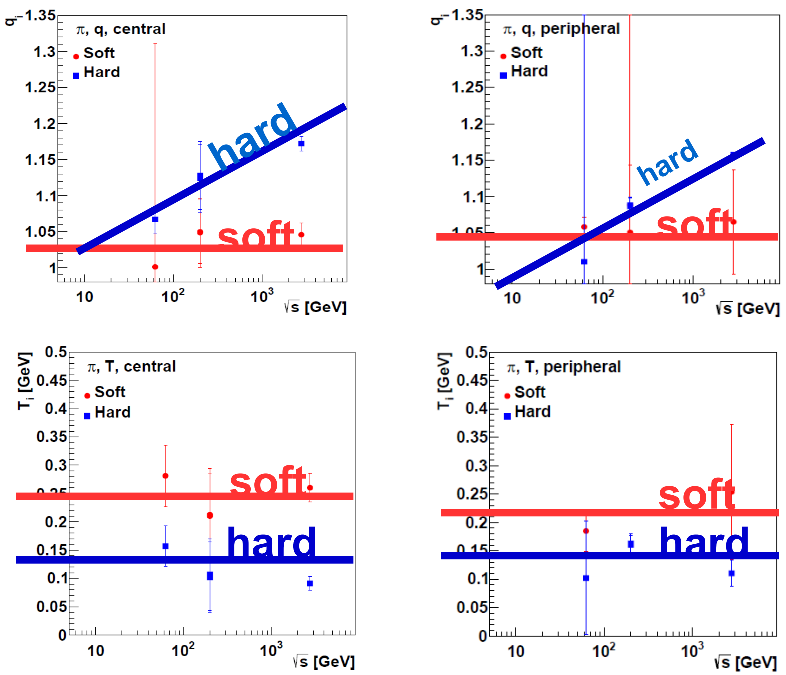

- Energy dependence
  - Parameters q & T present different values for centr./periph.
  - Above RHIC soft is BG-like and hard is more TP-like.

Can we connect this to asimuthal anisotropy?

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$p^0 \frac{dN}{d^3p}\bigg|_{y=0} = \int_{-\infty}^{+\infty} d\zeta \int_0^{2\pi} d\alpha\, f[u_\mu p^\mu] \longrightarrow \frac{dN}{2\pi p_T dp_T dy}\bigg|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi}\, p^0 \frac{dN}{d^3p}\bigg|_{y=0}$$

where we used parameters and assumptions

- Hadron momenum: $p^\mu = (m_T \cosh y, m_T \sinh y, p_T \cos\varphi, p_T \sin\varphi)$
- Cylindric symmetry: $u^\mu = (\gamma \cosh\zeta, \gamma \sinh\zeta, \gamma v \cos\alpha, \gamma v \sin\alpha)$

  where $\zeta = \frac{1}{2}\ln[(t+z)/(t-z)]$ and $\gamma = 1/\sqrt{1-v^2}$,
- Co-moving energy: $u_\mu p^\mu\big|_{y=0} = \gamma[m_T \cosh\zeta - v p_T \cos(\varphi - \alpha)]$
- Transverse flow: $v(\alpha) = v_0 + \sum_{m=1}^{\infty} \delta v_m \cos(m\alpha) \equiv v_0 + \delta v(\alpha)$
- Taylor expansion: $f[u_\mu p^\mu]\big|_{y=0} = \sum_{m=0}^{\infty} \frac{[\delta v(\alpha)]^m}{m!} \frac{\partial^m}{\partial v_0^m} f[u_\mu p^\mu]\big|_{y=0}^{v(\alpha)=v_0}$

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!} \frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O\left(\delta v^2\right)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int_0^{2\pi} d\alpha\, [\delta v(\alpha)]^m.$

- Azimuthal anisotropy:

$$v_n = \frac{\int_0^{2\pi} d\varphi \cos(n\varphi)\ p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int_0^{2\pi} d\varphi\ p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2} \frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O\left(\delta v^2\right)$$

  - Boltzmann–Gibbs: → $v_n^{BG} \approx \frac{\delta v_n \beta \gamma_0^3}{2}(p_T - v_0 m_T) + O\left(\delta v^2\right)$

    $f \sim \exp[-\beta E(v_0)].$

  - Tsallis–Pareto: → $v_n^{TS} \approx \frac{\delta v_n \beta \gamma_0^3}{2} \frac{p_T - v_0 m_T}{1 + (q-1)\beta \gamma_0 (m_T - v_0 p_T)} + O\left(\delta v^2\right)$

    $f \sim [1 + (q - 1)\beta E(v_0)]^{-1/(q-1)}$

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!} \frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O\left(\delta v^2\right)$$

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- Using the soft+hard model:

$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i} \frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i (m_T - v_i p_T) - m]}$$

Connecting spectra and v_2

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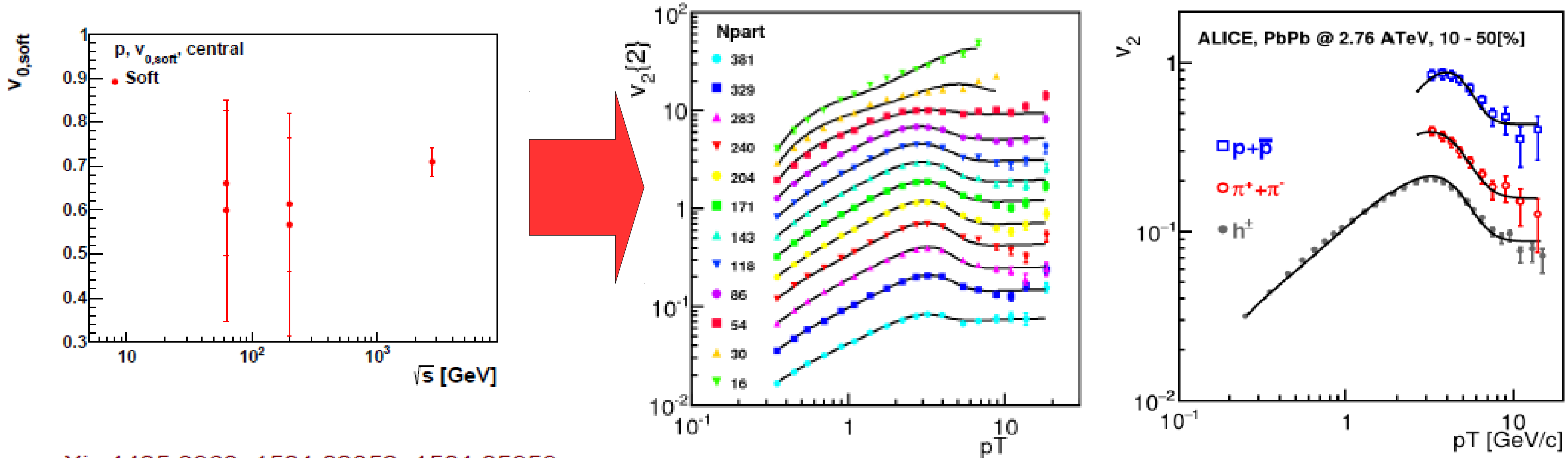
- Assuming v_0 only for the soft component v_2 can be obtained



arXiv:1405.3963, 1501.02352, 1501.05959
J.Phys.CS 612 (2015) 012048

S U M M A R Y

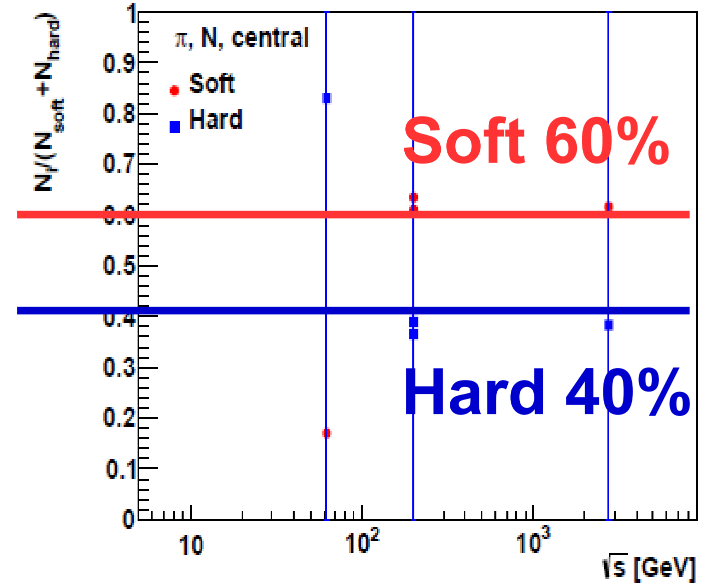
- Non-extensive statistical approach in e^+e^- & pp
  - Obtained Tsallis/Rényi entropies from the first principles.
  - Providing phyiscal meaning of $q=1-1/C + \Delta T^2/T^2$
  - *Boltzmann Gibbs limit* $C \rightarrow \infty$ & $\Delta T^2/T^2 \rightarrow 0$ $(q \rightarrow 1)$,
  - *Tsallis – Pareto fits on spectra in* e^+e^-, *pp*
  - *Not working for larger system, like pA, AA and no flow.*

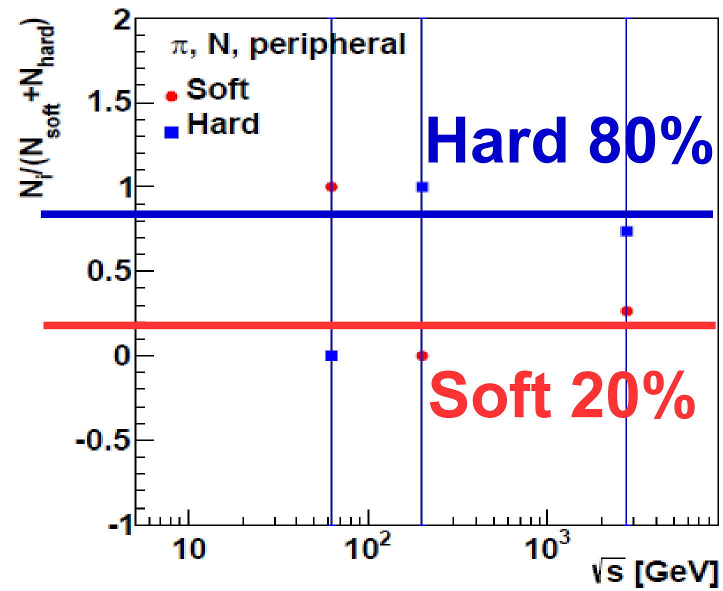
- Application of 'soft+hard' model in AA
  - Tsallis – Pareto + Exp does not working.
  - Double Tsallis – Pareto measures non-extensitivity
  - SOFT: $q \rightarrow 1$, suggest Boltzmann Gibbs (QGP)
  - HARD: $q > 1.1$, Tsallis – Pareto like
  - Asimuthal anisotropy can be obtained too.

B A C K U P

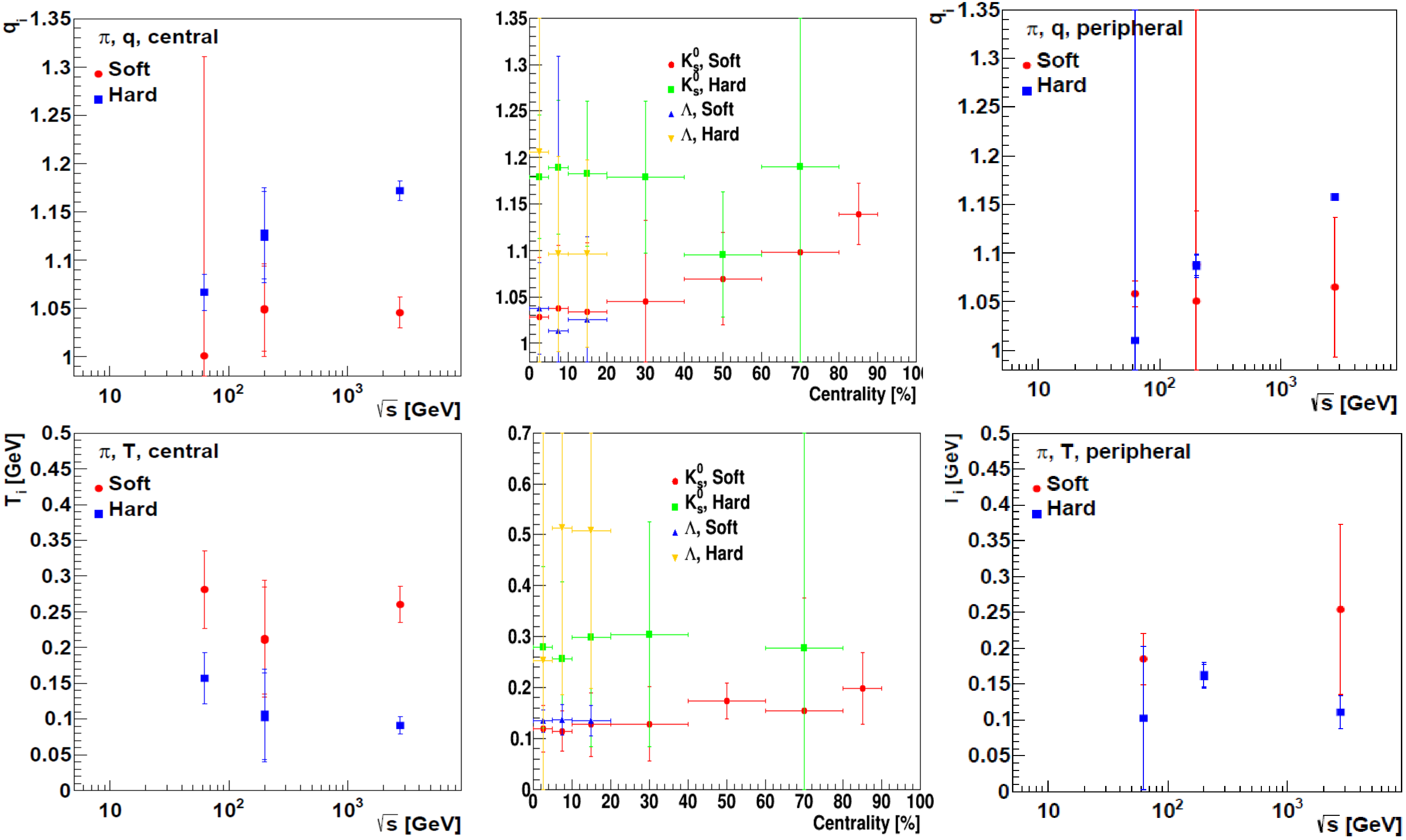
The c.m. Energy Dependence of N_{soft} & N_{hard}

- Energy dependence N_i/N_{tot}
  - Central
    - LHC: HARD 40% + SOFT 60%
    - RHIC: HARD 80% + SOFT 20%
  - Peripheral
    - LHC: HARD 80% + SOFT 20%
    - RHIC: HARD 10% + SOFT 90%





The c.m. Energy Dependence of *q* & *T*



Related publications..

1. arXiv:1409.5975: Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle

2. arXiv:1405.3963 Disentangling Soft and Hard Hadron Yields in PbPb Collisions at $\sqrt{s_{NN}}$ = 2.76 ATeV

3. arXiv:1405.3813 New Entropy Formula with Fluctuating Reservoir, Physica A (in Print) 2014

4. arXiv:Statistical Power-Law Spectra due to Reservoir Fluctuations

5. arXiv:1209.5963 Nonadditive thermostatistics and thermodynamics, Journal of Physics, Conf. Ser. V394, 012002 (2012)

6. arXiv:1208.2533 Thermodynamic Derivation of the Tsallis and Rényi Entropy Formulas and the Temperature of Quark-Gluon Plasma, EPJ A 49: 110 (2013)

7. arXiv:1204.1508 Microcanonical Jet-fragmentation in proton-proton collisions at LHC Energy, Phys. Lett. B, 28942 (2012)

8. arXiv:1101.3522 Pion Production Via Resonance Decay in a Non-extensive Quark-Gluon Medium with Non-additive Energy Composition Rule

9. arXiv:1101.3023 Generalised Tsallis Statistics in Electron-Positron Collisions, Phys.Lett.B701:111-116,2011

10. arXiv:0802.0381 Pion and Kaon Spectra from Distributed Mass Quark Matter, J.Phys.G35:044012,2008

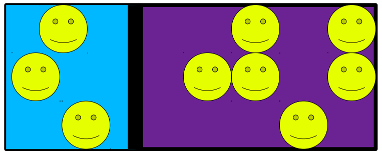
General derivation as inproved canonical

The story is about...

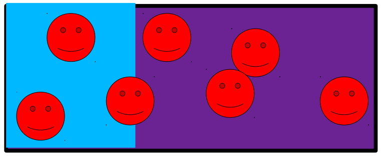
- Two body thermodynamics:

  1 subsystem *(E_1)* +one reservoir *(E-E_1)*

- Finite system, finite energy → microcanonical description
  - microcanonical $\sum_j \epsilon_j = E$



  - canonical $\sum_j <\epsilon_j> = E$



- Maximize a monotonic function of the Boltzmann-Gibbs entropy, *L(S)* (0^{th} law of thermodynamics)

- Taylor expansion of the *L(S)* = max, principle beyond $-\beta E$

Description of a system & reservoir

- For generalized entropy function $L(S_{12}) = L(S_1) + L(S_2)$
- In order to exist β of the system $L\left(S(E_1)\right) + L\left(S(E - E_1)\right) = \max$

TS Biró P. Ván: Phys Rev. E84 19902 (2011)

- Thermal contact between system *(E_1)* & reservoir *(E-E_1)*, requires to eliminate E_1 :

$$\beta_1 = L'\left(S(E_1)\right) \cdot S'(E_1)$$
$$= L'\left(S(E - E_1)\right) \cdot S'(E - E_1)$$

- This is usually handled in canonical limit, but now, we keep higher orders in the Taylor-expansion in E_1/E

$$\beta_1 = L'\left(S(E)\right) \cdot S'(E) - \left[S'(E)^2 L''(S(E)) + S''(E)L'(S(E))\right] E_1 + \ldots$$

Description of a system & reservoir

- Assuming $\beta_1=\beta$, the Lagrange multiplicator become familiar for us:

$$\beta = L'(S(E)) \cdot S'(E) = L'(S) \cdot \frac{1}{T}$$

- To satisfy this, need simply to solve

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}$$

- *Universal Thermostat Independence (UTI)* Principle: l.h.s. must be as an *S*-independent constant for solving *L(S)*,

$$\frac{L''(S)}{L'(S)} = a$$

- Based on *L(S)* →*S* for small *S*, coming from 3rd law of the thermodynamics *L'(0)=1* and *L(0)=0*

$$L(S) = \frac{e^{aS} - 1}{a}$$

- EoS derivatives do have physical meaning:

$$S'(E) = 1/T$$
$$S''(E) = -1/CT^2$$

Description of a system & reservoir

- Assuming β_1=β, the Lagrange multiplicator become familiar for us:

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$$L(S) = \frac{e^{aS} - 1}{a}$$

- Simly the heat capacity of the reservoir:

$$a = 1/C$$

From two system to many...

- Analogue to Gibbs ensamble generalize

  $$S = -\sum_i P_i \ln P_i \quad \rightarrow \quad L(S) = \sum_i P_i L(-\ln P_i)$$

-

- The *L*-additive form of a generally non-additive entropy, given by:

  $$L(S(E_1)) - \beta E_1 = \frac{1}{a}\left(e^{aS(E_1)} - 1\right) - \beta E_1 = \max.$$

- Introducing $a = 1/C(E) \quad \rightarrow \quad L(S(E_1)) = L(-\ln P_1) = \frac{1}{a}\left(P_1^{-a} - 1\right)$

- we need to maximize: $$\frac{1}{a}\sum_i \left(P_i^{1-a} - P_i\right) - \beta \sum_i P_i E_i - \alpha \sum_i P_i = \max.$$

which, results Tsallis:

$$S_{\text{Tsallis}} := L(S) = \frac{1}{q-1}\sum_i \left(P_i - P_i^q\right)$$

and its inverse Rényi:

$$S_{\text{Rényi}} := S = \frac{1}{1-q} \ln \sum_i P_i^q$$

The temperature slope

- Taking P_i weights of system, E_i , results cut power law:

$$P_i = \left(Z^{1-q} + (1-q)\frac{\beta}{q}E_i\right)^{\frac{1}{q-1}} = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}\,e^{S/C}}{C-1}\frac{E_i}{T}\right)^{-C}$$

- Partition sum is related to Tsallis entropy, $L(S_1)$ and E_1

$$\ln_q Z := C\left(Z^{1/C} - 1\right) = L\left(S_1\right) - \frac{1}{1-1/C}\beta E_1$$

- In $C \to \infty$ limit, the inverse log slope of the energy distribution:

$$T_{\text{slope}}(E_i) = \left(-\frac{d}{dE_i}\ln P_i\right)^{-1} = T_0 + E_i/C, \quad \text{with} \quad T_0 = T e^{-S/C} Z^{1/C}(1-1/C)$$