The Tsallis Distribution at Large Transverse Momenta

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Work done in collaboration with:
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C.Y. Wong, G. Wilk, J.L. Cirto, C. Tsallis
Phys. Rev. D91 (2015) 114027
C.Y. Wong, G. Wilk,
Acta Physica Polonica, B43 (2012) 2047-2054

Outline

Tsallis Thermodynamics

The Tsallis distribution is given by

$$f(E) = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}},$$

and the thermodynamic quantities N, E, P, S, ... are integrals over this distribution.
Asymptotically

$$\lim_{E\to\infty} f(E) = \left(\frac{E}{T}\right)^{-\frac{1}{q-1}}$$

scale is set by T
asymptotic behaviour is set by q.

For high energy physics a consistent form of Tsallis thermodynamics for the particle number, energy density and pressure is given by

$$
\begin{aligned}
N &= gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \\
\epsilon &= g \int \frac{d^3p}{(2\pi)^3} E \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \\
P &= g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}.
\end{aligned}
$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. This introduces only one new parameter q which for transverse momentum spectra is always close to 1.

Thermodynamic consistency

$$dE = -pdV + TdS + \mu dN$$

Inserting $E = \epsilon V$, $S = sV$ and $N = nV$ leads to

$$d\epsilon = Tds + \mu dn$$

$$dP = nd\mu + sdT$$

In particular

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied.

Thermodynamic consistency: an example

$$
\begin{aligned}
\frac{\partial P}{\partial \mu} &= gV \int \frac{d^3p}{(2\pi^3)} \frac{p^2}{3E} \frac{\partial}{\partial \mu} f^q \\
&= -gV \int \frac{d^3p}{(2\pi^3)} \frac{p^2}{3E} \frac{d}{dE} f^q \\
&= -gV \frac{4\pi}{(2\pi^3)} \int_0^\infty dp \, \frac{p^4}{3E} \frac{d}{dE} f^q \\
&= -gV \frac{4\pi}{(2\pi^3)} \int_0^\infty dp \, \frac{p^3}{3} \frac{d}{dp} f^q \quad \text{using} \quad EdE = pdp \\
&= gV \frac{4\pi}{(2\pi^3)} \int_0^\infty dp \, p^2 f^q \\
&= n
\end{aligned}
$$

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,$$

which, in terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, becomes (at mid-rapidity $y = 0$ and for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} ,$$

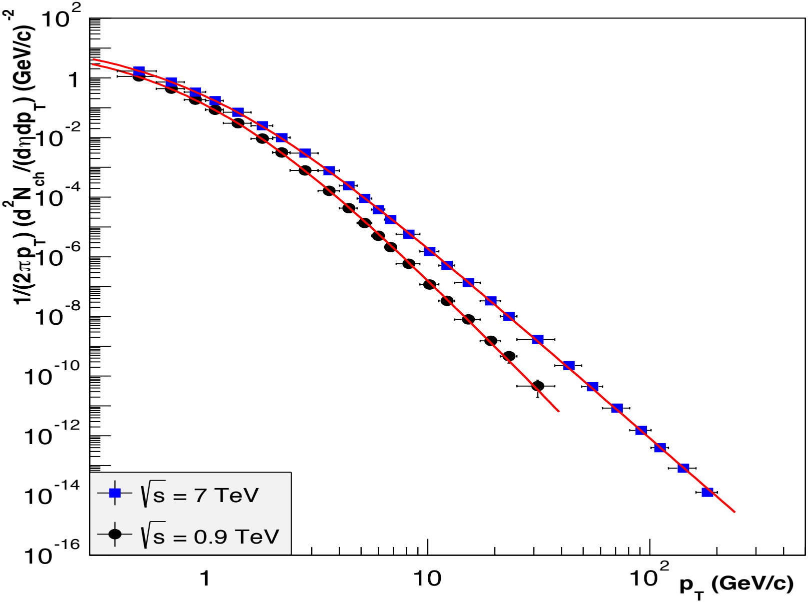
J.C. and D. Worku, J. Phys. **G39** (2012) 025006; arXiv:1203.4343[hep-ph].

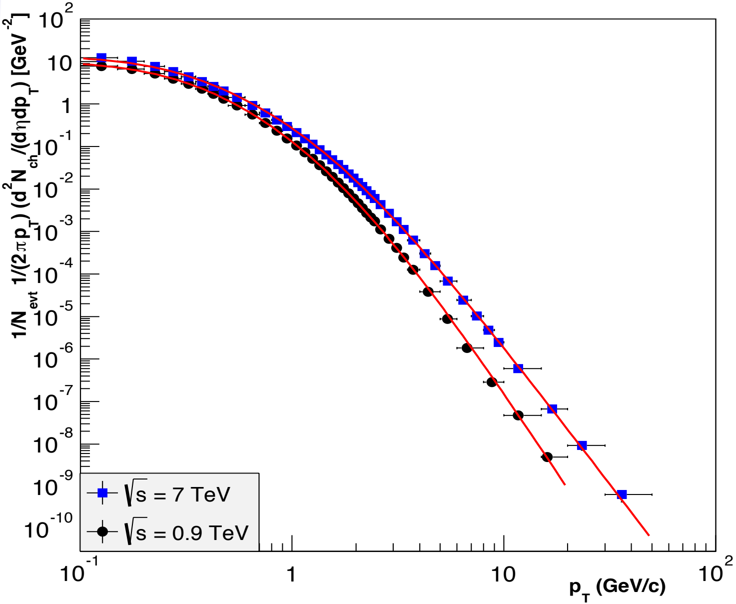
For charged particles use:

$$\left.\frac{d^2N(\text{charged})}{dp_T\ dy}\right|_{y=0} = \sum_{i=\pi,K,p,\ldots} g_i V \frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}},$$

M.D. Azmi and J.C. , arXiv:1501.07217v3[hep-ph].

Tsallis Distribution p-p CMS





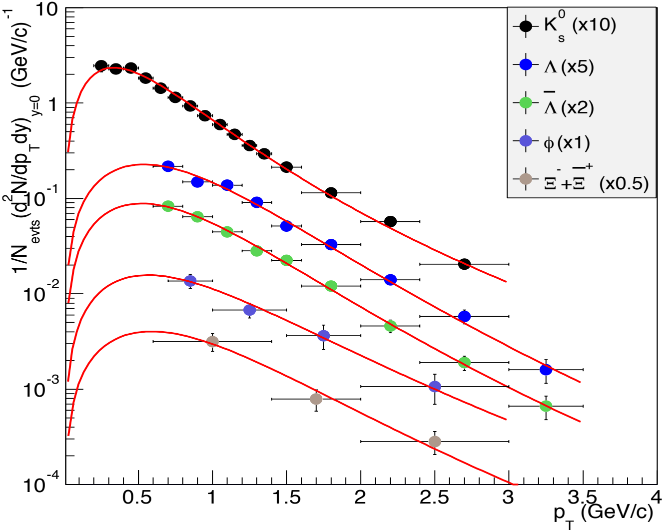
$1/N_{evt}\ 1/(2\pi p_T)\ (d^2N_{ch}/(d\eta dp_T))\ [GeV^{-2}]$
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$\sqrt{s}$ = 7 TeV
$\sqrt{s}$ = 0.9 TeV
10^{-1}
1
10
10^2
p_T (GeV/c)

Tsallis Distribution p-p

Experiment	$\sqrt{s}$ **(TeV)**	q	T **(MeV)**
ATLAS	0.9	1.129 ± 0.005	74.21 ± 3.55
ATLAS	7	1.150 ± 0.002	75.00 ± 3.21
CMS	0.9	1.129 ± 0.003	76.00 ± 0.17
CMS	7	1.153 ± 0.002	73.00 ± 1.42

Values of the q and T parameters to fit the p_T spectra measured by the ATLAS and CMS collaborations.

Tsallis fits to strange particles

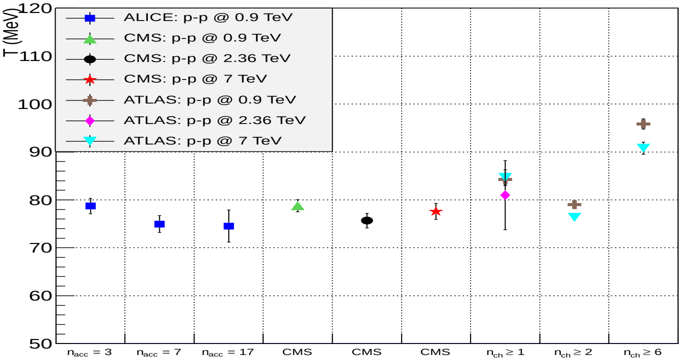


Fit parameters vs ALICE measurements

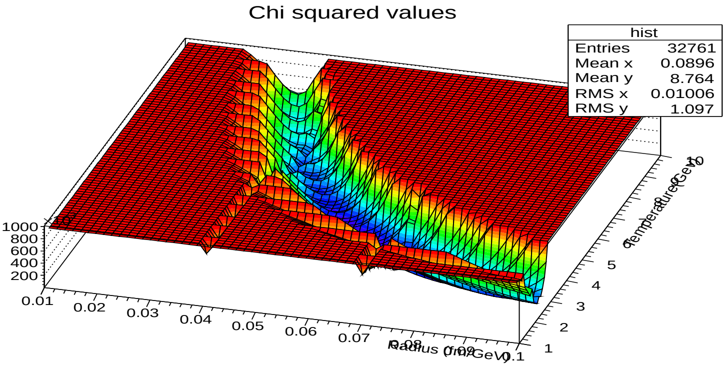
Particles	q	T (MeV) Tsallis	T (MeV) ALICE	dN/dy Tsallis	dN/dy ALICE	X^2/NDF Tsallis	X^2/NDF ALICE
K_s^0	1.15 ± 0.03	73.67 ± 3.85	168 ± 5	0.182	0.184	2.01/13	10.8/13
Φ	1.14 ± 0.03	79.99 ± 6.12	164 ± 91	0.019	0.021	0.12/1	0.6/1
Λ	1.11 ± 0.008	79.99 ± 5.63	229 ± 15	0.049	0.048	1.38/6	9.6/6
Λ(bar)	1.11 ± 0.008	70.00 ± 9.8	210 ± 15	0.047	0.047	0.42/6	3.7/6
$\Xi^- + \Xi(bar)^+$	1.11 ± 0.03	75.00 ± 7.5	175 ± 50	0.0096	0.0101	0.189/0*	- #

* The number of data sets is 3 and the number of fit parameters are also 3. So, the NDF = 0

p-p collisions: Summary of results for parameter T



Tsallis: problem in determining parameters T and V



The Tsallis distribution provides an
excellent description of the transverse momentum spectra over 14 orders of magnitude
up to 200 GeV. **Use**

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}},$$

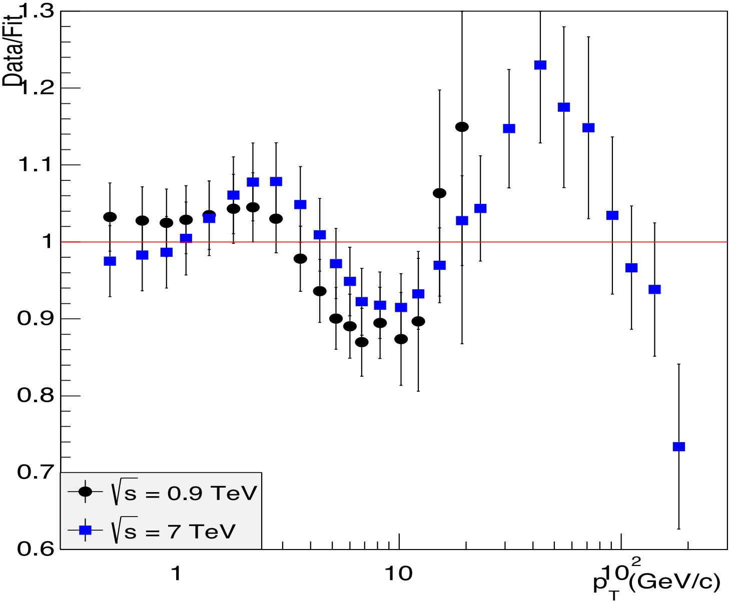
Advantages : thermodynamic conistency:

$$n = \frac{\partial P}{\partial \mu} \quad \text{etc...,}$$

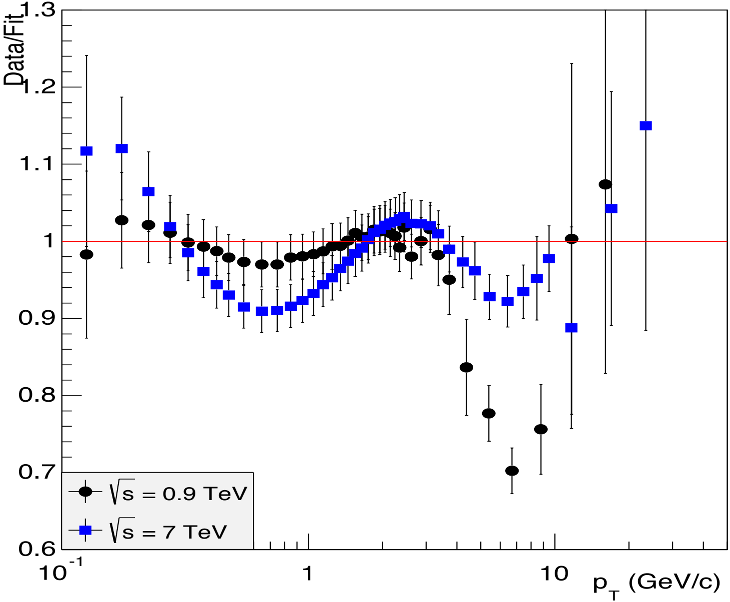
and the parameter T deserves its name since

$$T = \frac{\partial S}{\partial E} \quad \ldots$$

Tsallis Distribution p-p



Tsallis Distribution p-p



Tsallis Distribution p-p

Experiment	$\sqrt{s}$ **(TeV)**	R **(fm)**	χ^2/NDF
ATLAS	3.55	4.62 ± 0.29	0.657503/36
ATLAS	3.21	5.05 ± 0.07	4.35145/41
CMS	0.9	4.32 ± 0.29	0.648806/17
CMS	7	5.04 ± 0.27	0.521746/24

Values of R and χ^2/NDF parameters to fit the p_T spectra measured by the ATLAS and CMS collaborations.