Flow Viscosity with a Running Coupling

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(With many thanks to André Peshier)

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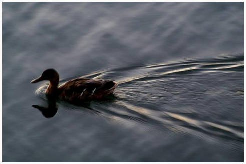
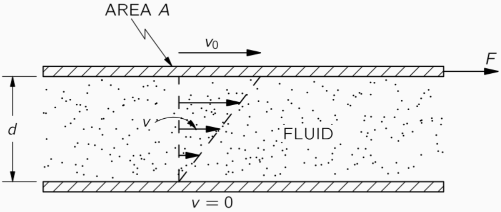


Fluids are

M$^{A}_{I}$CRO*scopic* { bulk properties
particle interactions



An easy problem: ←

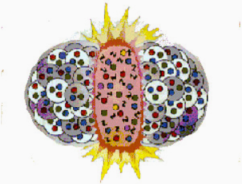
→ parallel plates

$$\frac{F}{A} = \boxed{\eta} \frac{v_0}{d}$$

“how good” the fluid is at moving momentum around: $\eta \sim n \, \Delta p \, \ell_{\text{mfp}}$

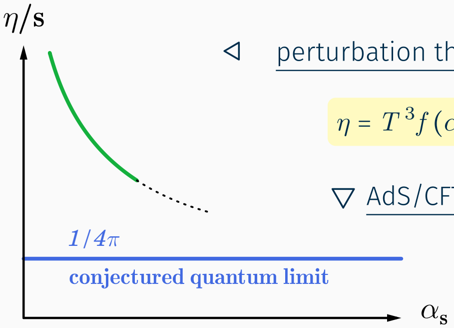
A stressful problem: ⇝ Heavy Ion Collisions

What is the shear viscosity in **QCD** ?



$\mathcal{L}$agrangian $\rightsquigarrow$ transport coefficients

QCD is a theory! (NOT A MODEL) Calculate from 1st principles...?



◁ perturbation theory: [Arnold, Moore, Yaffe (2001)]

$$\eta = T^3 f(\alpha_s) \xrightarrow{\text{leading-log}} \frac{a\ T^3}{\alpha_s^2\ \ln(b/\alpha_s)}$$

▽ AdS/CFT duality: [Kovtun, Son, Starinets (2004)]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$ „ uncertainty principle "

FREE theories $(0 \simeq \alpha_s)$ } ← interaction → { **IDEAL** hydrodynamics $(\alpha_s \to \text{“}\infty\text{”})$

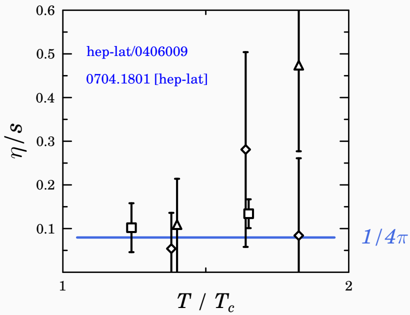
Consensus*: $\eta/s \lesssim 0.5$

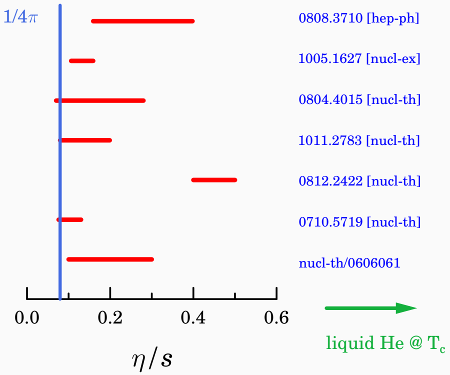
phenomonology: ▷

[heavy ion]

lattice QCD: ▽

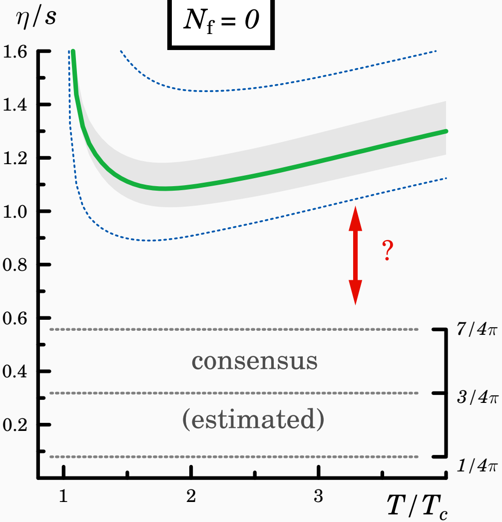
[Kubo formula]





*terms & conditions apply:

many aspects
large uncertainties



leading-log "pocket formula" with $\alpha \to \alpha(2\pi T)$

sensitive to CUT-OFF

IMPRESSION :

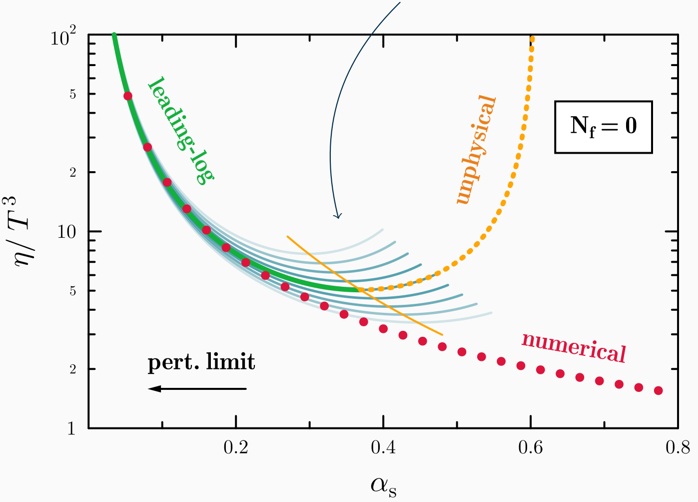
cannot *understand*

$\eta/s \ll 1$ from pQCD

interacting entropy, $s(T)$ from lQCD: [Boyd *et al*, (1996)]

Issues with expansion

back of the envelope: $\eta^{-1} \propto \sigma_{\rm tr} \sim \int_{\# m_D^2}^{T^2} \mathrm{d}t\, t \left[\frac{\alpha}{t}\right]^2 = \alpha^2 \ln\left(\frac{T^2}{m_D^2}\right)$ ← [fixed coupling]



expansion in $\ln(1/\alpha_s)$

small angle scatterings

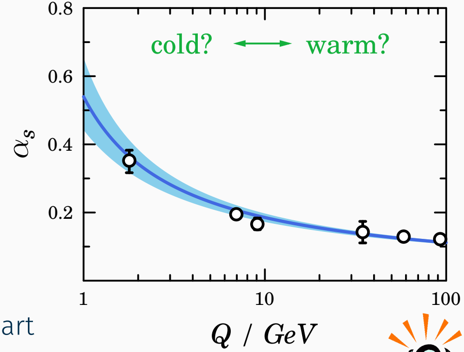
screening
"soft"
$m_D^2 \sim \alpha T^2$

What's cooking?

Physics:

thermal contribution	vacuum fluctuations
screening	running coupling

Run, α_s , run!

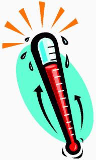


Recipe:
[old]

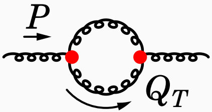
drop vacuum part

fixed coupling calculation

guess scale dep. $\alpha_s(Q)$ ... $Q \simeq 2\pi T$



Re^3 = Regularise, Resum, Renormalise...



dim. reg.	Π_{vac}	$\sim$	$P^2\left(\frac{1}{\epsilon} - \ln\left[\frac{P^2}{\mu^2}\right]\right)$
HTL	Π_{mat}	$\sim$	$T^2 f\,(p_0, \boldsymbol{p})$

PROPAGATOR: (1-loop, resummed)

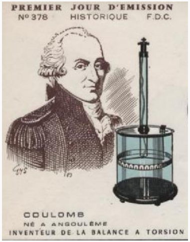
in vacuum $\quad \dfrac{\alpha}{P^2 - \alpha\,\Pi_{\mathrm{vac}}} \xrightarrow{\text{renorm.}} \dfrac{\alpha(P^2)}{P^2}$

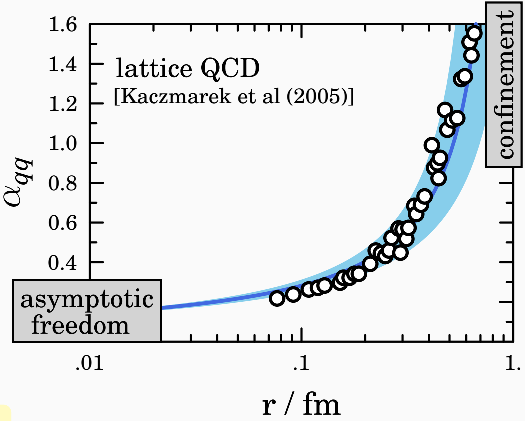
in heat bath $\quad \dfrac{\alpha}{P^2 - \alpha\,\Pi_{\mathrm{vac}} - \alpha\,\Pi_{\mathrm{mat}}} \xrightarrow{\text{renorm.}} \dfrac{\alpha(P^2)}{P^2 - \alpha(P^2)\Pi_{\mathrm{mat}}}$

Heavy quarks at $T = 0$

step 1: measure “force” to ADJUST parameters

$$\frac{4}{3}\frac{\alpha_{qq}(r)}{r^2} := -\frac{\mathrm{d}V}{\mathrm{d}r}$$





$\Lambda \simeq 0.2$–0.3 GeV

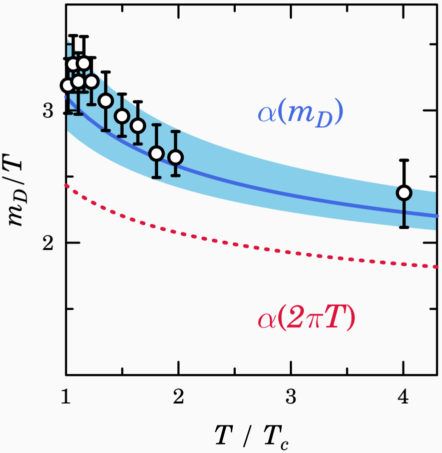
($N_f = 2$)

for *some* observables

pQCD MAY work *quantitatively* @ $\alpha \sim 1$.

step 2: compare in order to VERIFY approximation scheme

(following [André Peshier, (2006)])



$$V \to F_1 = \int_k e^{ikr} \;[\text{diagram}]\; \sim \frac{e^{-m_D r}}{r}$$

Debye mass is a crucial *regulator*

@ finite temperature

($\triangledown$ works near T_c!)

self

$$m_D^2 = 4\pi\alpha_s(m_D^2)\,T^2\left(\frac{N_c}{3} + \frac{N_f}{6}\right)$$

consistent

Back to viscosity/entropy

step 3: PREDICT physical observables

(following **[AMY, (2003)] & [Baym, (1990)]**)

---

1) Boltzmann eq: $(\partial_t + \boldsymbol{u}\cdot\nabla)\, f \;=\; \mathcal{C}\,[f\,]$

2) linearise: $f = f^{(0)} + \delta f$, ... find $S = \mathcal{C}\,[\delta f\,]$

3) variational treatment:

$$\mathcal{Q}\,[\delta f\,] = \langle \delta f\,|S\,\rangle - \frac{1}{2}\,\langle \delta f\,|\mathcal{C}|\delta f\,\rangle$$

$$\eta = \frac{2}{15}\mathrm{Max}\,[\mathcal{Q}] \propto \left\langle S\middle|\mathcal{C}^{-1}\middle|S\right\rangle$$

...in "Hilbert space" of δf test functions

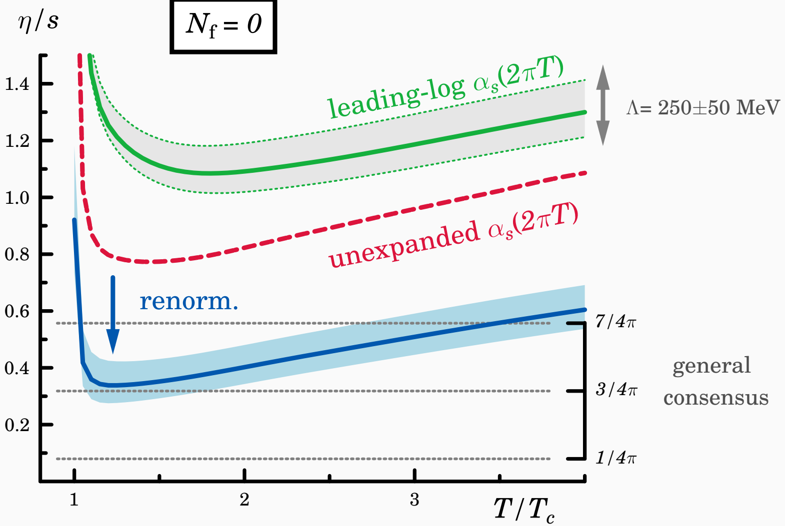
← **checklist:**

✓	kinematics
✓	quantum stat.
✓	effective cutoff
?	running $\alpha(Q)$

Two cents:

renormalise collision operator $\mathcal{C}\,[\delta f\,]$

(f)low viscosity with a running coupling!



PREDICTION : gradual increase with T and $\eta/s \sim 0.5$

Summary

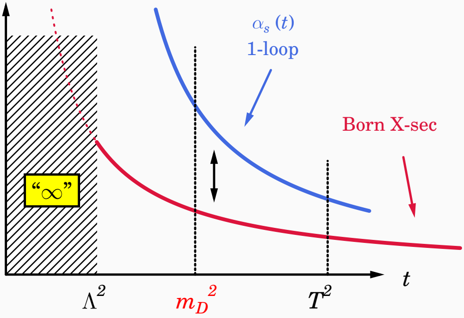
identify inconsistency:

vacuum corrections "dropped"

(re)-calculate:

1-loop, resummed framework
(no 1→2 processes yet)

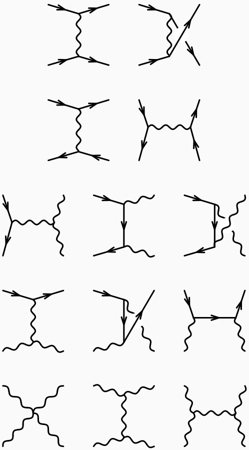
Coupling runs with different scales!



$$\eta^{-1} \propto \sigma_{\rm tr} \sim \int_{m_D^2}^{T^2} {\rm d}t\, t \left[\frac{\alpha(t)}{t}\right]^2 \quad = \quad \alpha(m_D^2)\alpha(T^2)\ln\left(\frac{T^2}{m_D^2}\right)$$

→ BACKUP SLIDES

Matrix elements



Møller
$qq \to qq$

Bhabha
$q\bar{q} \to q\bar{q}$

Annihilation
$q\bar{q} \to gg$

Compton
$qg \to qg$

Delbrück
$gg \to gg$

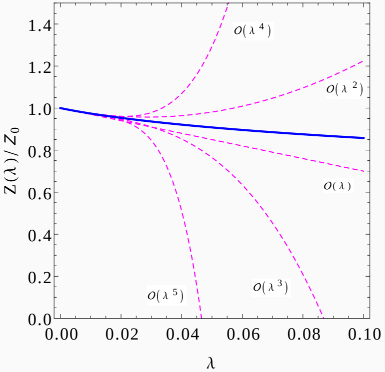
$ab \leftrightarrow cd$	$\left\vert\mathcal{M}^{ab}_{cd}\right\vert^2 / g^4$
$q_1 q_2 \leftrightarrow q_1 q_2$, $q_1 \bar{q}_2 \leftrightarrow q_1 \bar{q}_2$, $\bar{q}_1 q_2 \leftrightarrow \bar{q}_1 q_2$, $\bar{q}_1 \bar{q}_2 \leftrightarrow \bar{q}_1 \bar{q}_2$	$8\,\frac{d_F^2\, C_F^2}{d_A}\left(\frac{s^2+u^2}{\underline{t^2}}\right)$
$q_1 q_1 \leftrightarrow q_1 q_1$, $\bar{q}_1 \bar{q}_1 \leftrightarrow \bar{q}_1 \bar{q}_1$	$8\,\frac{d_F^2\, C_F^2}{d_A}\left(\frac{s^2+u^2}{\underline{t^2}} + \frac{s^2+t^2}{\underline{u^2}}\right) + 16\, d_F\, C_F \left(C_F - \frac{C_A}{2}\right)\frac{s^2}{tu}$
$q_1 \bar{q}_1 \leftrightarrow q_1 \bar{q}_1$	$8\,\frac{d_F^2\, C_F^2}{d_A}\left(\frac{s^2+u^2}{\underline{t^2}} + \frac{t^2+u^2}{s^2}\right) + 16\, d_F\, C_F \left(C_F - \frac{C_A}{2}\right)\frac{u^2}{st}$
$q_1 \bar{q}_1 \leftrightarrow q_2 \bar{q}_2$	$8\,\frac{d_F^2\, C_F^2}{d_A}\left(\frac{t^2+u^2}{s^2}\right)$
$q_1 \bar{q}_1 \leftrightarrow g\, g$	$8\, d_F\, C_F^2 \left(\frac{u}{\underline{t}} + \frac{t}{\underline{u}}\right) - 8\, d_F\, C_F\, C_A \left(\frac{t^2+u^2}{s^2}\right)$
$q_1\, g \leftrightarrow q_1\, g$, $\bar{q}_1\, g \leftrightarrow \bar{q}_1\, g$	$-8\, d_F\, C_F^2 \left(\frac{u}{s} + \frac{s}{\underline{u}}\right) + 8\, d_F\, C_F\, C_A \left(\frac{s^2+u^2}{\underline{t^2}}\right)$
$g\, g \leftrightarrow g\, g$	$16\, d_A\, C_A^2 \left(3 - \frac{su}{\underline{t^2}} - \frac{st}{\underline{u^2}} - \frac{tu}{s^2}\right)$

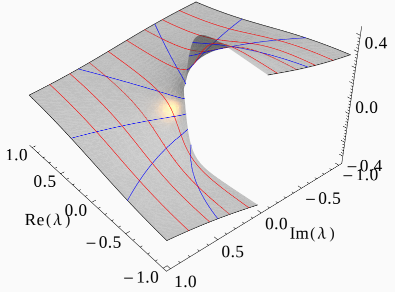
... on asymptoyic series

$$L(\lambda) = \frac{1}{2}x^2 + \lambda x^4$$

toy "0-dim" QFT

$$\left[\text{partition function}\right] \rightarrow Z(\lambda) = \int \mathrm{d}x \exp[-L(\lambda)]$$

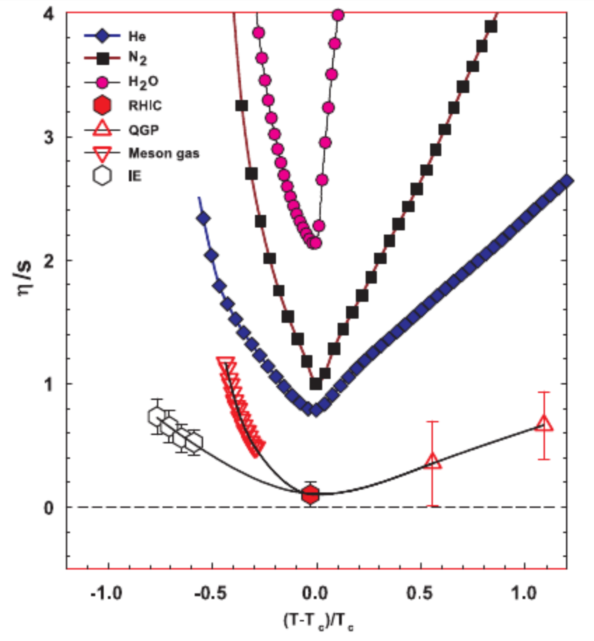




At *LARGE* coupling...

truncate @ $n \sim \lambda^{-1}$

pQCD still useful!



4
3
2
1
0
η/s
He
N_2
H_2O
RHIC
QGP
Meson gas
IE
-1.0
-0.5
0.0
0.5
1.0
$(T-T_c)/T_c$