Structure of compact stars in a pion superfluid phase

- Pion superfluid quark matter
- Mass-radius relation of compact stars

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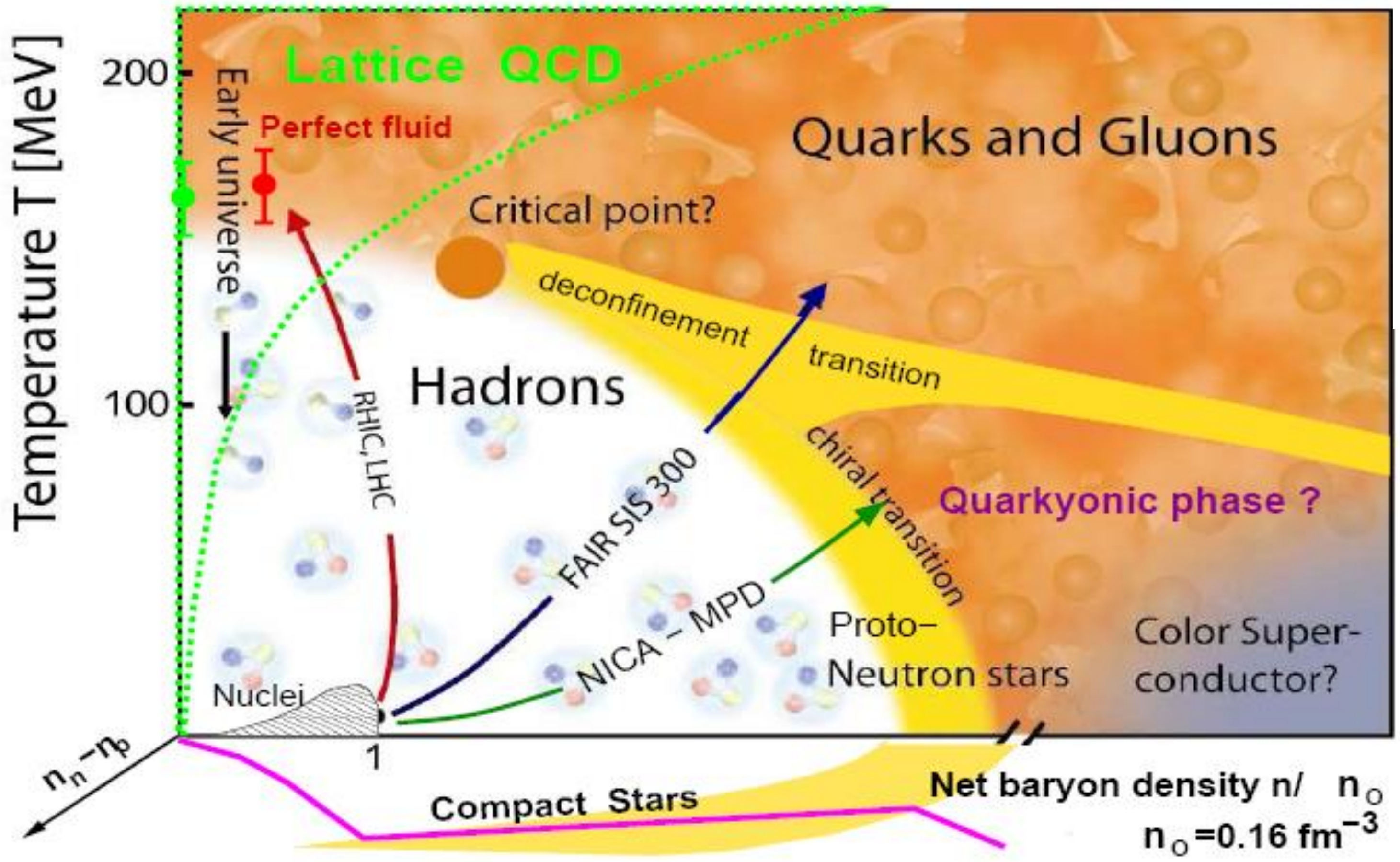
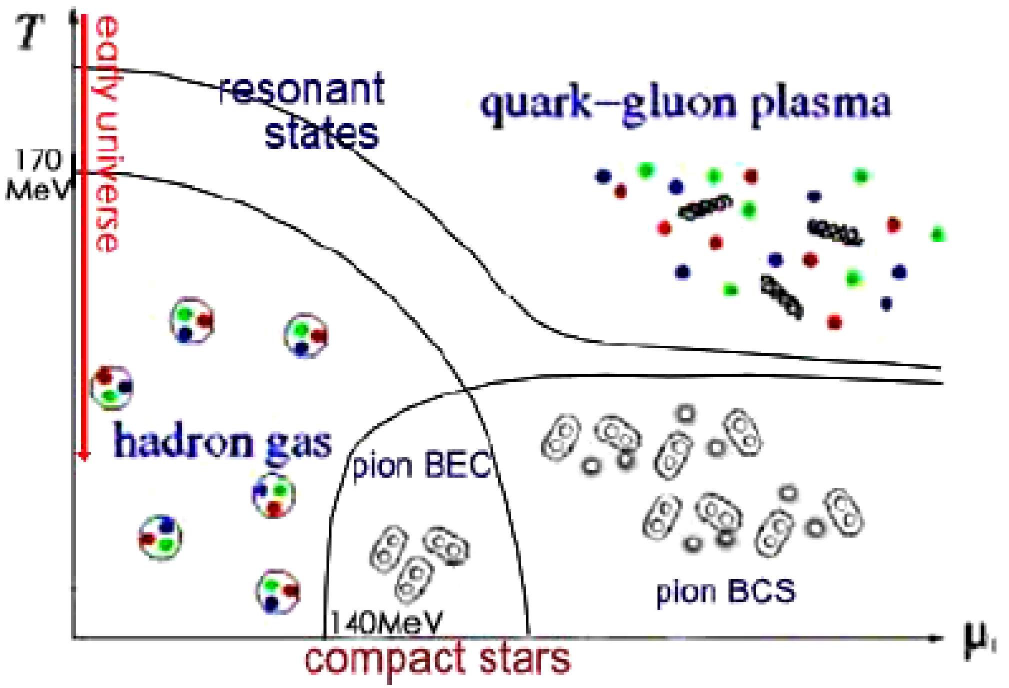
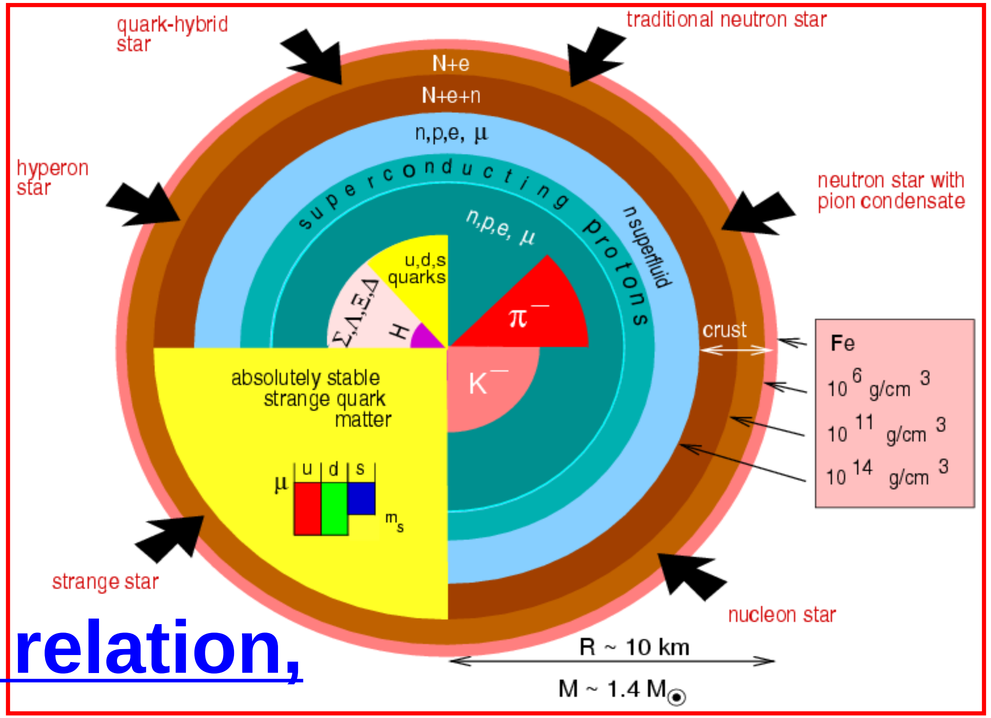
1. Introduction

QCD matter

low T, high density







Mass-Radius relation,

2. Pion superfluid

SU(2) Nambu—Jona-Lasinio Model

$$SU_L(2) \otimes SU_R(2)$$

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau_i\gamma_5\psi)^2\right]$$

$$\mu = diag(\frac{\mu_B}{3} + \frac{\mu_I}{2}, \frac{\mu_B}{3} - \frac{\mu_I}{2}),$$

μ_I: **$SU_I(2) \rightarrow \rightarrow U_I(1)$**

μ_B: Fermi surface mismatch

chiral condensate: $\sigma = \langle \bar{\psi}\psi \rangle,$

pion condensate: $\pi_+ = \sqrt{2}\langle \bar{u}i\gamma_5 d \rangle = \frac{\pi}{\sqrt{2}} e^{-2i\vec{q}\cdot\vec{x}}, \quad \pi_- = \sqrt{2}\langle \bar{d}i\gamma_5 u \rangle = \frac{\pi}{\sqrt{2}} e^{2i\vec{q}\cdot\vec{x}}$

q=0, homogeneous state (BCS)

q≠0, inhomogeneous state (LOFF)

Mean Field Approximation

quark propagator:

$$S^{-1}(p,\vec{q}) = \begin{pmatrix} \gamma^{\mu} p_{\mu} - \vec{\gamma}\cdot\vec{q} + \mu_u \gamma_0 - m & -i\Delta\gamma_5 \\ -i\Delta\gamma_5 & \gamma^{\mu} p_{\mu} + \vec{\gamma}\cdot\vec{q} + \mu_d \gamma_0 - m \end{pmatrix}$$

$$\Delta = -2G\pi$$

$$m = m_0 - 2G\sigma$$

thermodynamic potential: $\Omega(\mathrm{m},\Delta,q) = \frac{m^2+\Delta^2}{4\mathrm{G}} - \frac{T}{V}\mathrm{Tr}\,\mathrm{Ln}\,S^{-1}$

Gap equation: $\frac{\partial\Omega}{\partial m} = 0, \quad \frac{\partial\Omega}{\partial\Delta} = 0, \quad \frac{\partial\Omega}{\partial q} = 0.$

Ground state: minimum of Ω .

EoS:

$$P = -\Omega, \quad s = -\frac{\partial\Omega}{\partial T}, \quad n_B = -\frac{\partial\Omega}{\partial\mu_B}, \quad n_I = -\frac{\partial\Omega}{\partial\mu_I},$$

$$\epsilon = -P + Ts + \mu_I n_I + \mu_B n_B.$$

Regularization

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right]$$

1. Hard cutoff: $\bar{p}^2 < \Lambda^2$

2. <u>Pauli-Villars regularization</u>:

$$m \rightarrow m_j = \sqrt{m^2 + a_j\Lambda^2}.$$
$$E \rightarrow E_j = c_j E$$

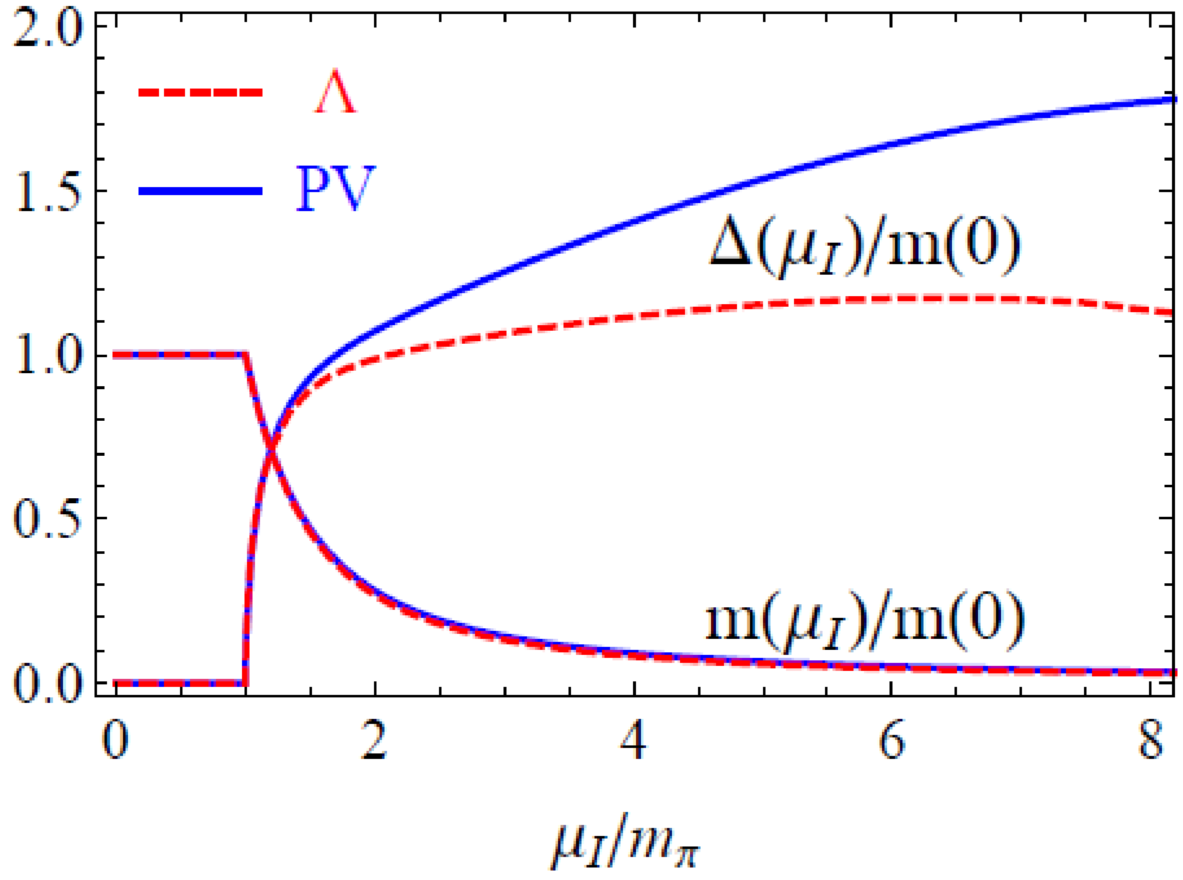
$$a_0 = 0,\ c_0 = 1, \sum_{i=0}^{N} c_i = 0, \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2) = 0, \ldots \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2)^{(N-1)} = 0$$

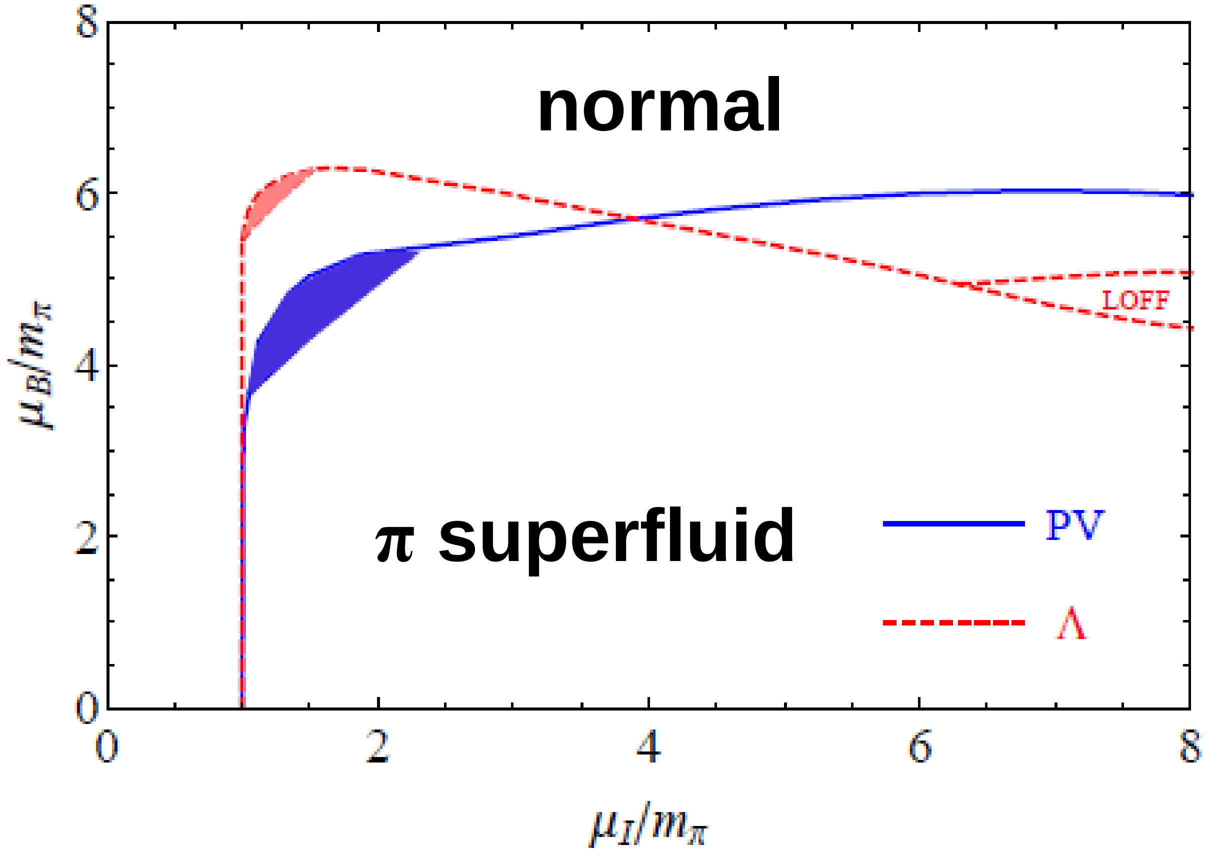
Order parameters

Phase diagram in μ_I- μ_B

quark mass: m
pion condensate: Δ

$T = 0; \mu_B = 0$

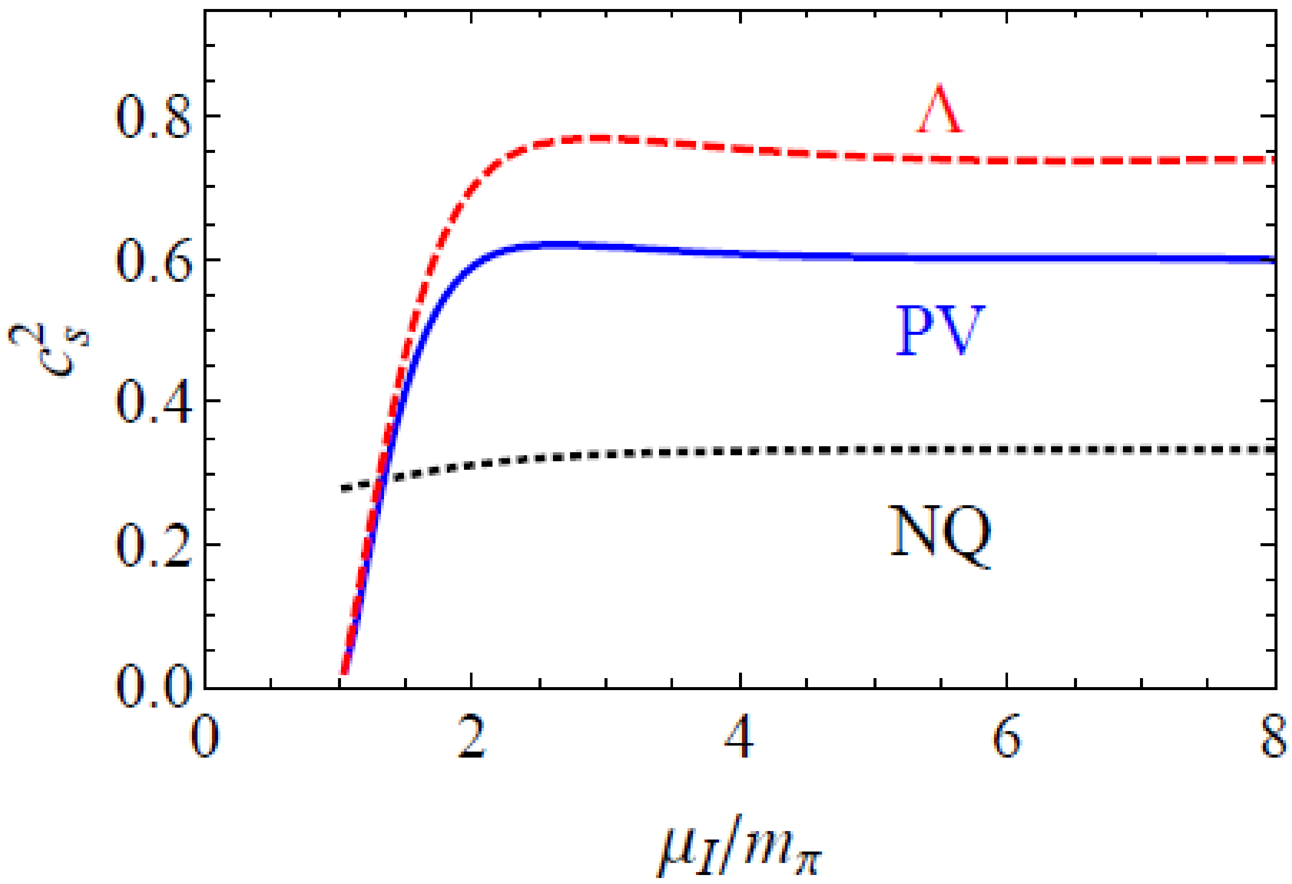




LOFF disappears!

Sound velocity: $c_s^2 = \frac{dP}{d\varepsilon}$

$T = 0; \mu_B = 600$ MeV



PV: $c_s^2 = 0.63$;

Λ: $c_s^2 = 0.73$;

CL: $c_s^2 = 1$;

NQ: $c_s^2 = 1/3$;

Color superconductor:

$\mathbf{c_s^2 \approx 1/3}$

3. Mass-radius relation

Can massive compact star be in pion superfluid state??

TOV + EoS ⟶ Mass-Radius relation,

static, spherical stars: **Tolman-Oppenheimer-Volkoff (TOV) equation**

$$\frac{dP}{dr} = -\frac{G_N(\epsilon + P)\left(M + 4\pi r^3 P\right)}{r^2\left(1 - 2G_N M/r\right)},$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon,$$

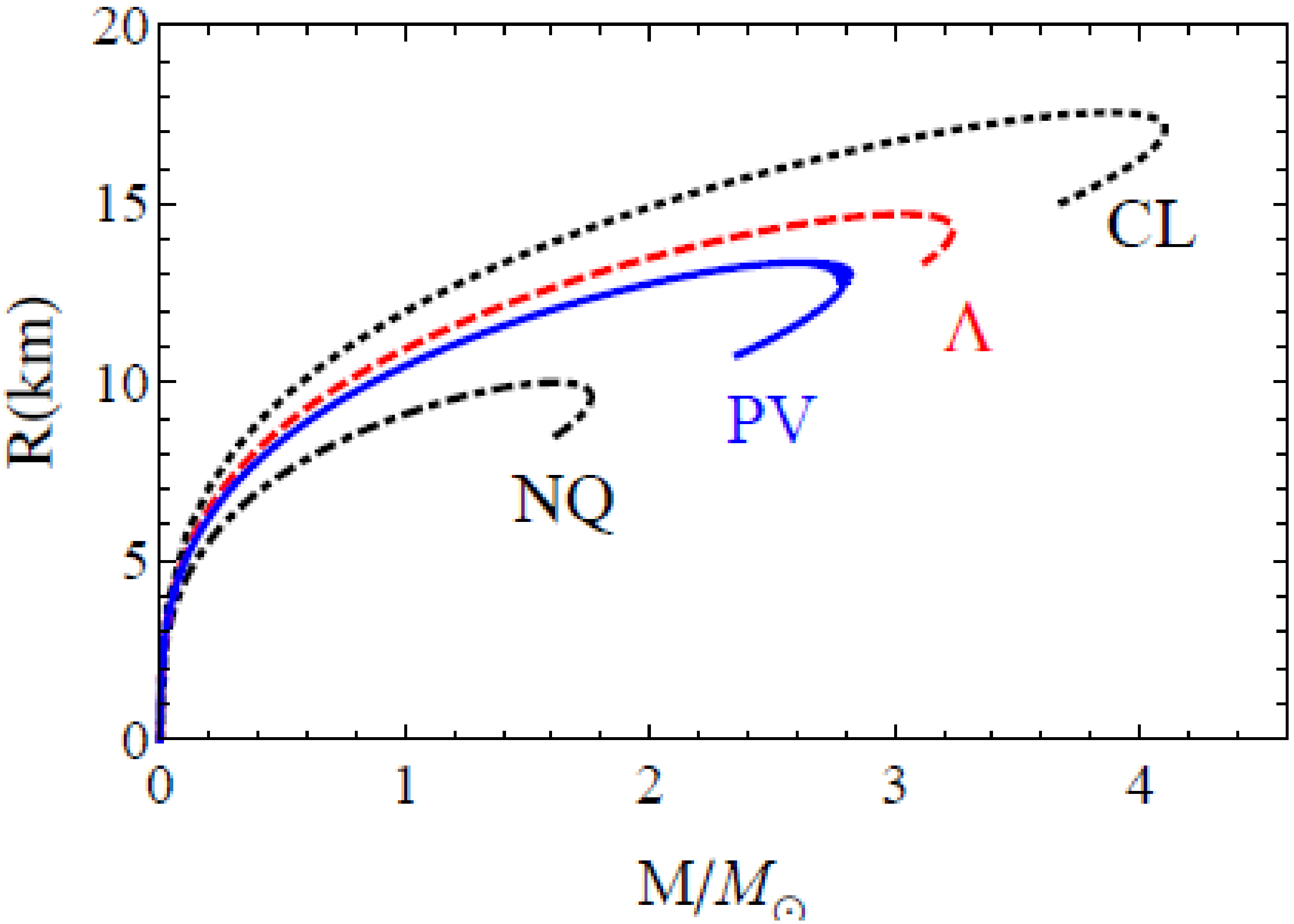
Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi\kappa T_{\mu\nu}$$

+ EoS of quark matter in pion superfluid phase

$$P(r=0) = P_c; \quad P(r=R) = B \longrightarrow \text{Mass}, \quad \textit{Radius}$$

M-R relation



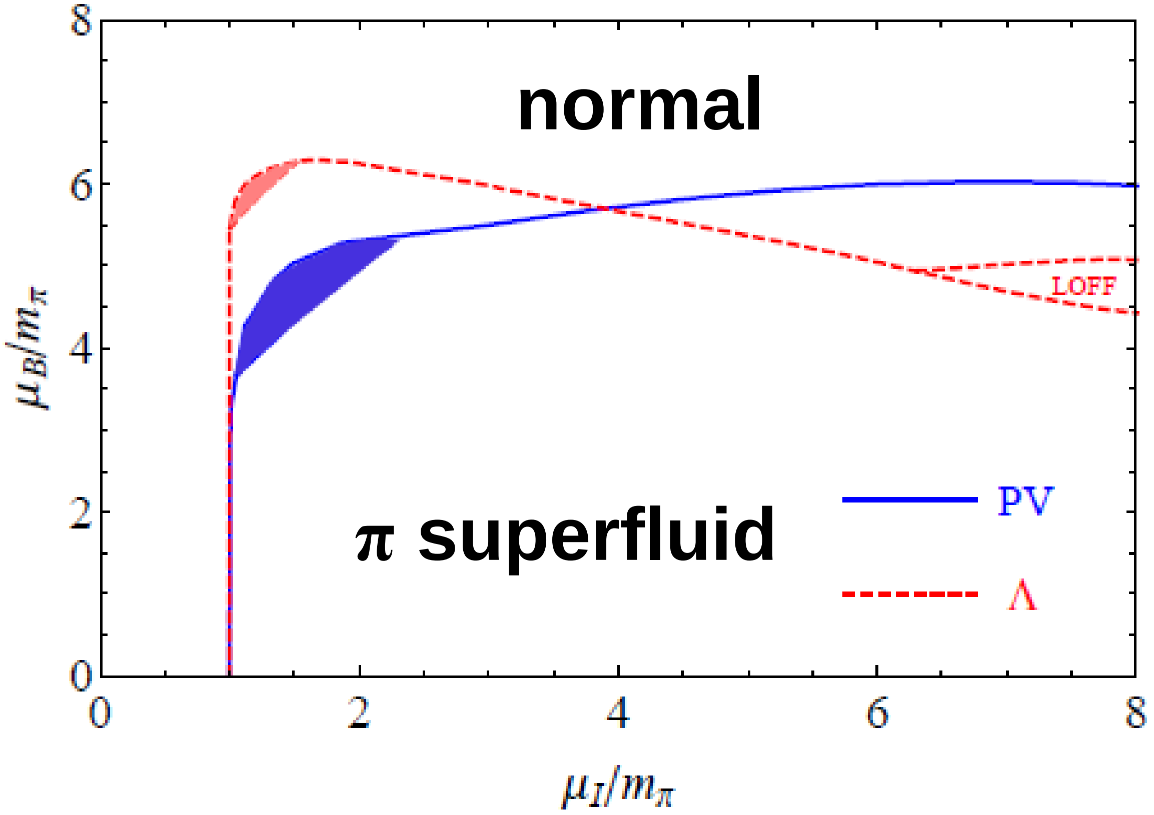
πS : $M_{max} \approx 3.0 M_{\odot}$, $R_{max} \approx 14.0$ km;

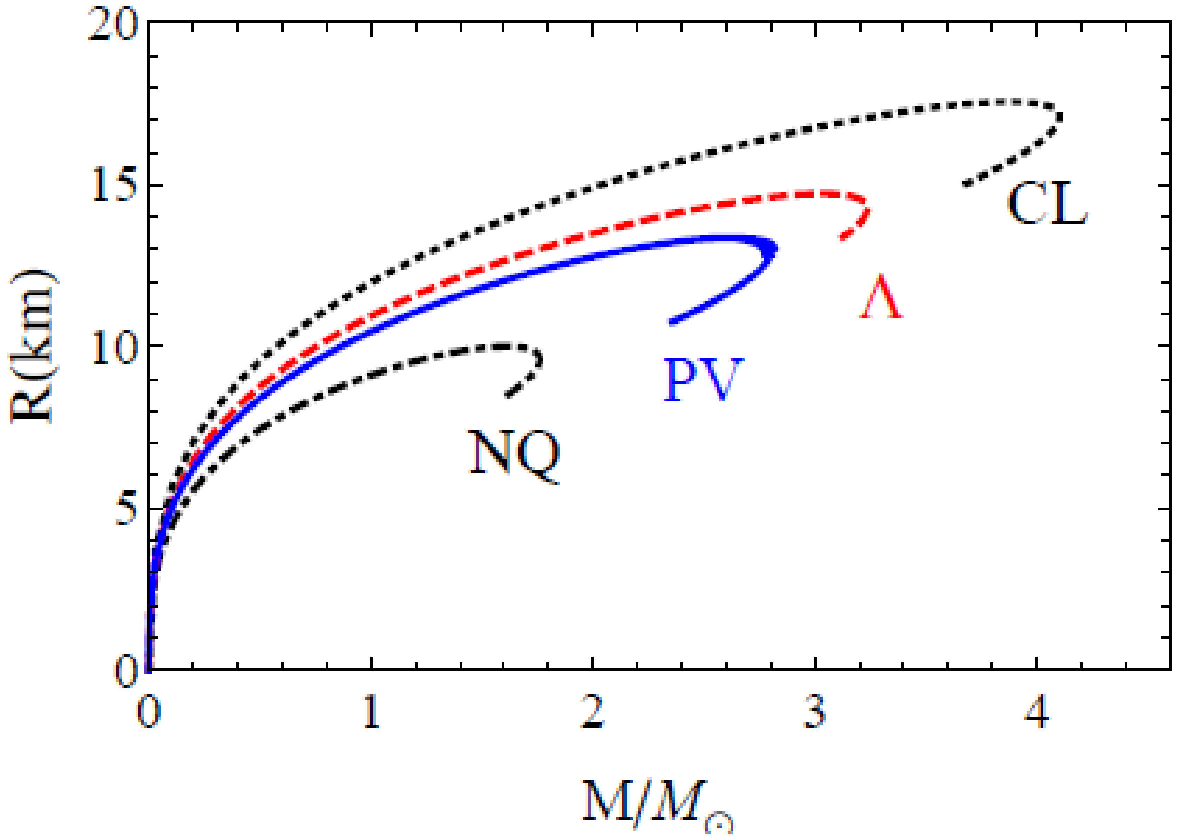
NQ : $M_{max} \approx 1.8 M_{\odot}$, $R_{max} \approx 10.0$ km;

larger and massive star
in pion superfluid phase

summary

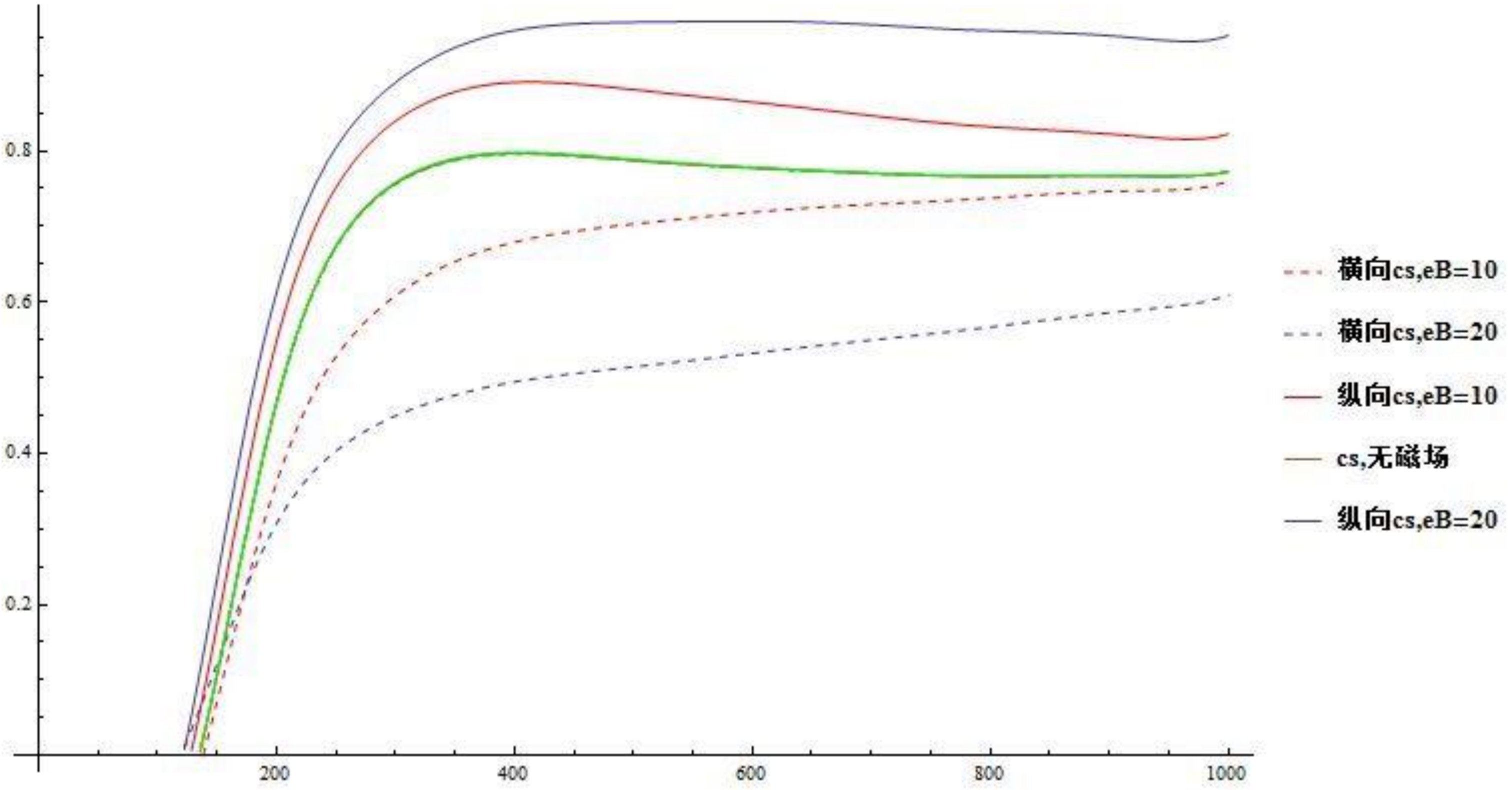
- Can massive compact star be in pion superfluid state?
- The star in π superfluid phase supports a larger M & R.

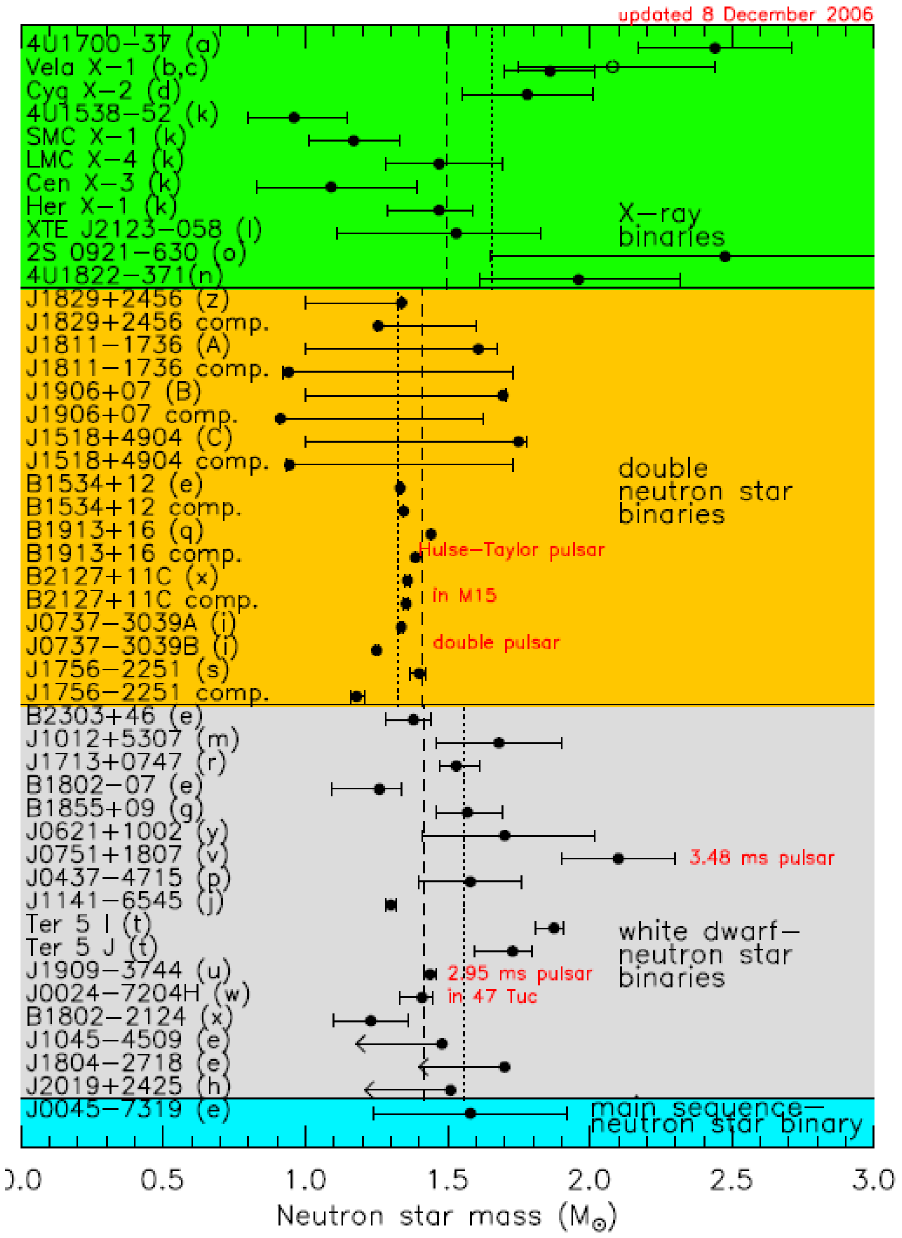




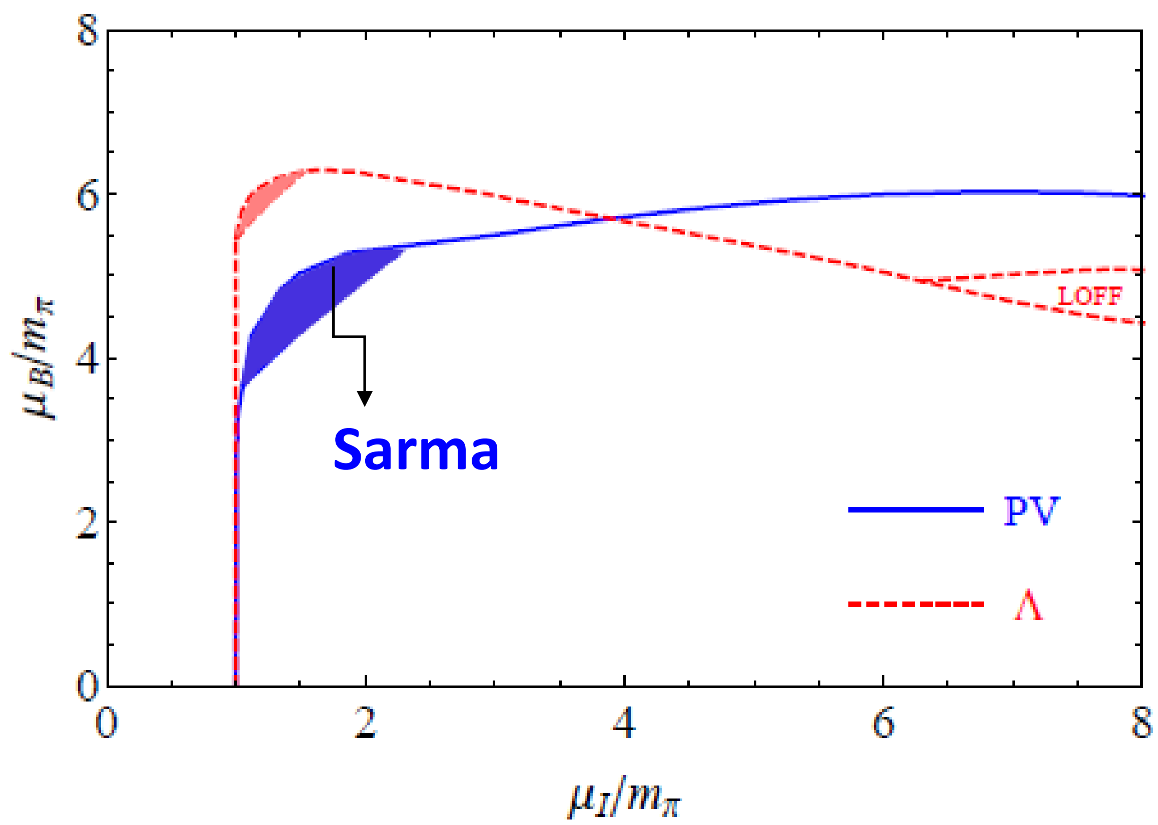
Back-up

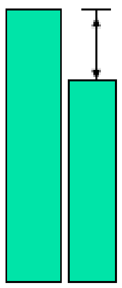
$$\begin{aligned}\Omega(q,0,m)-\Omega(0,0,m) &= q\frac{\partial\Omega}{\partial q}\Big|_{q=0}+\frac{q^2}{2}\frac{\partial^2\Omega}{\partial q^2}\Big|_{q=0}+\ldots \\ &= -\frac{3q^2}{\pi^2}\int_0^\Lambda\frac{p^2dp}{\sqrt{p^2+m^2}}+\ldots \\ &\neq 0. \qquad (8)\end{aligned}$$

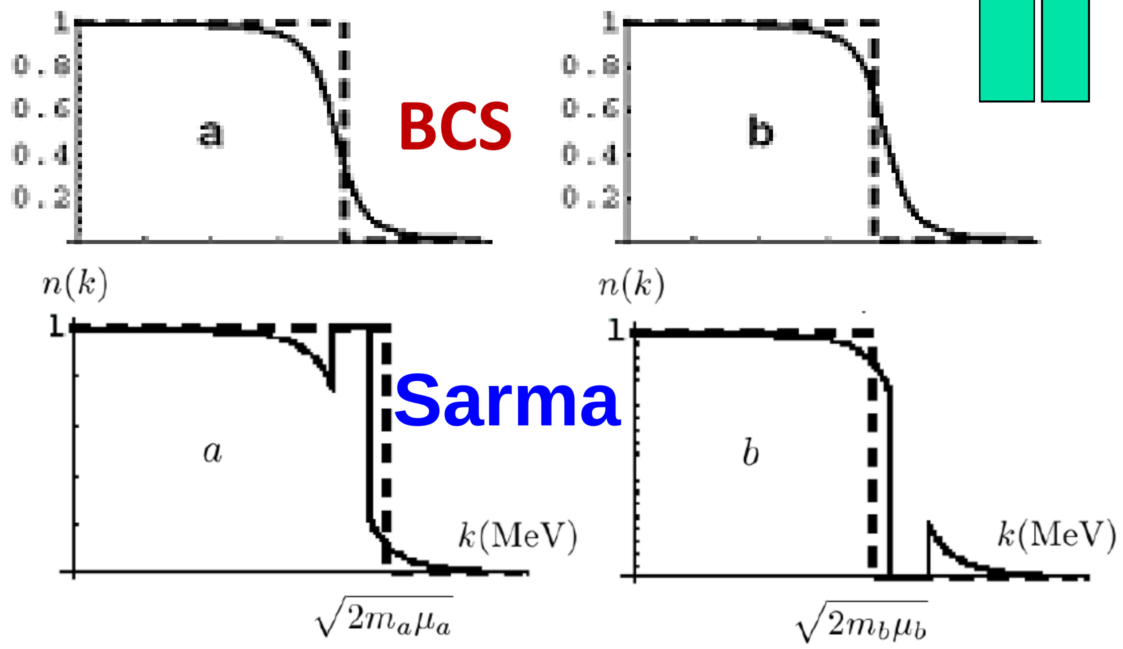




Phase structure in μ_I- μ_B Plane (T=0)

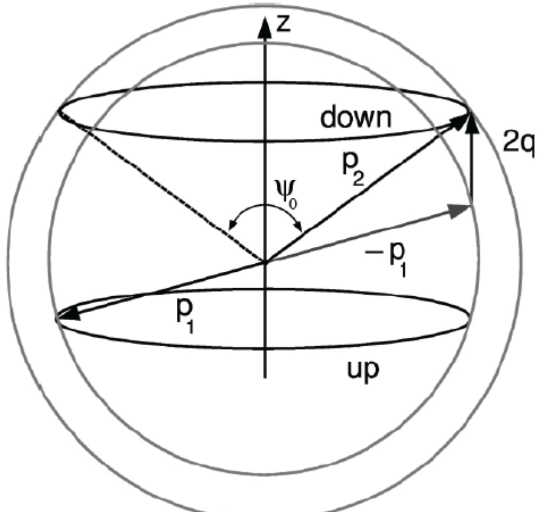






LOFF disappears!

LOFF(q≠0)



Regularization

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right]$$

1. Hard cutoff: $\bar{p}^2 < \Lambda^2$

✓ homogeneous state, T, μ << Λ

2. Pauli-Villars regularization:

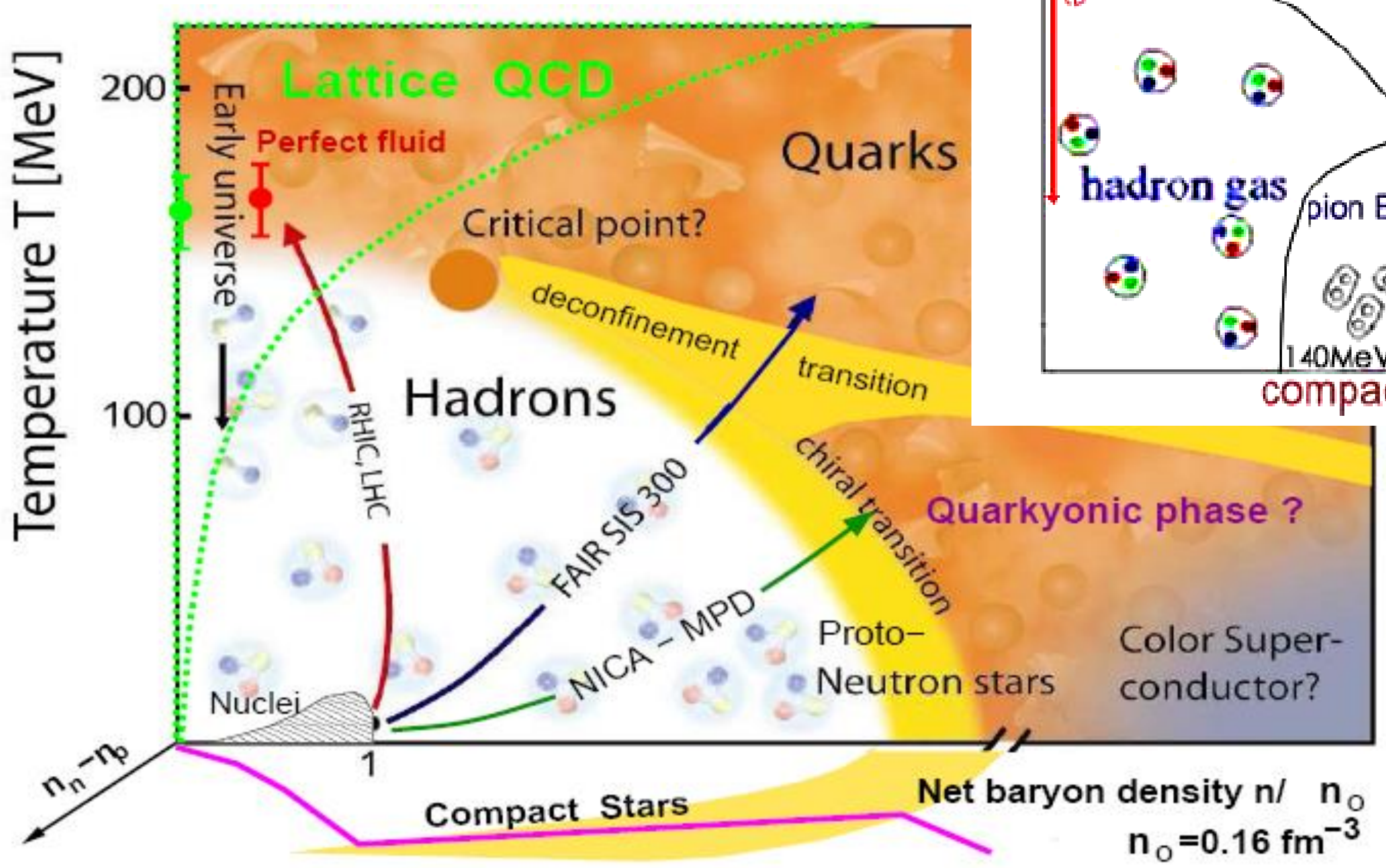
$$m \rightarrow m_j = \sqrt{m^2 + a_j\Lambda^2}.$$
$$E \rightarrow E_j = c_j E$$

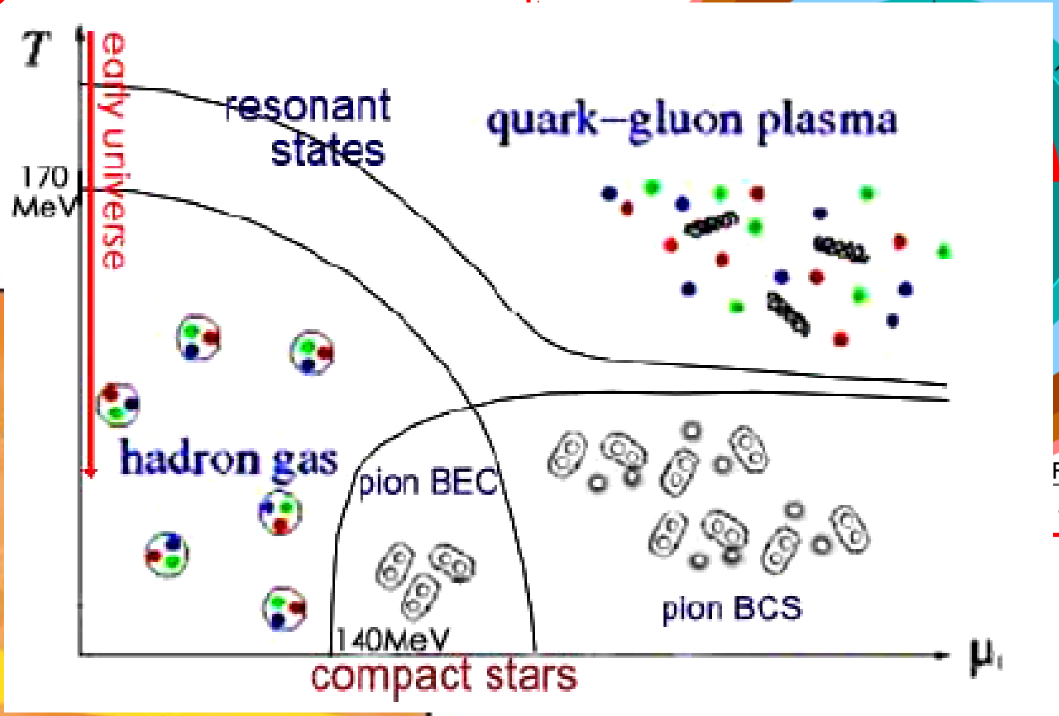
$$a_0 = 0,\ c_0 = 1, \sum_{i=0}^{N} c_i = 0, \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2) = 0, \ldots \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2)^{(N-1)} = 0$$

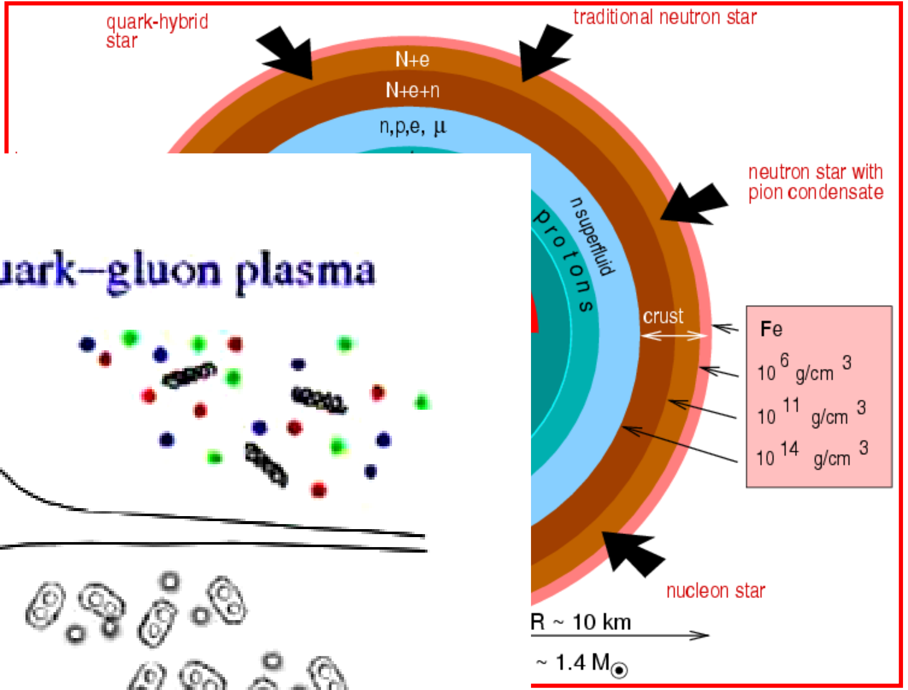
✓ homogeneous and inhomogeneous state

1. Compact Stars

low T, high density







glich, kick, cooling,

r-mode instability,

Mass-Radius relation,

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