Approaches to QCD phase diagram; effective models, strong coupling lattice QCD, and compact stars

Akira Ohnishi (YITP, Kyoto U.)

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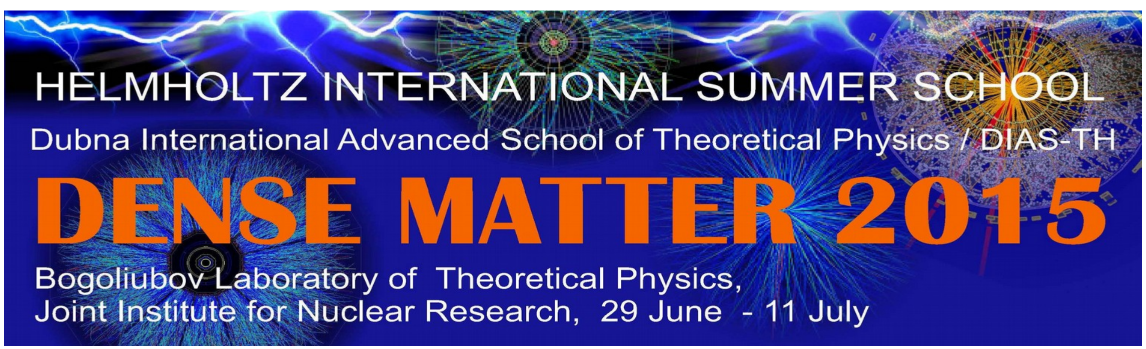
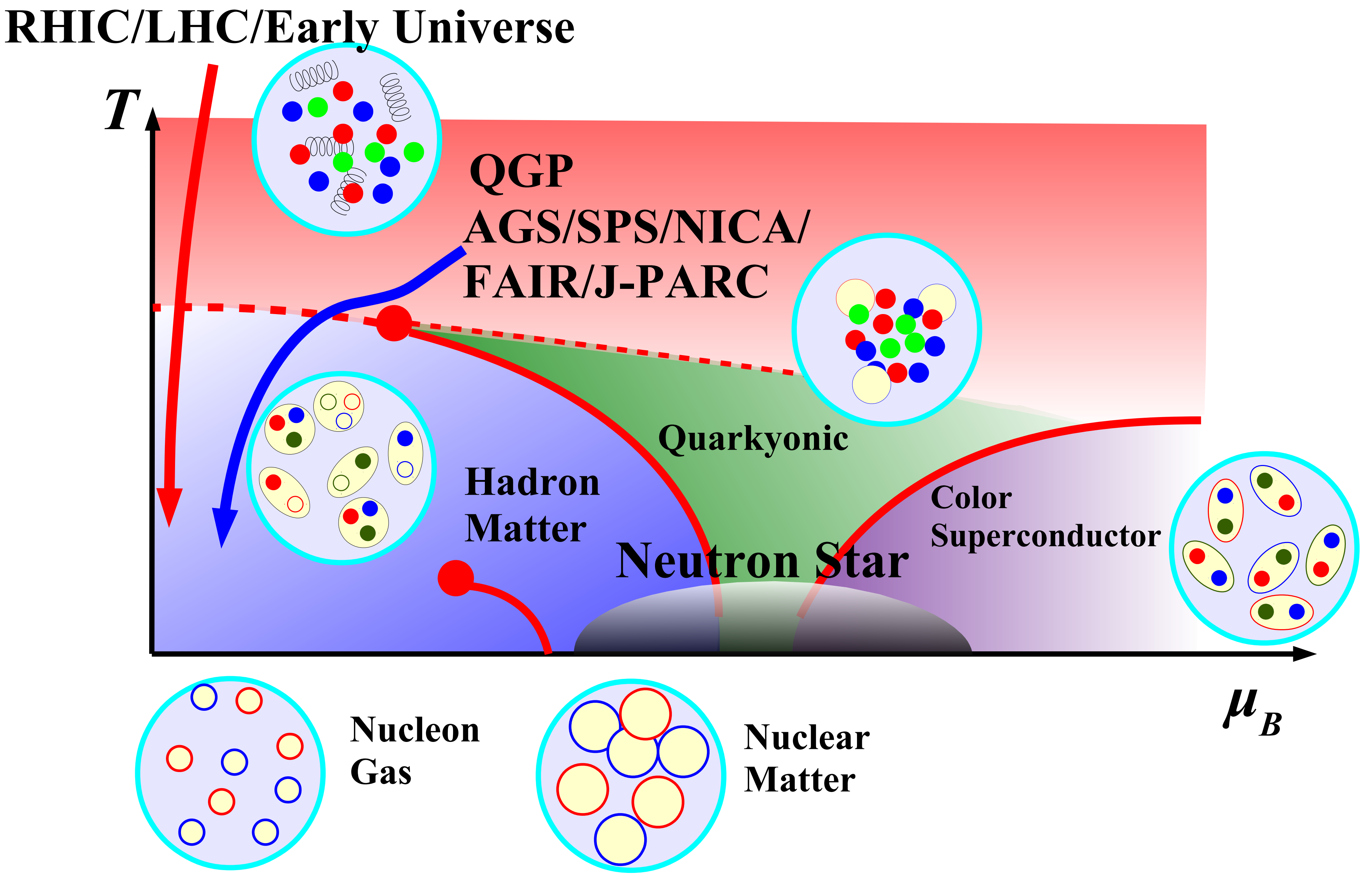
Helmholtz Int. Summer School & Dubna Int. Adv. School on Theor. Phys. / DIAS-TH, Bogoliubov Lab. of Theor. Phys,, Joint Inst. for Nucl. Research, Russia.



SQM DUBNA 2015

QCD Phase Diagram

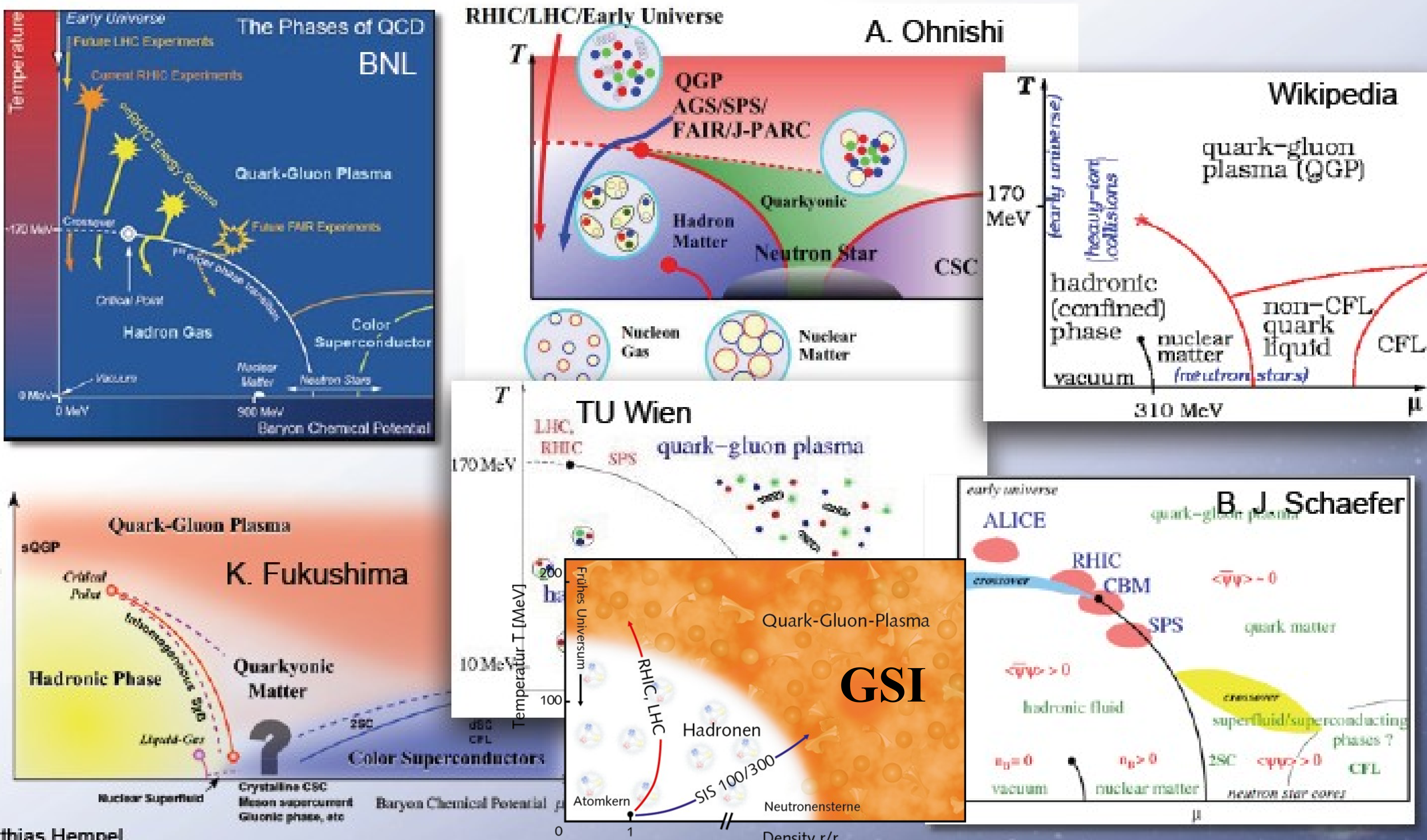


AO, PTPS 193('12)1

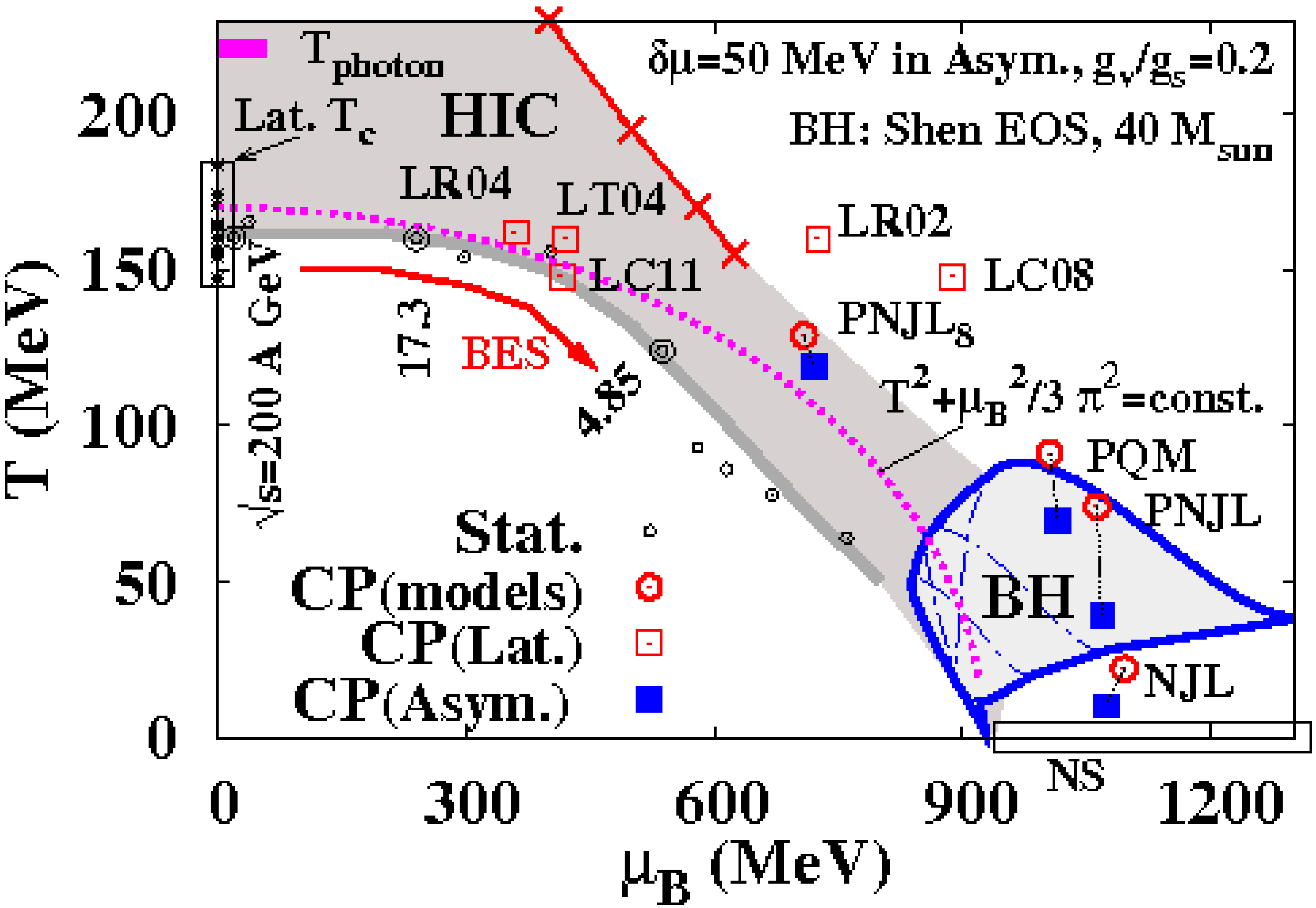
Introduction – QCD phase diagrams

by M. Hempel

- fundamental question: phase diagram of strongly interacting matter
- typical examples in T-μ, first order phase transitions (PT) as lines:



QCD phase diagram (Exp. & Theor. Studies)



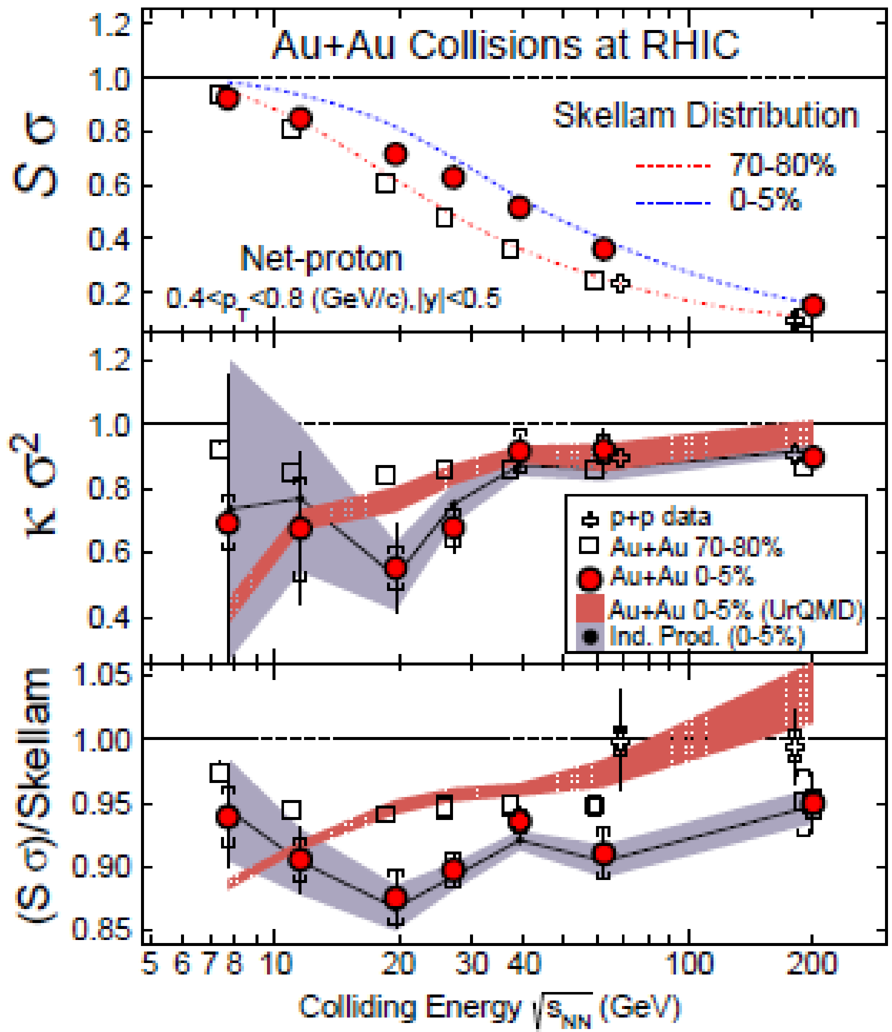
Hempel, Cleymans, Castorina, Randrup, and many others

> ***QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars***

AO, PTPS 193('12)1

Net-Proton Number Moments & Directed Flow

- **Non-monotonic behavior of $\kappa\sigma^2$ and dv_1/dy. CP signal ?**



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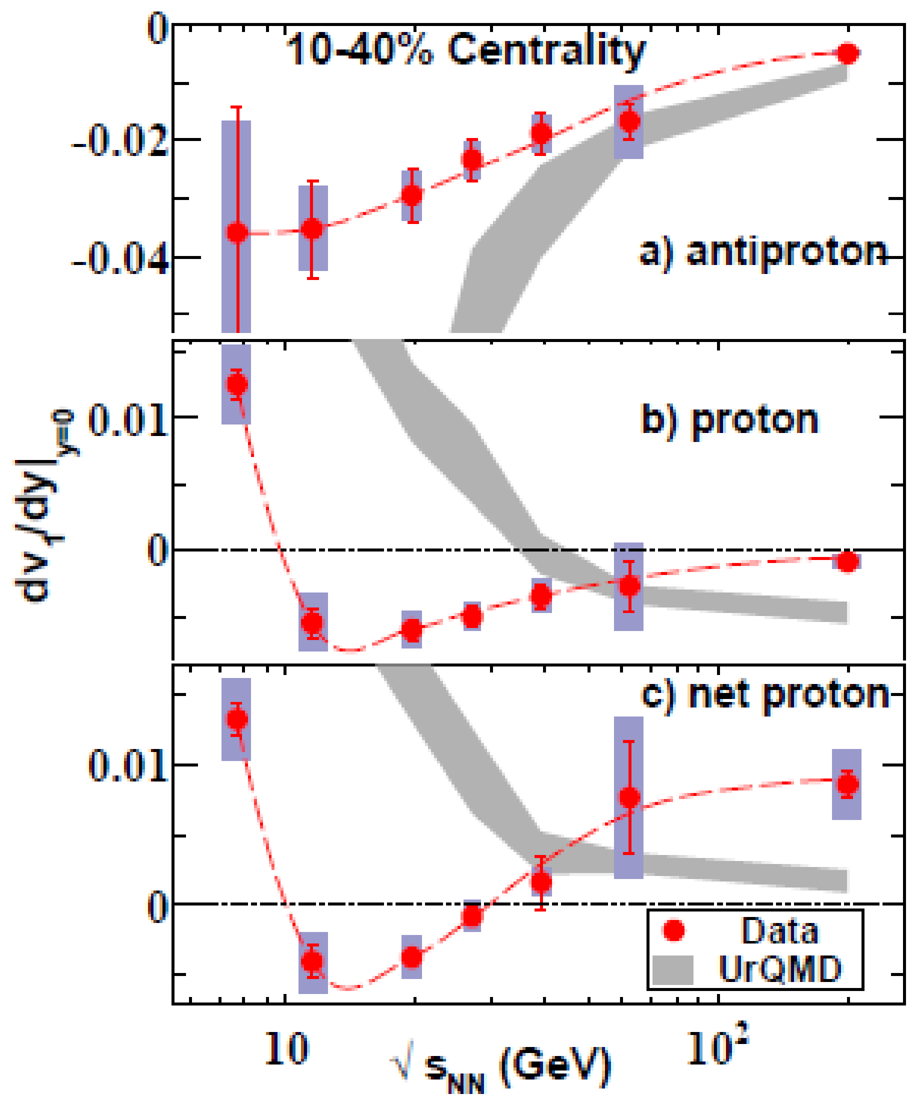
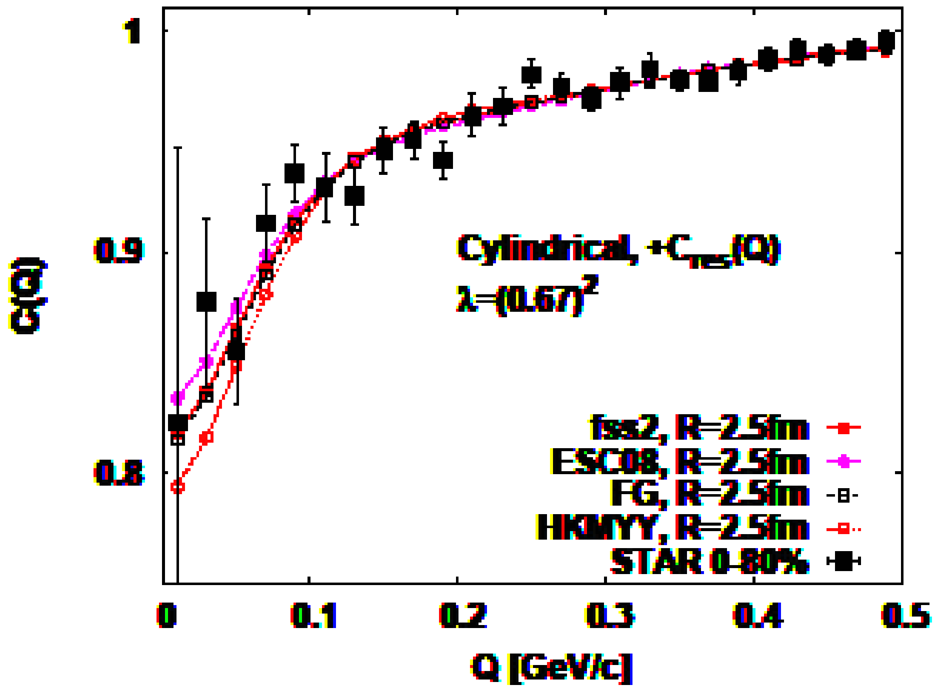
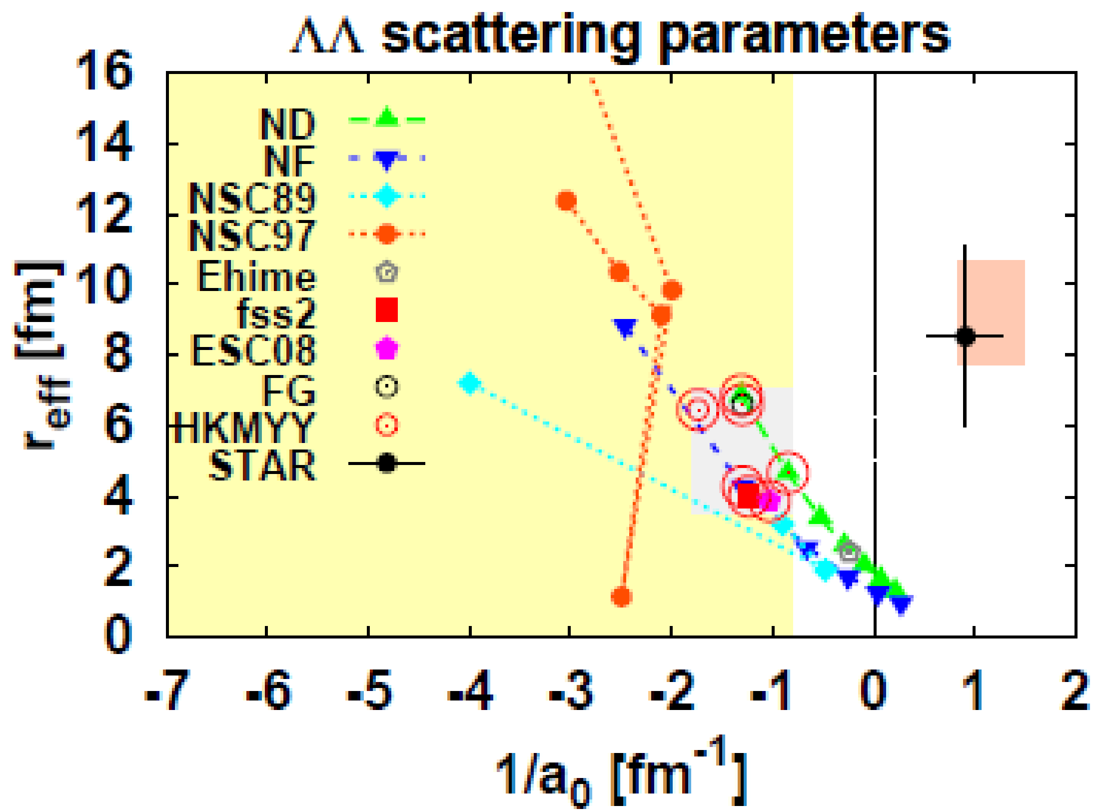


STAR Collab., PRL 112('14)162301.

ΛΛ interaction from ΛΛ correlation at RHIC

ΛΛ correlation with long. and transverse flow effects, Σ^0 feed down, and unknown long tail effects
→ Constraints on ΛΛ interaction





K.Morita, T.Furumoto, AO, PRC91('15)024916 [arXiv:1408.6682]
Data: Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

Physics of Dense Matter

- "Dense Matter" ($\rho_B > \rho_0$) and QCD phase diagram would be probed in heavy-ion collisions and compact star phenomena.
- Theoretical approaches to QCD phase diagram
  - Lattice QCD Monte-Carlo simulations (Sign problem)
  - Effective models (Lec.1, prediction is model dependent)
  - Approximation in LQCD, e.g. Strong-coupling lattice QCD
- Dense matter in compact star phenomena
  - Neutron Stars, Supernova, Black Hole formation, Binary Neutron Star Merger, ….
  - Key variable = Y_Q = Q(of hadrons) / B (Nuclear matter Y_Q=Y_e)
    → Phase diagram of isospin-asymmetric matter

Contents

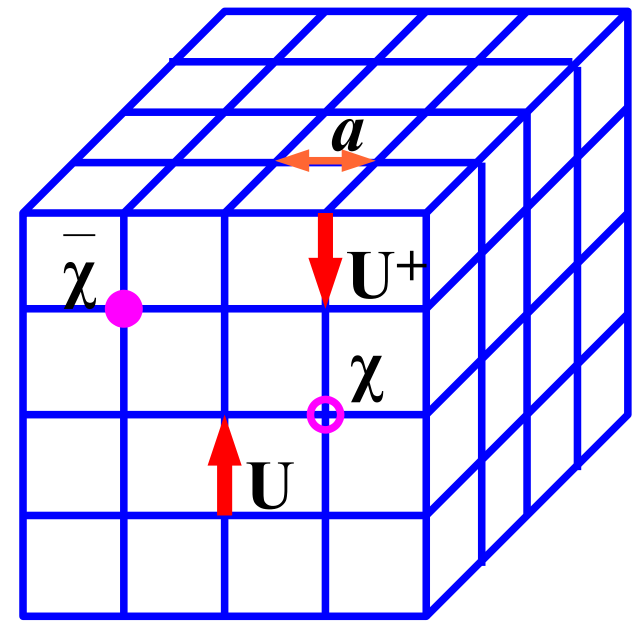
QCD phase diagram in strong-coupling lattice QCD

Lattice QCD

- **Space-time discretization of fields**
- **Quarks = Grassmann number on sites**

$$\chi_i \chi_j = -\chi_j \chi_i, \quad \int d\chi\, 1 = 0, \quad \int d\chi\, \chi = 1$$

$$\rightarrow \int d\chi_1 d\chi_2 \cdots d\bar{\chi}_1 d\bar{\chi}_2 \cdots \exp(\bar{\chi} D \chi) = det(D)$$



- **Gluons → Link variable**

$$U_\mu(x) = \exp\left[ig \int_x^{x+\hat{\mu}} dx\, A(x) \right] \sim \exp(i g A_\mu)$$

$$\int dU\, U_{ab} = 0, \quad \int dU\, U_{ab} U^+_{cd} = \delta_{ad}\delta_{bc}/N_c, \quad \int dU\, U_{ab} U_{cd} U_{ef} = \varepsilon_{ace}\varepsilon_{bdf}/N_c!$$

- **Gauge transf.**

$$\chi(x) \rightarrow V(x)\chi(x), \quad \bar{\chi}(x) \rightarrow \bar{\chi}(x) V^+(x),$$
$$U_\mu(x) \rightarrow V(x) U_\mu(x) V(x+\hat{\mu})$$
$$\bar{\chi}(x) U_\mu(x) \chi(x+\hat{\mu}) = \text{invariant}$$

Lattice spacing = a
→ Lattice unit: a=1

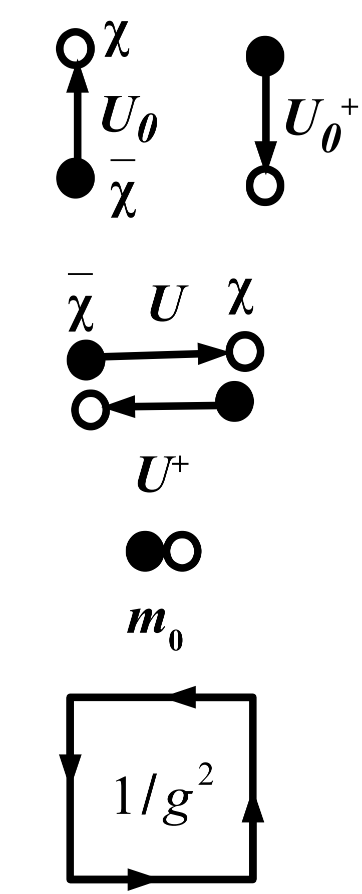
Lattice QCD action

- **Lattice QCD action (unrooted staggered fermion)**

$$L=\frac{1}{2}\sum_x\left[\bar{\chi}_x U_0(x) e^{\mu}\chi_{x+\hat{0}}-\bar{\chi}_{x+\hat{0}} U_0^{+}(x) e^{-\mu}\chi_x\right]$$
$$+\frac{1}{2}\sum_{x,j}\eta_j(x)\left[\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}} U_j^{+}(x)\chi_x\right]$$
$$+m_0\sum_x\bar{\chi}_x\chi_x$$
$$+\frac{2N_c}{g^2}\sum_{plaq.}\left[1-\frac{1}{N_c}\mathrm{Re\,tr}\,U_{\mu\nu}(x)\right]$$

$\rightarrow \chi\,(\partial + i\,gA)\,\chi$

Stokes theorem → rotation



- **Staggered sign factor (~ γ matrix)**
  $\eta_j(x)=(-1)^{**}(x_0+\ldots+x_{j-1})$
- **Chiral transf.**
  $\chi_x \rightarrow \exp[i\,\theta\varepsilon(x)]\,\chi_x$, $\varepsilon(x)=(-1)^{**}(x_0+x_1+x_2+x_3)$

χ quark (Grassmann #)
U link ~ exp(igA)

Sign problem in lattice QCD

- **Fermion determinant (= stat. weight of MC integral) becomes complex at finite μ in LQCD.**

$$Z=\int D[U,q,\bar{q}]\ \exp(-\bar{q}D(\mu,U)q-S_G(U))$$
$$=\int D[U]\ \mathrm{Det}(D(\mu,U))\ \exp(-S_G(U))$$

$$[\gamma_5 D(\mu)\gamma_5]^{+}=D(-\mu^*)\rightarrow[\mathrm{Det}(D(\mu))]^*=\mathrm{Det}(D(-\mu^*))$$
$$(\gamma_5\ \ \text{hermiticity})$$

  - **Note: Euclidean $D=\gamma_\mu D_\mu+m-\mu\gamma_0$ (γ = Hermite, D_μ= anti-Hermite)**
  - **Fermion det. (Det D) is real for zero μ (and pure imag. μ)**
  - **Fermion det. is complex for finite real μ.**

- **Approximate methods:**
  - **Taylor expansion, Imag. μ , Canonical, Re-weighting, Fugacity expansion, Histogram method, Complex Langevin, Strong-coupling lattice QCD**

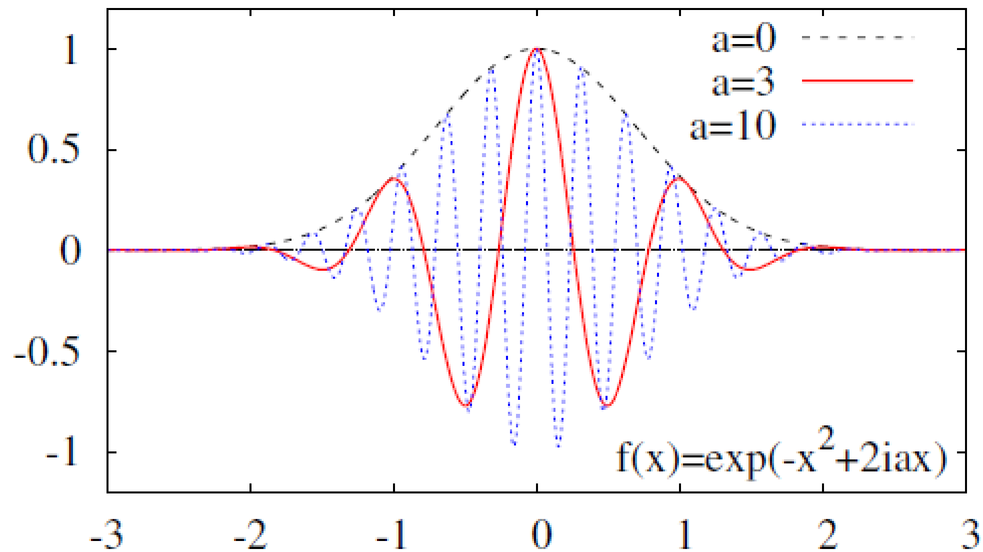
Sign Problem

- **Monte-Carlo integral of oscillating function**

$$Z=\int dx \exp(-x^2+2iax)=\sqrt{\pi}\exp(-a^2)$$

$$\langle O\rangle=\frac{1}{Z}\int dx\, O(x)\, e^{-x^2+2iax}$$

Easy problem for human is not necessarily easy for computers.



- **Complex phase appears from fluctuations of H and N.**
  de Forcrand

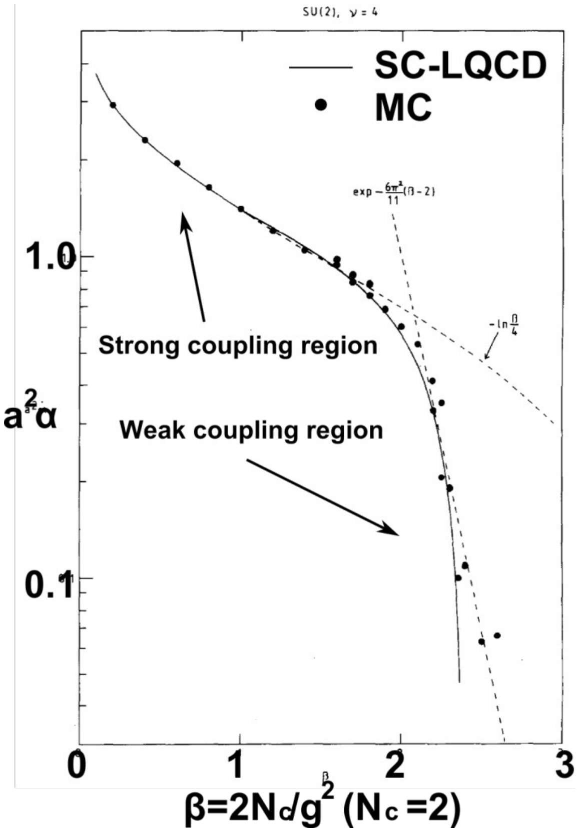
$$Z=\sum\langle\psi|\exp[-(H-\mu N)/T]|\psi\rangle=\sum\prod\langle\psi_\tau|\exp[-(H-\mu N)/(N_\tau T)]|\psi_{\tau+1}\rangle$$

→ Description based on "Hadronic" (color singlet) action would be helpful to reduce fluctuations.

→ Strong coupling lattice QCD

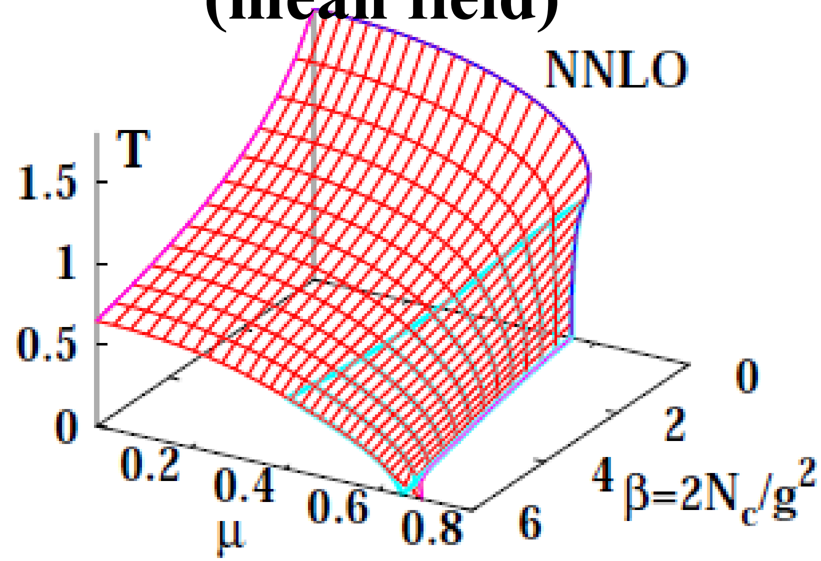
Strong Coupling Lattice QCD

Pure YM



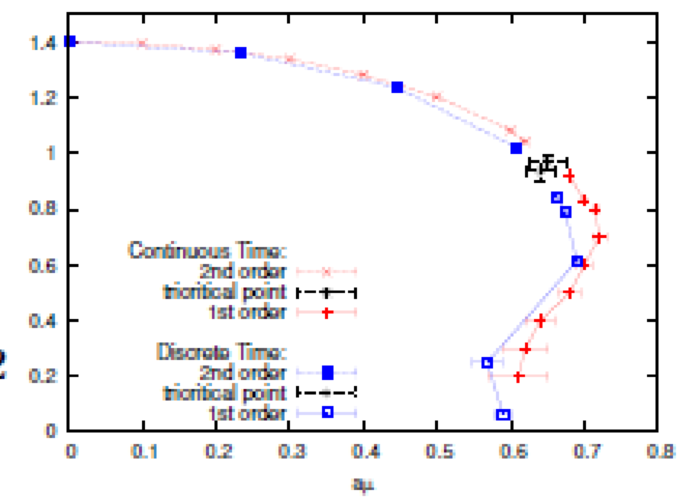
Wilson ('74), Creutz ('80), Munster ('80, '81), Lottini, Philipsen, Langelage's ('11)

Phase diagram (mean field)



Kawamoto ('80), Kawamoto, Smit ('81), Damagaard, Hochberg, Kawamoto ('85), Ilgenfritz, Kripfganz ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03); Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10)

Fluctuations



Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), AO, Ichihara, Nakano ('12), Ichihara, Nakano, AO ('14), de Forcrand, Langelage, Philipsen, Unger ('14)

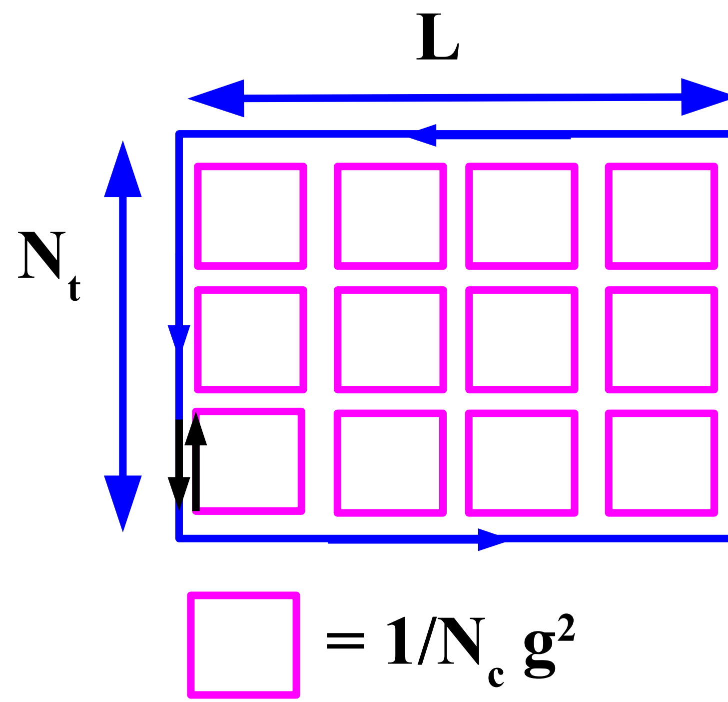
Area Law

Wilson ('74), Creutz ('80), Munster ('80, '81)

- **Wilson loop in pure Yang-Mills theory**

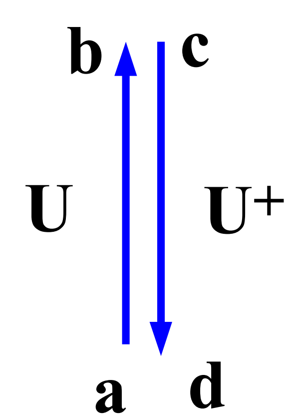
$$\langle W(C=L\times N_\tau)\rangle$$
$$=\frac{1}{Z}\int DU\, W(C)\exp\left[\frac{1}{g^2}\sum_P \mathrm{tr}\left(U_P+U_P^+\right)\right]$$
$$=\exp\left(-V(L)N_\tau\right)$$

V(L)= heavy-qq pot.



- **One-link integral**

$$\int dU\, U_{ab}\, U^+_{cd}=\frac{1}{N_c}\delta_{ad}\,\delta_{bc}$$



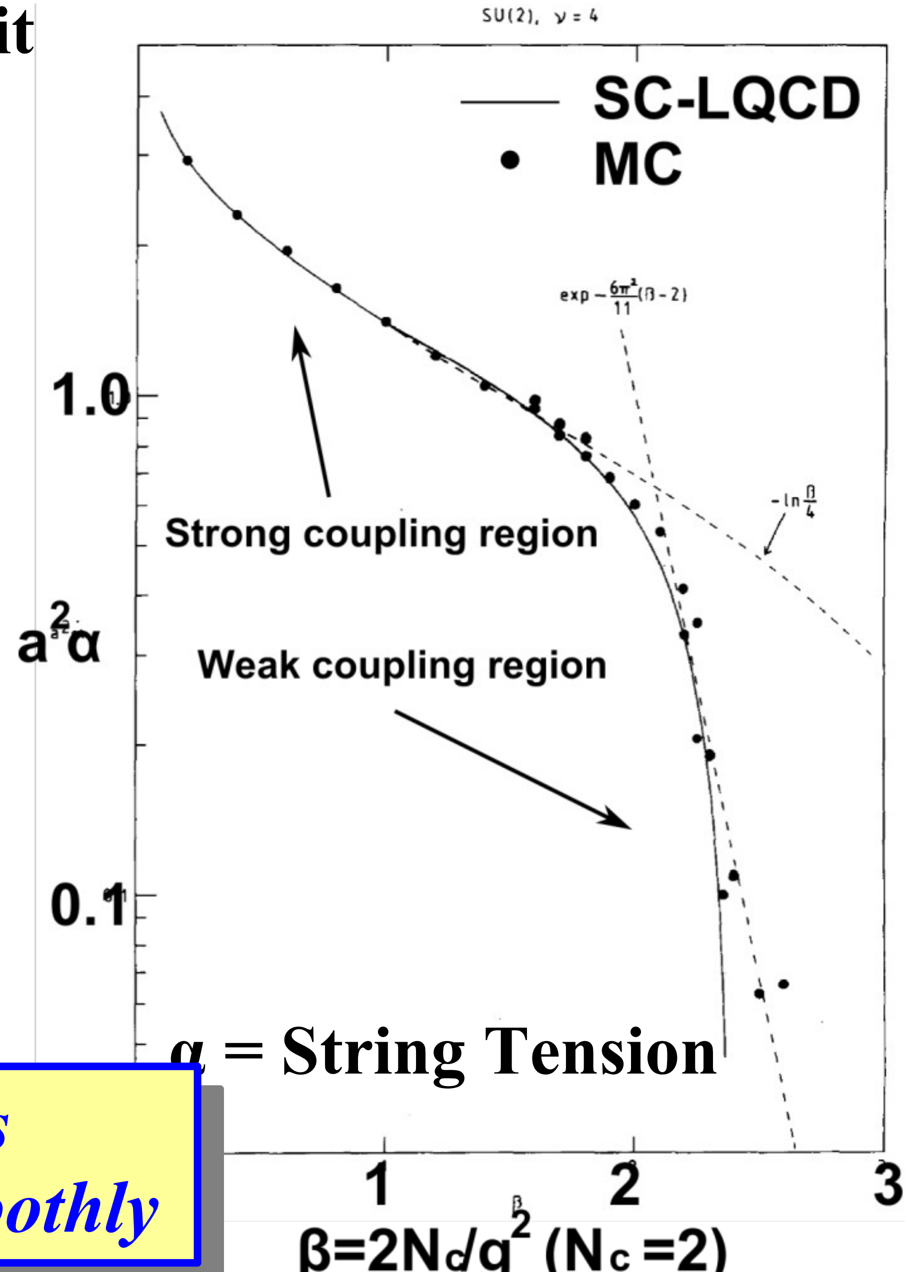
- **In the strong coupling limit**

$$\langle W(C)\rangle=N\left(\frac{1}{g^2 N}\right)^{L N_\tau}\rightarrow V(L)=L\log\left(g^2 N\right)$$

> ***Linear potential between heavy-quarks***
> ***→ Confinement (Wilson, 1974)***

Area Law

- **Area law in the strong coupling limit**
  Wilson ('74)
- **Verification of the area law in Lattice MC simulation**
  Creutz ('80)
- **Strong coupling expansion to higher orders**
  Munster ('80, '81),
  Lottini, Philipsen, Langelage ('11)
- **Weak coupling region**
  $\to g^2/4\pi = 1 / \beta_0 \log (q^2/\Lambda^2)$
  $\to a \sim 1/q \sim \exp(2\pi/g^2\beta_0)/\Lambda$



α = String Tension

Strong coupling expansion connects SCL and Weak coupling region smoothly

Strong Coupling Lattice QCD

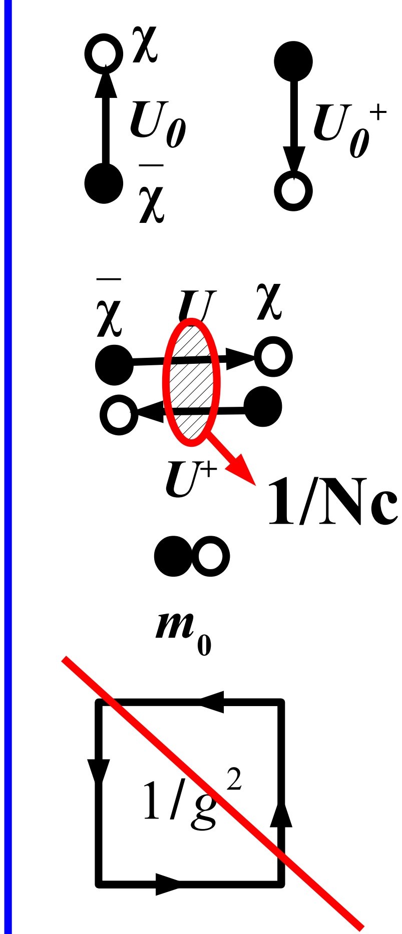
- **Strong coupling limit**
  Damgaard, Kawamoto, Shigemoto ('84)

$$S_{\mathrm{SCL}} = S_F^{(t)} - \frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}} + m_0 \sum_x M_x$$

$$\left(M_x = \bar{\chi}_x \chi_x\right)$$

  - **Integrate out spatial links using one-link formula, and pick up diagrams with min. quark numbers.**

$$\int dU\, U_{ab} U^{+}_{cd} = \delta_{ad}\delta_{bc}/N_c$$



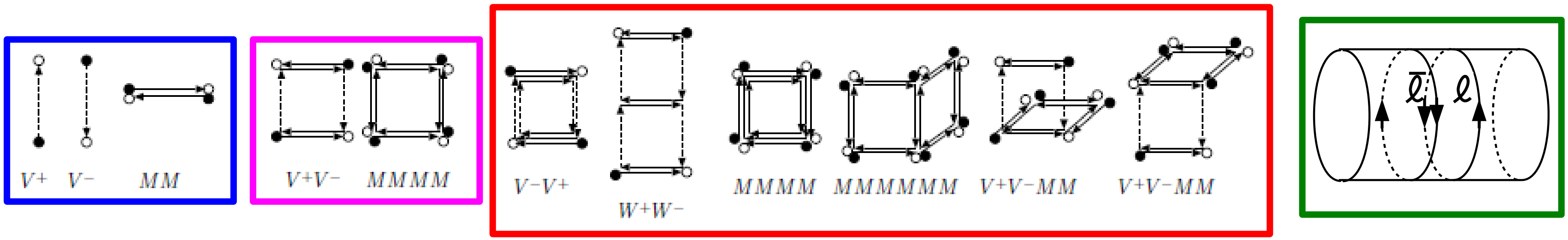
Lattice QCD in SCL
→ Fermion action with nearest neighbor four Fermi interaction

Finite Coupling Effects

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → $S_{\rm eff}$

$$S_{\rm eff} = S_{\rm SCL} - \log\langle \exp(-S_G)\rangle = S_{\rm SCL} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n \rangle_c$$

$\langle S_G^n \rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\rm eff} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

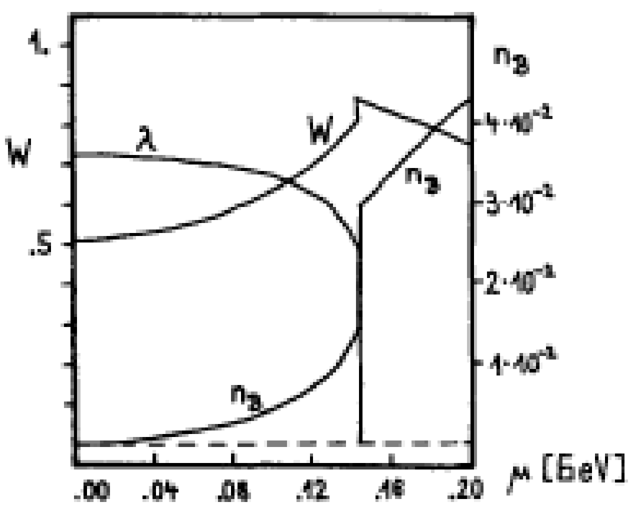
$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

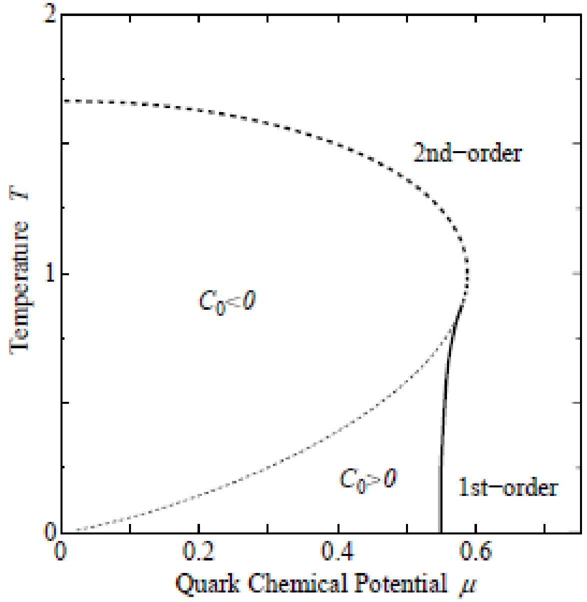
$$-\left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\mathbf{x},j>0}\left(\bar{P}_{\mathbf{x}} P_{\mathbf{x}+\hat{j}} + h.c.\right)$$ *Polyakov loop (Gocksch, Ogilvie ('85), Fukushima ('04) Nakano, Miua, AO ('11))*

Phase diagram in SC-LQCD (mean field)

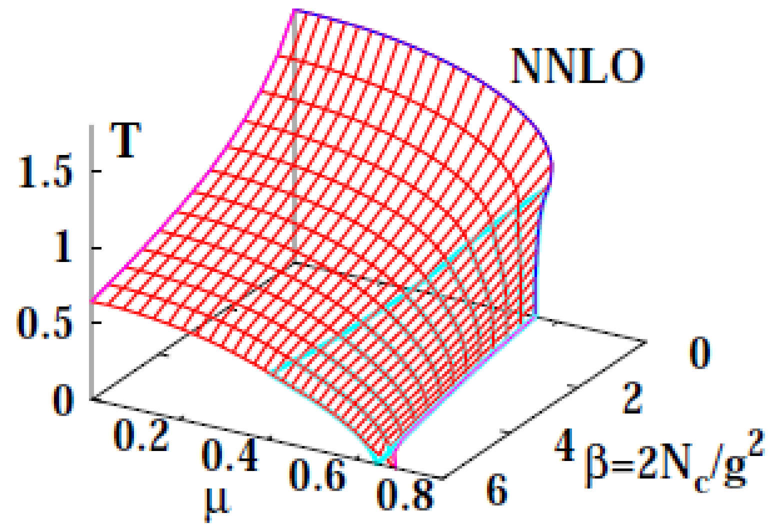
- "Standard" simple procedure in Fermion many-body problem
  - Bonosize interaction term (Hubbard-Stratonovich transformation)
  - Mean field approximation (constant auxiliary field)
  - Fermion & temporal link integral
    Damgaard, Kawamoto, Shigemoto ('84); Ilgenfritz, Kripfganz ('85); Faldt, Petersson ('86); Bilic, Karsch, Redlich ('92); Fukushima ('04); Nishida ('04); Miura, Nakano, AO, Kawamoto ('09); Nakano, Miura, AO ('10, '11)



Ilgenfritz, Kripfganz ('85)



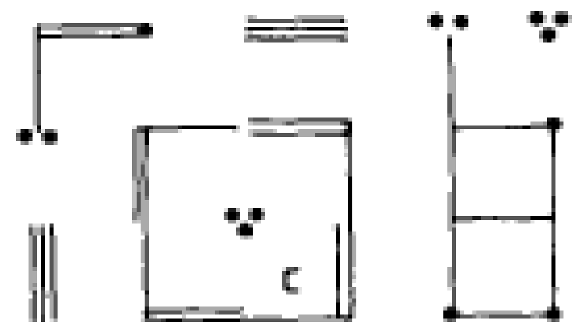
Fukushima ('04)



AO, Miura, Nakano, Kawamoto ('09)

SC-LQCD with Fluctuations

- **Monomer-Dimer-Polymer (MDP) simulation**
  Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11)
  - **Integrating out all links**
    → Z= weight sumof monomer, dimer, polymer configurations



$$Z(m,\mu) = \sum_{\{n_x, n_b, C_B\}} \prod_b \frac{(N_c - n_b)!}{N_c! n_b!} \prod_x \frac{N_c!}{n_x!}(2m)^{n_x} \prod_{C_B} w(C_B) \qquad w(C_B, \pm) = \varepsilon(C_B) \exp(\pm 3\ell L_t \mu)$$

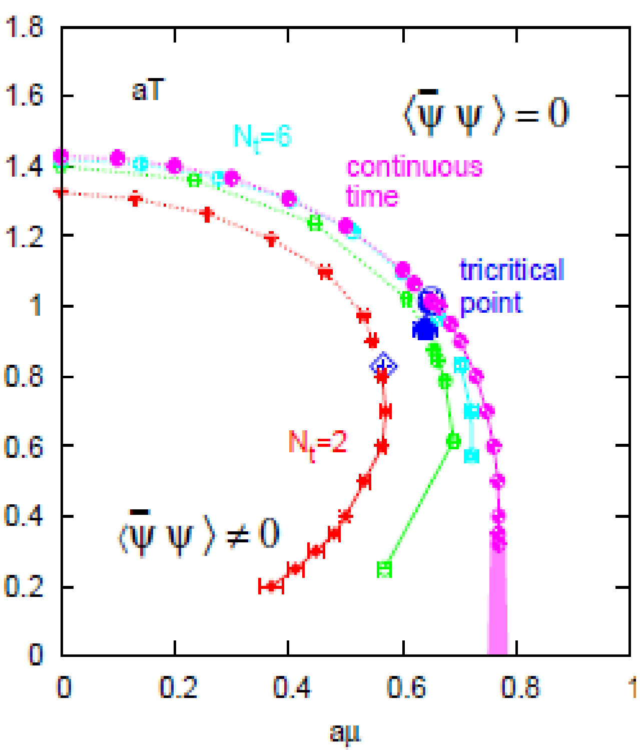
- **Auxiliary Field Monte-Carlo (AFMC) method**
  Ichihara, AO, Nakano ('14)
  - **Bonsonize the effective action, and MC integral over aux. field.**

$$S_{\text{eff}} = S_F^{(t)} + \sum_x m_x M_x + \frac{L^3}{4N_c} \sum_{\boldsymbol{k},\tau} f(\boldsymbol{k}) \left[|\sigma_{\boldsymbol{k},\tau}|^2 + |\pi_{\boldsymbol{k},\tau}|^2 \right]$$

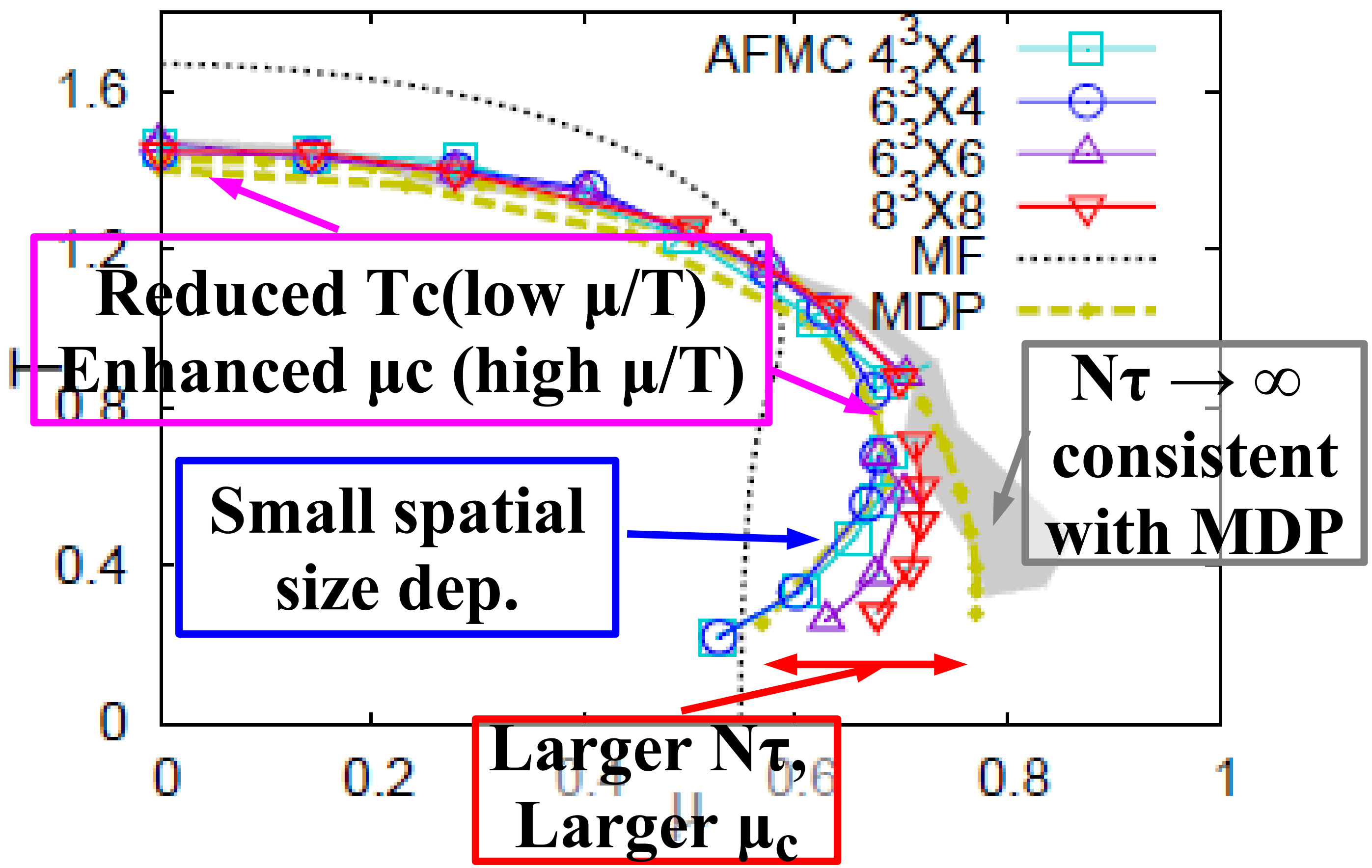
$$m_x = m_0 + \frac{1}{4N_c} \sum_j (\sigma + i\varepsilon\pi)_{x\pm\hat{j}}, \quad f(\boldsymbol{k}) = \sum_j \cos k_j, \quad \varepsilon = (-1)^{x_0+x_1+x_2+x_3}$$

Phase diagram

- **Phase diagrams in two independent methods (MDP & AFMC) agree with each other in the strong coupling limit.**
  → SCL phase diagram is determined !



de Forcrand, Fromm ('10), de Forcrand, Langelage, Philipsen, Unger ('14)



Ichihara, AO, Nakano ('14)

Cumulant Ratio: Phase transition signal ?

- **Cumulants** **c.f. Kaczmarek**

$$\chi^{(n)} = \frac{\partial^n (P/T^4)}{\partial \hat{\mu}^n}, \quad \hat{\mu} = \mu_B/T$$

$$\chi^{(4)}/\chi^{(2)} = \kappa \sigma^2 \quad (\kappa : \mathrm{kurtosis})$$

  - **$\kappa\sigma^2$ shows DOF at μ=0, and criticality at μ>0.**

- **Lattice MC at μ=0**
  Bazarov, .., Kaczmarek, et al.('14), Bellwied et al.('13), …. Gavai, Gupta ('05), Allton et al. ('05),

- **Lattice MC at μ>0 but large m_q**
  Jin, Kuramashi, Nakamura, Takeda, Ukawa ('15)

- **Scaling function analysis**
  Friman, Karsch, Redlich, Skokov ('11)

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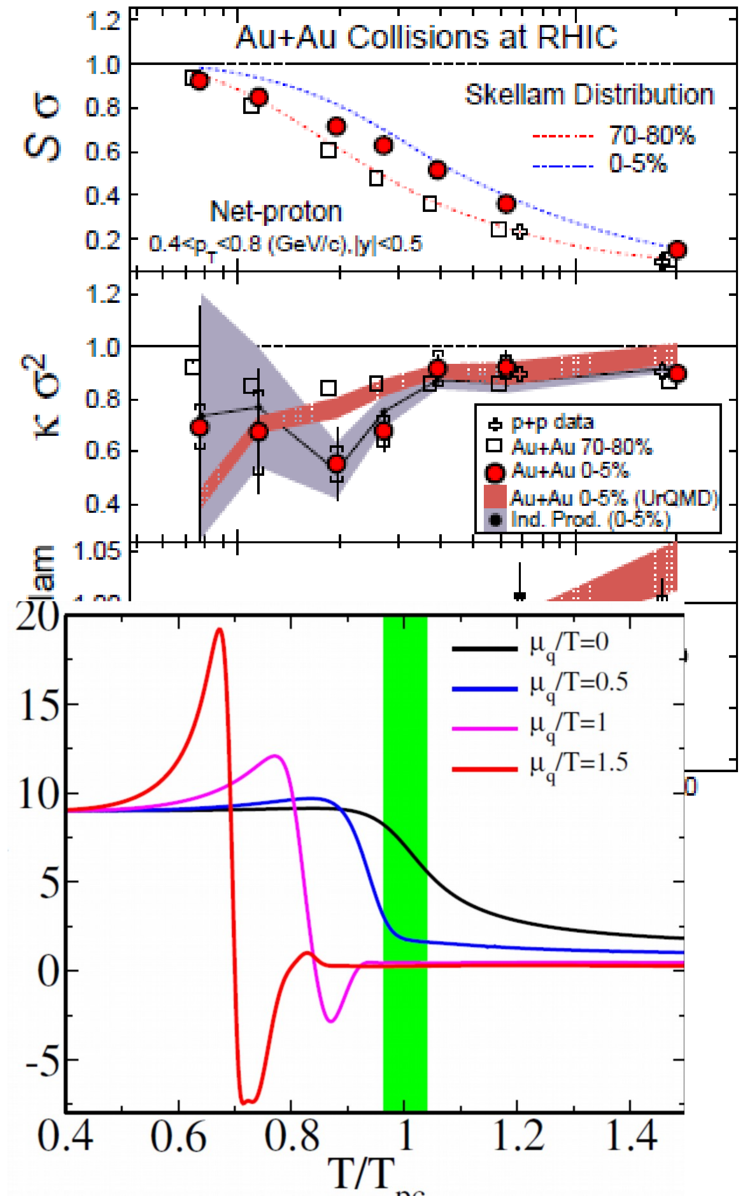
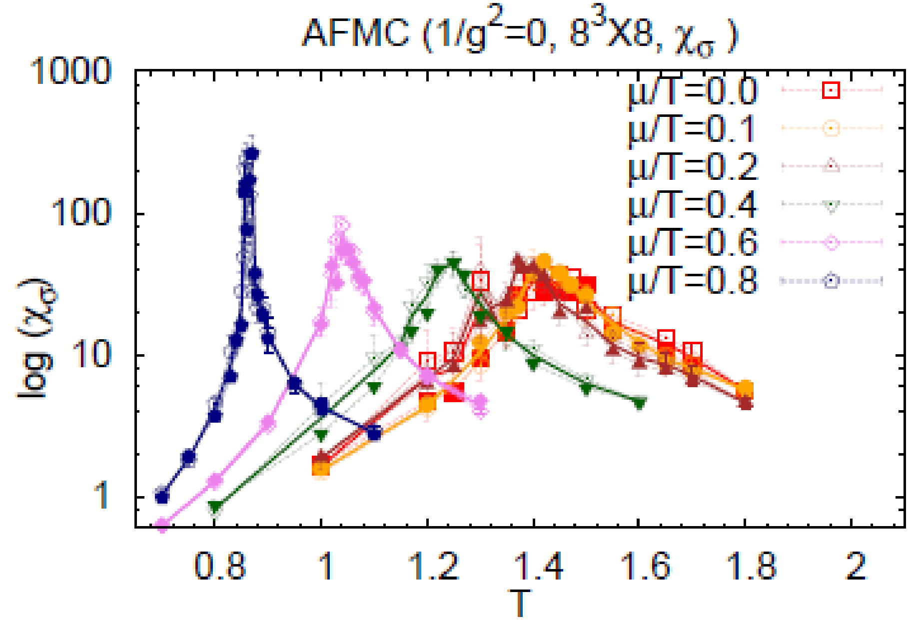


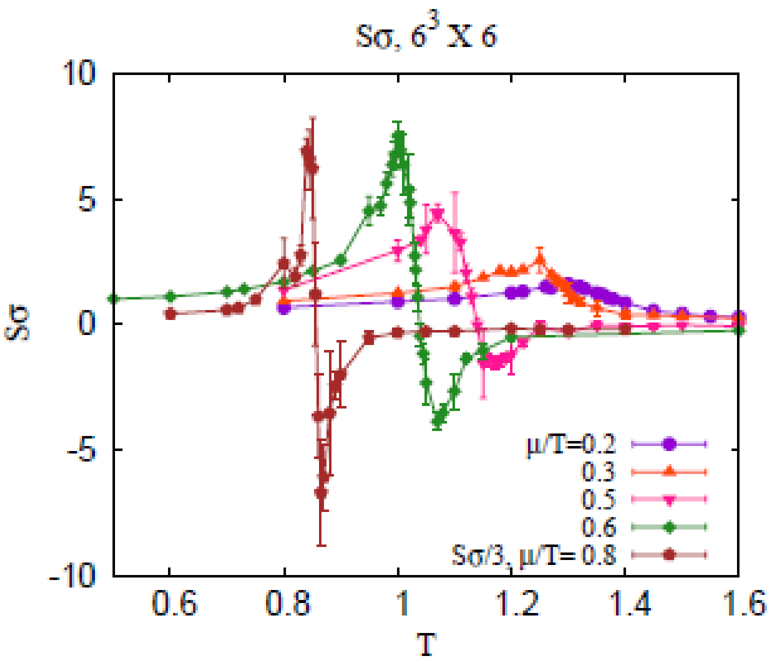
Friman et al. ('11)

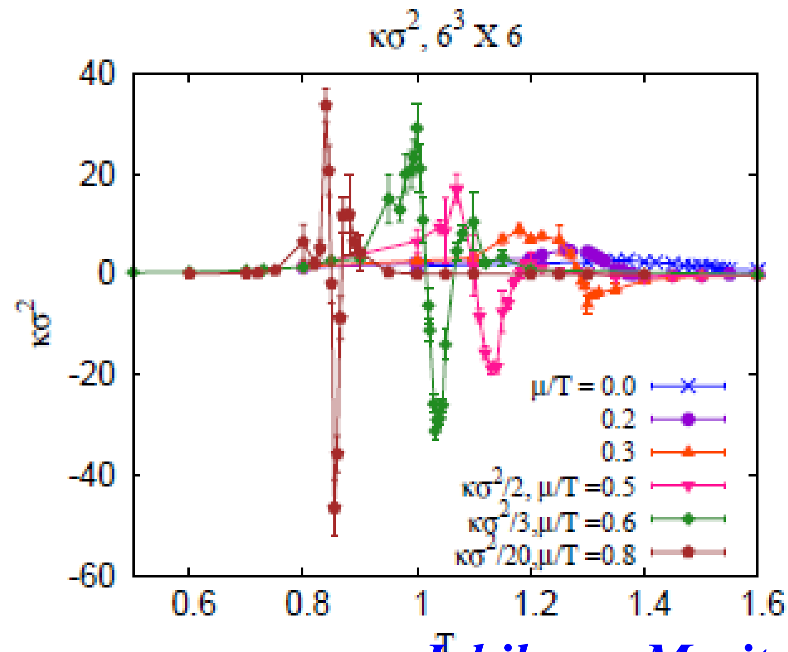
Susceptibilities, Skewness, Kurtosis, ...

- **Chiral susceptibility**
  → Divergent at V → ∞
- **Net baryon skewness**
  Sσ → +∞ from below
  - ∞ from above
- **Net baryon kurtosis**
  $\kappa\sigma^2$ → +-+ structure



Ichihara, AO, Nakano ('14)





Ichihara, Morita, AO, in prep.

Caveats

- **One species of unrooted staggered fermion corresponds to Nf=4 in the continuum limit, and should show the first order phase transition at μ=0. Second order transition shown here comes from O(2) chiral symmetry remaining also at coarse lattice spacing.**
- **We have worked in the leading order of 1/d expansion, where the MM term is assumed to remain finite at large spatial dim., d. Under this assumption, we quark field scales as $\chi \propto d^{-1/4}$, then terms with larger number of quarks such as spatial baryon hopping are suppressed. (MDP includes those terms.)**
- **Positive slope of the first order phase boundary comes from the saturated quark matter at high density, $\rho \sim Nc$. In this case, entropy is carried by the holes rather than particles, and can be smaller in the high density phase. Thus the Clausius-Clapeyron relation is not violated.**

$$P_H = P_Q \rightarrow \rho_H d\mu + s_H dT = \rho_Q d\mu + s_Q dT$$

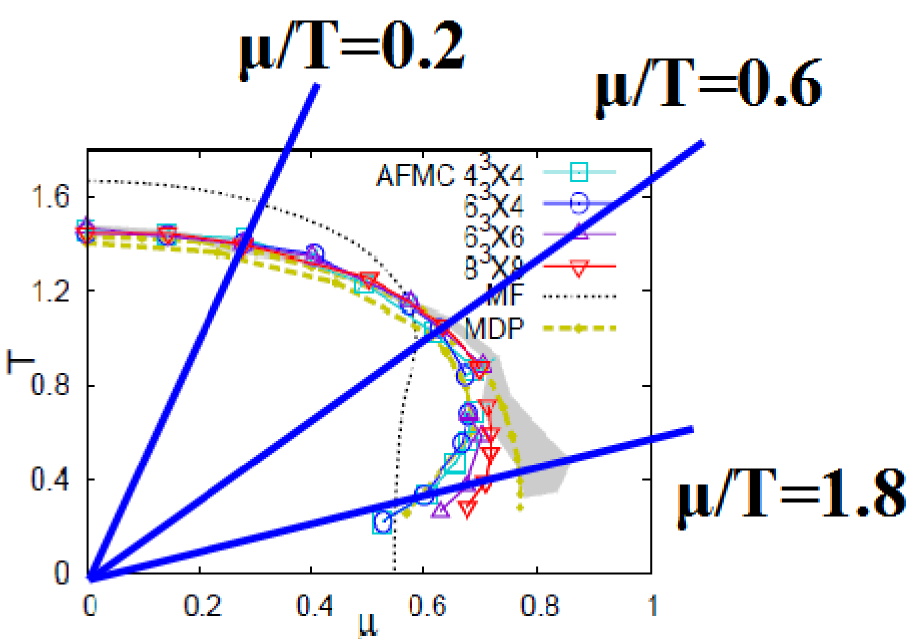
- **The sign problem exists in SC-LQCD when fluctuations are included, but it is not very severe and $V \rightarrow \infty$ limit may be obtained.**

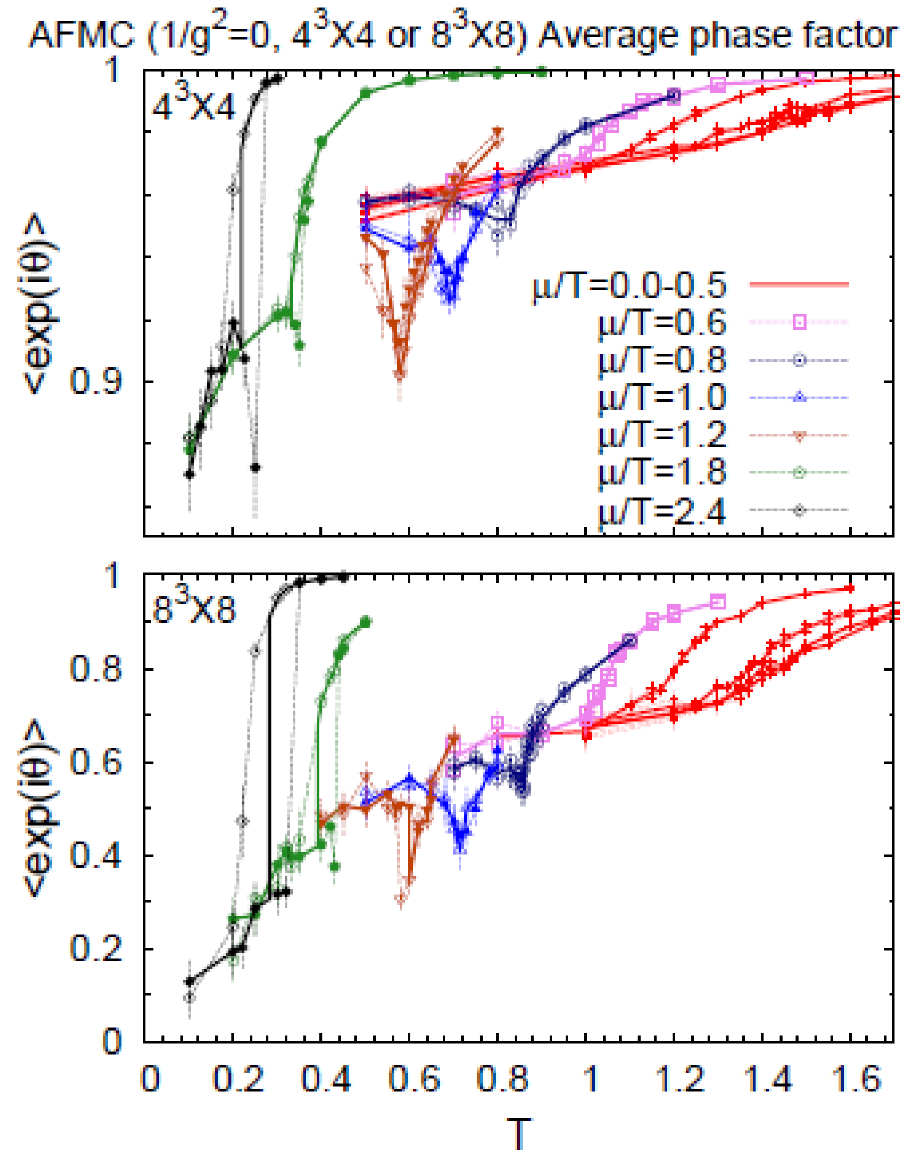
Average Phase Factor

- **Average phase factor = Weight cancellation**

$$\langle e^{i\theta} \rangle = Z_{\text{phase quenched}} \; / \; Z_{\text{full}}$$

- **AFMC results**
  - **$\langle e^{i\theta} \rangle > 0.9$ on 4^4 lattice**
  - **$\langle e^{i\theta} \rangle > 0.1$ on 8^4 lattice**

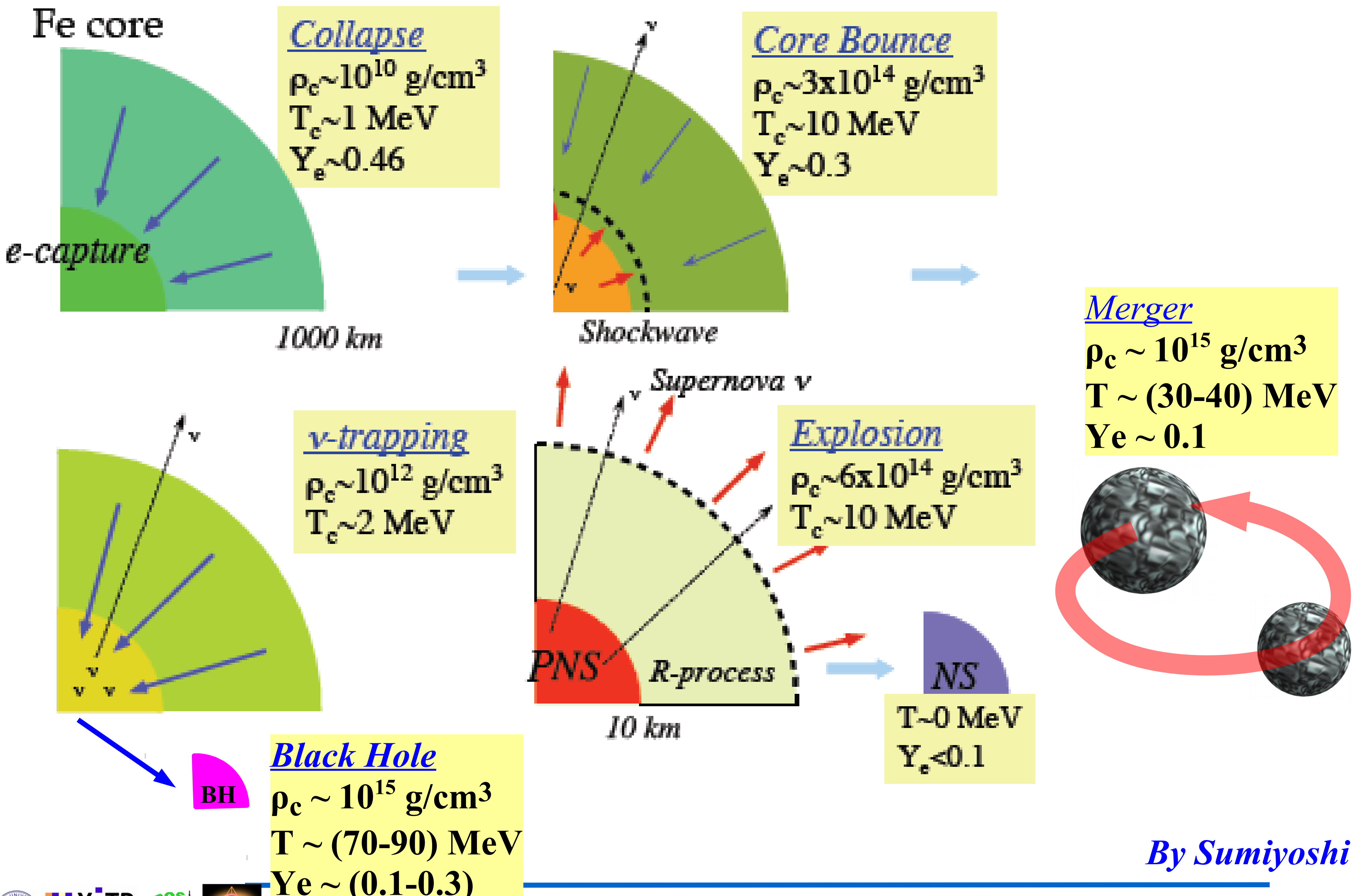




Ichihara, AO, Nakano ('14)

Dense Matter in Compact Star Phenomena

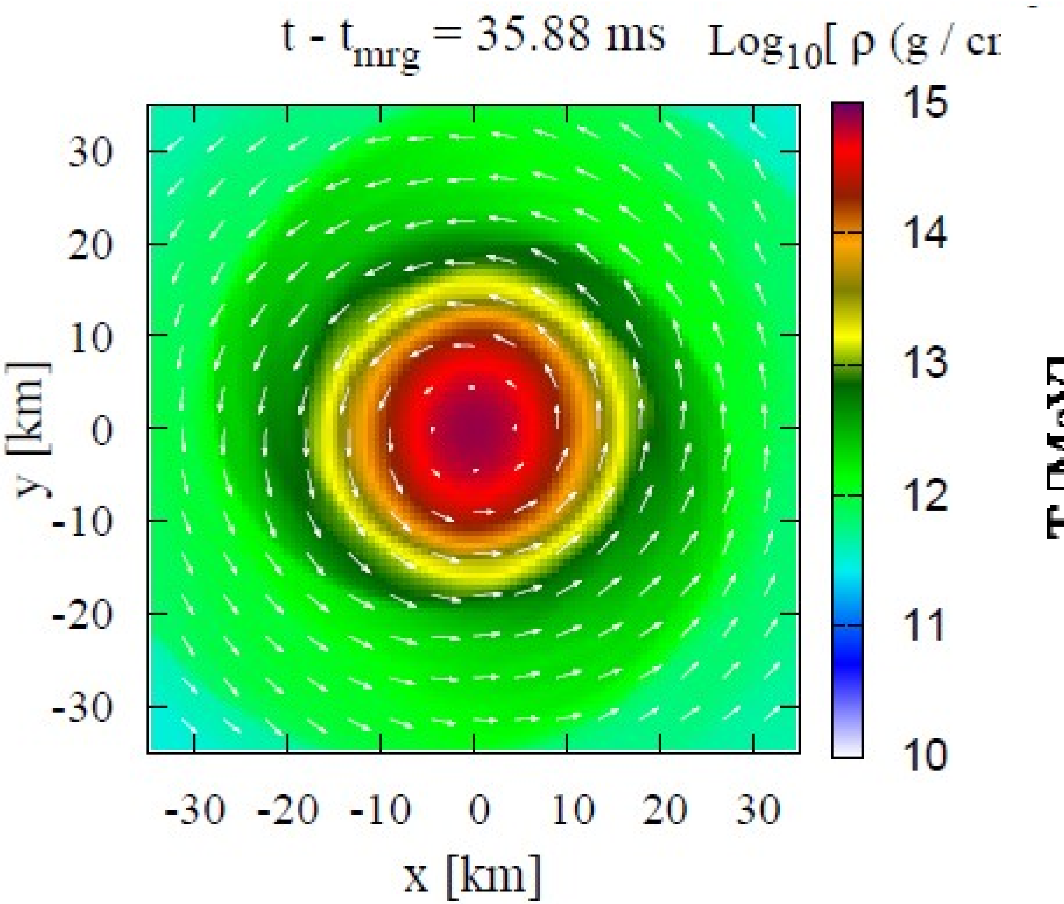
Gravitational Collapse of Massive Star

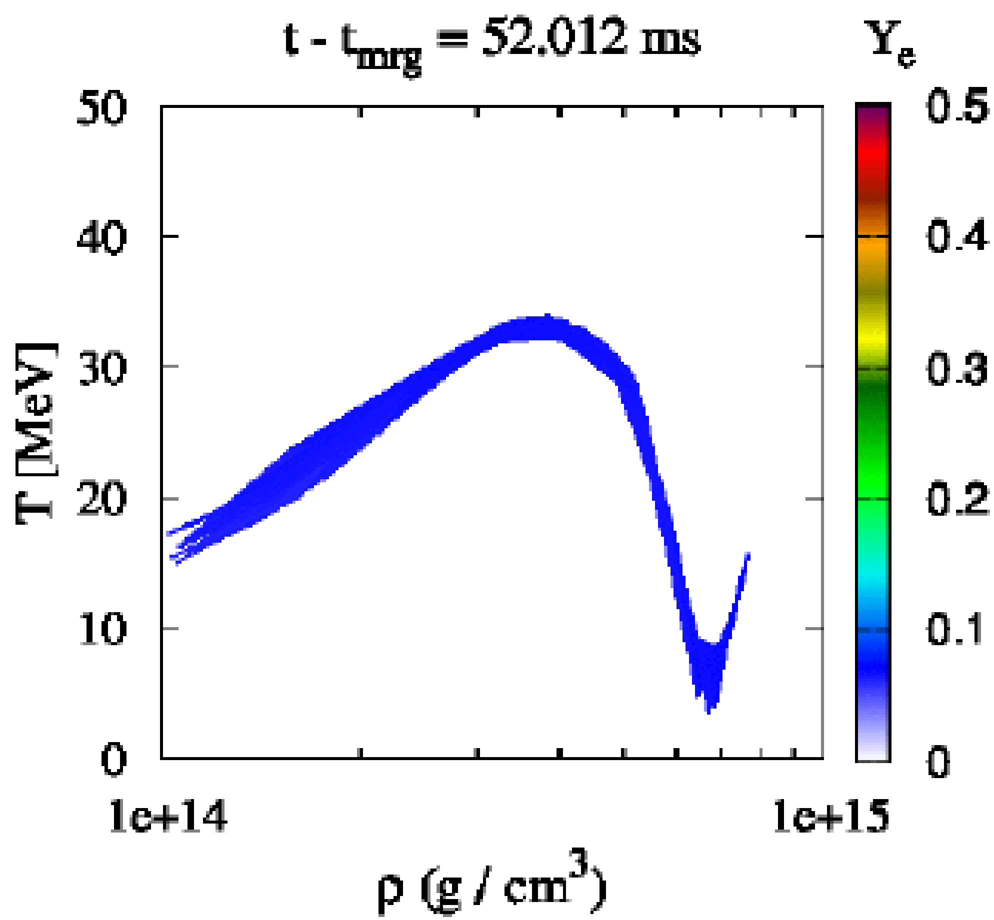


By Sumiyoshi

Binary Neutron Star Merger

- $T \sim 40$ MeV, $\rho_B \sim 10^{15}$ g/cm^3 $\sim 4\,\rho_0$ ($\rho_0 \sim 2.5 \times 10^{14}$ g/cm^3), $Ye \sim 0.1$





Courtesy of K. Kiuchi

Data are from Y. Sekiguchi, K. Kiuchi, K. Kyotoku, M. Shibata, PRD91('15)064059.

Quark Matter in Compact Stars

- **Neutron Star**
  E.g. N. Glendenning, "Compact Stars";F. Weber, Prog.Part.Nucl.Phys.54('05)193
  - Cold (T~0), Dense (ρ_B~5 ρ_0), Highly Asymmetric (Y_p ~ (0.1-0.2))
- **Supernova** *T. Hatsuda, MPLA2('87)805; I. Sagert et al., PRL102 ('09) 081101.*
  - Warm (T~20 MeV), Dense (ρ_B~1.8 ρ_0), mildly asym. (Y_p~ (0.3-0.4))
- **Binary Neutron Star Merger**
  Sekiguchi, Kiuchi, Kyotoku, Shibata, PRD91('15)064059.
  - Hot (T~30-40 MeV), Dense (ρ_B~(4-5) ρ_0), Highly Asymmetric (Y_p ~ (0.1-0.2))
- **Dynamical black hole formation**
  K. Sumiyoshi, et al., PRL97('06) 091101;K.Sumiyoshi, C.Ishizuka, AO, S.Yamada, H.Suzuki, ApJL690('09),L43; Nakazato et al. ('10); Hempel et al. ('12); ...
  - Hot (T~(70-90)MeV), Dense(ρ_B~(4-5)ρ_0), and Asymmetric (Y_p ~ (0.1-0.3))

Comparison of conditions in NS, SN, and HIC

M. Hempel

	neutron stars	supernovae	heavy ion collisions
dynamic timescales	(d - yrs)	ms	fm/c
equilibrium	full	weak eq. only partly	only strong eq.
temperatures	0	0 - 100 MeV	10 - 200 MeV
charge neutrality	yes	yes	no
asymmetry	high	moderate	low
highest densities	$< 9\ \rho_0$	$< 2\text{-}4\ \rho_0$	$< 4\text{-}5\ \rho_0$

weak equilibrium

$\mu_i = B_i\mu_B + Q_i\mu_Q + L_i\mu_L$; $\mu_S=0$

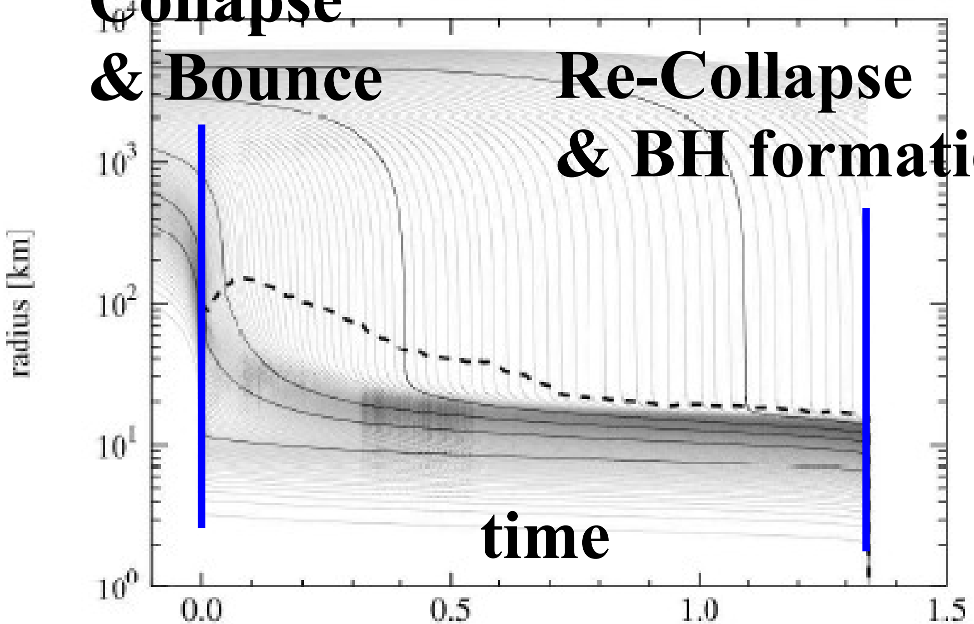
charge neutrality:

$Y_Q = Y_e+Y_\mu \Leftrightarrow n_Q = n_e+n_\mu$

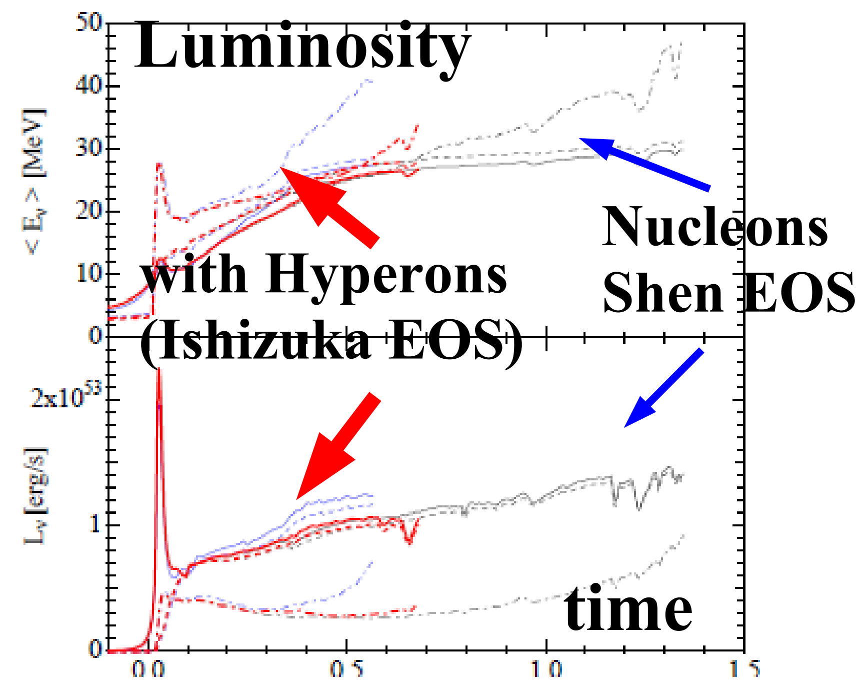
- matter in SN: no weak equilibrium, finite temperature
  → somewhere between cold neutron stars and heavy-ion collisions

Dynamical Black Hole Formation

- **Gravitational collapse of heavy (e.g. 40 $M_\odot$) progenitor would lead to BH formation.**
  - **Shock stalls, and heating by ν is not enough to take over strong accretion. → failed supernova**
  - **ν emission time ~ (1-2) sec w/o exotic matter.**
  - **emission time is shortened by exotic dof (quarks, hyperons, pions).**



Sumiyoshi, Yamada, Suzuki, Chiba, PRL 97('06)091101.

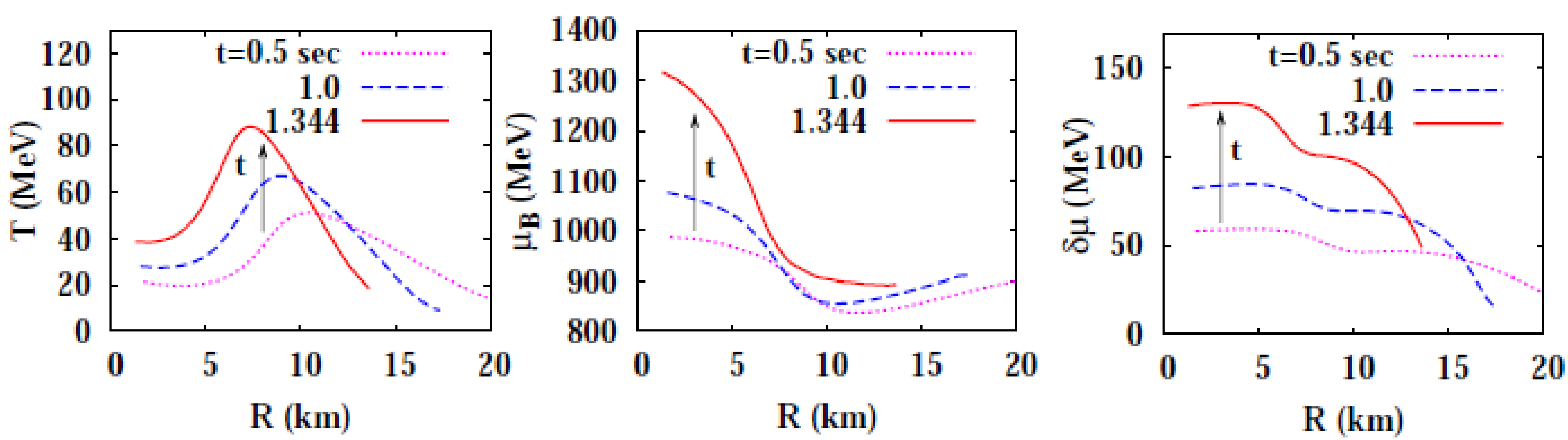


Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, ApJL 690('09)43.

Thermal Condition during BH formation

- **Quark-hadron and nuclear physicists are interested in (T, μ) !**
  - **Maximum T ~ 90 MeV (off-center) (Heated by shock propagation)**
  - **Maximum μ_B ~ 1300 MeV (center)**
  - **Maximum $\delta\mu = (\mu_n - \mu_p)/2$ ~ 130 MeV (center)**

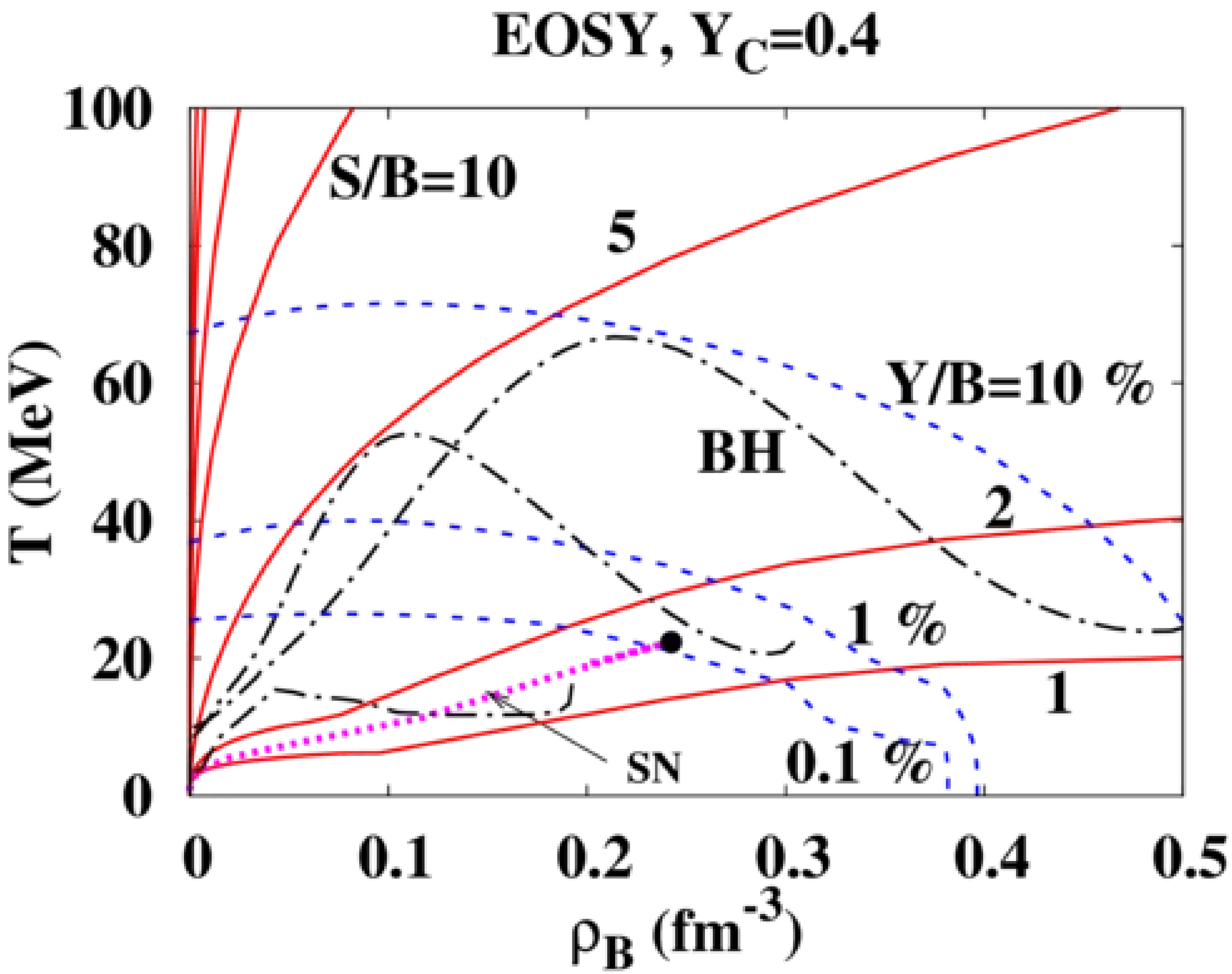
Can we reach CP ? What is the effects of δμ ?



Nucleon+leptons+photon (Shen EOS), 40 Msun

AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

Thermal Condition during BH formation



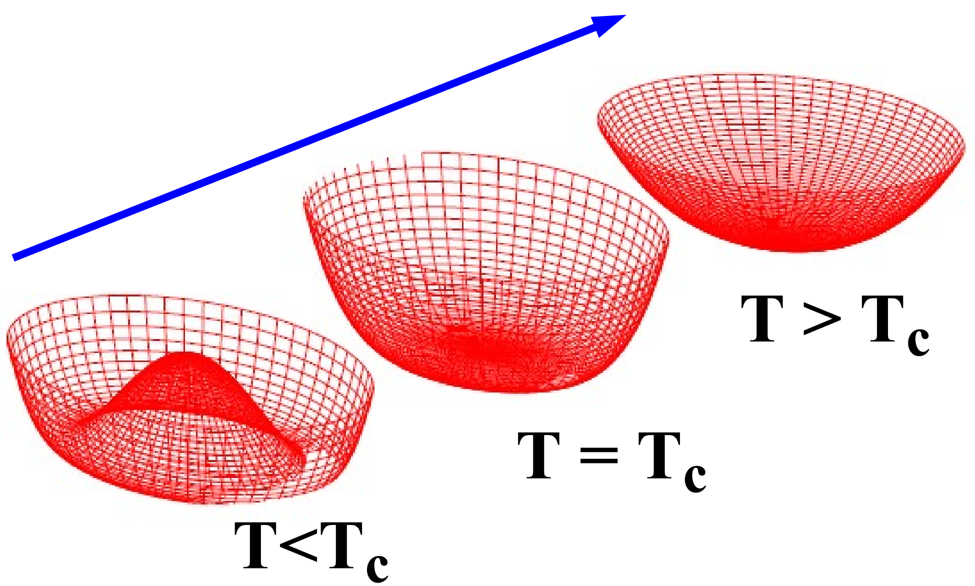
Ishizuka,AO, Tsubakihara, Sumiyoshi, Yamada, JPG 35('08) 085201;
AO et al., NPA 835('10) 374.

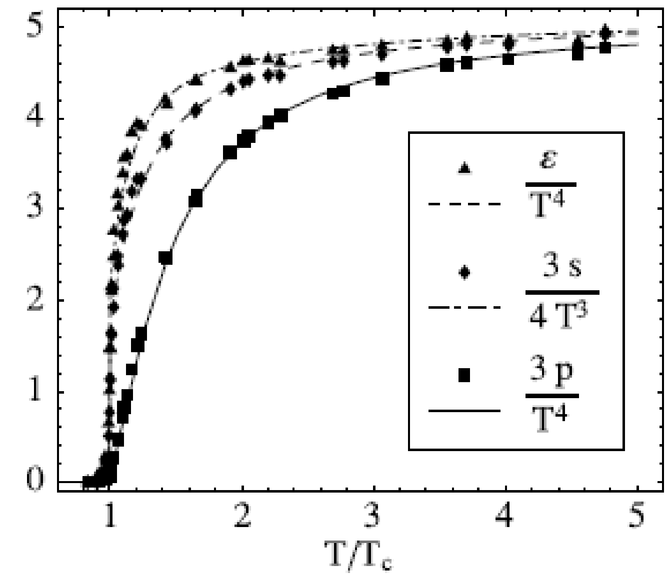
Chiral Effective Models

- **Chiral Effective models: NJL, PNJL, PQM**
  NJL=Nambu-Jona-Lasinio model,
  PNJL=Polyakov loop extended NJL,
  PQM=Pol. loop ext. Quark Meson model

Nambu, Jona-Lasinio ('61), Fukushima('03), Ratti, Thaler, Weise ('06), B.J.Schafer, Pawlowski, Wambach ('07); Skokov, Friman, E.Nakano, Redlich('10)

  - **Spontaneous breaking & restoration of chiral symmetry**
  - **Polyakov loop extension → Deconf. transitions**





Roessner et al.('07)

Chiral Effective Models (N_f=2)

- **Lagrangian (PQM, as an example)**

$$L=\bar{q}\left[i\gamma^{\mu}\underline{D_{\mu}}-g_{\sigma}\underline{\left(\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)}\right]q+\frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma+\frac{1}{2}\partial^{\mu}\boldsymbol{\pi}\cdot\partial_{\mu}\boldsymbol{\pi}$$

q-Pol. **quark-meson**

$$-\underline{U_{\sigma}(\sigma,\pi)}-\underline{U_{\Phi}(\Phi,\bar{\Phi})}$$

chiral **Polyakov**

$$F_{\rm eff}\equiv\Omega/V=U_{\sigma}(\sigma,\pi=0)+U_{\Phi}(\Phi,\bar{\Phi})+\underline{F_{\rm therm}}+\underline{U_{\rm vac}(\sigma,\Phi,\bar{\Phi})}$$

particle exc. **q zero point**

- **Polyakov loop effective potential from Haar measure**
  U_Φ ~ -log (Haar Measure) **(Fit lattice data to fix parameters).**

- **Vector coupling is not known well → Comparison of g_V/g_S=0, 0.2**

$$L_V=-g_v\bar{q}\gamma_{\mu}\left(\omega^{\mu}+\boldsymbol{\tau}\cdot\boldsymbol{R}^{\mu}\right)q-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}-\frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu}+\frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu}+\frac{1}{2}m_{\rho}^2R_{\mu}R^{\mu}$$

- **8 Fermi interaction**

T. Sasaki, Y. Sakai, H. Kouno, M. Yahiro ('10)

$$G_{\sigma 8}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]^2$$

Model Details

- **BH formation calculation**

  Sumiyoshi, Yamada, Suzuki, Chiba, PRL 97('06)091101.

  - **ν radiation 1D (spherical) Hydrodynamics**
  - **ν transport is calculated exactly by solving the Boltzmann eq.**
  - **Gravitational collapse of 40 $M_\odot$ star**
  - **Initial condition: WW95**
    S.E.Woosley, T.A.Weaver, ApJS 101 ('95) 181
  - **Shen EOS (npeμ)**

- **QCD effective models**
  - **NJL, PNJL, PNJL with 8 quark int., PQM**
  - **N_f=2**
  - **Vector coupling → G_V/G_s (g_V/g_s in PQM)=0, 0.2**

Isospin chemical potential

- **Isospin chemical potential δμ**

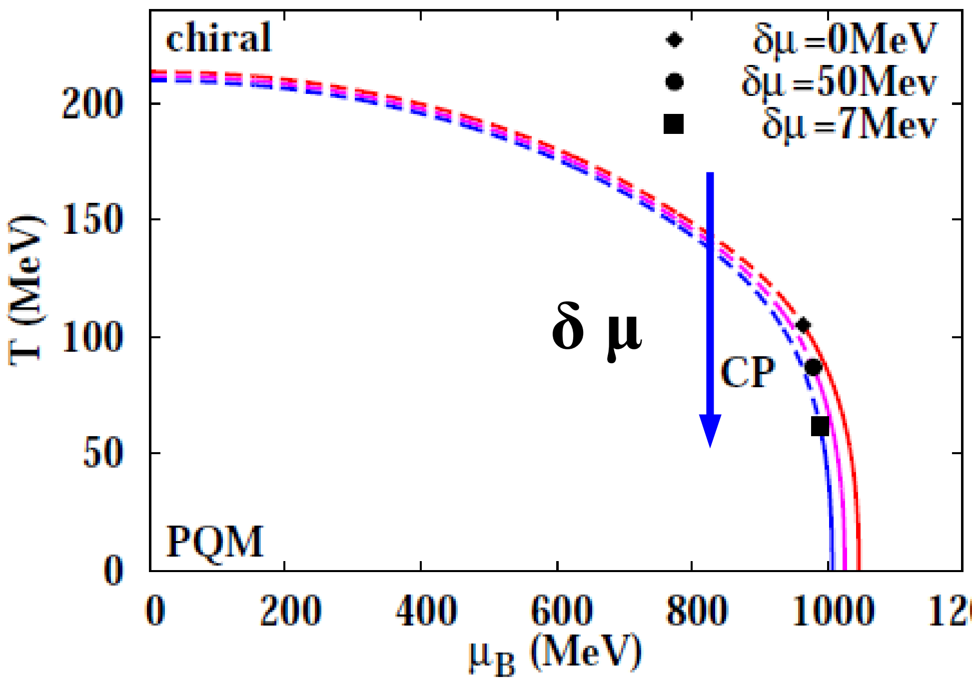
  $\delta\mu = (\mu_d - \mu_u)/2 = (\mu_n - \mu_p)/2 \rightarrow \mu_d = \mu_q + \delta\mu,\ \ \mu_u = \mu_q - \delta\mu$

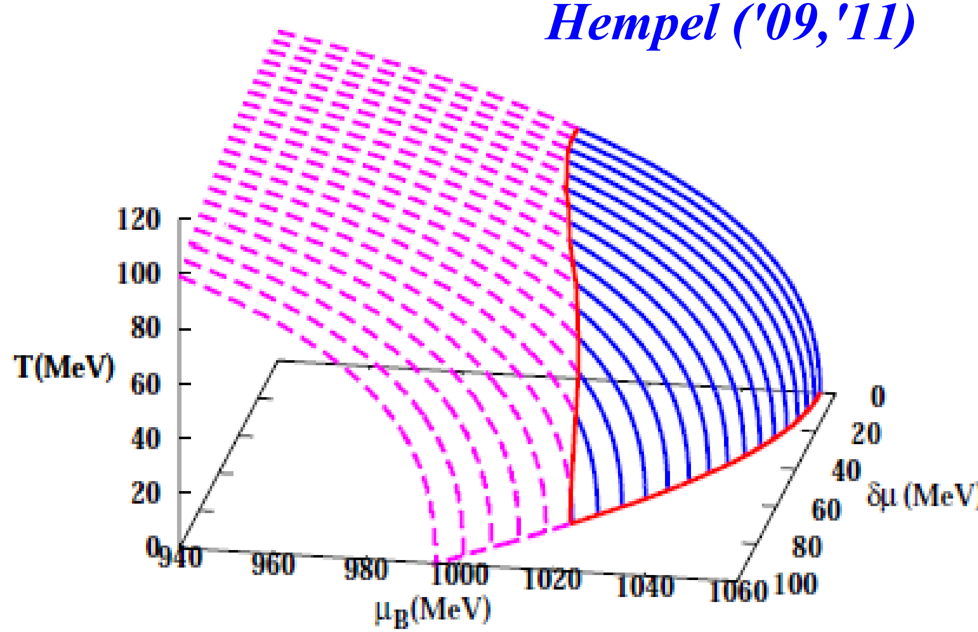
  - **Finite δμ → (Isospin) Asymmetric matter $N_u \neq N_d$**

    → Smaller "Effective" number of flavors

    → Weaker phase transition → smaller T_{CP}

c.f. Hempel's Lec.
Sagert, Pagliara, Schaffner-Bielich, Hempel ('09,'11)

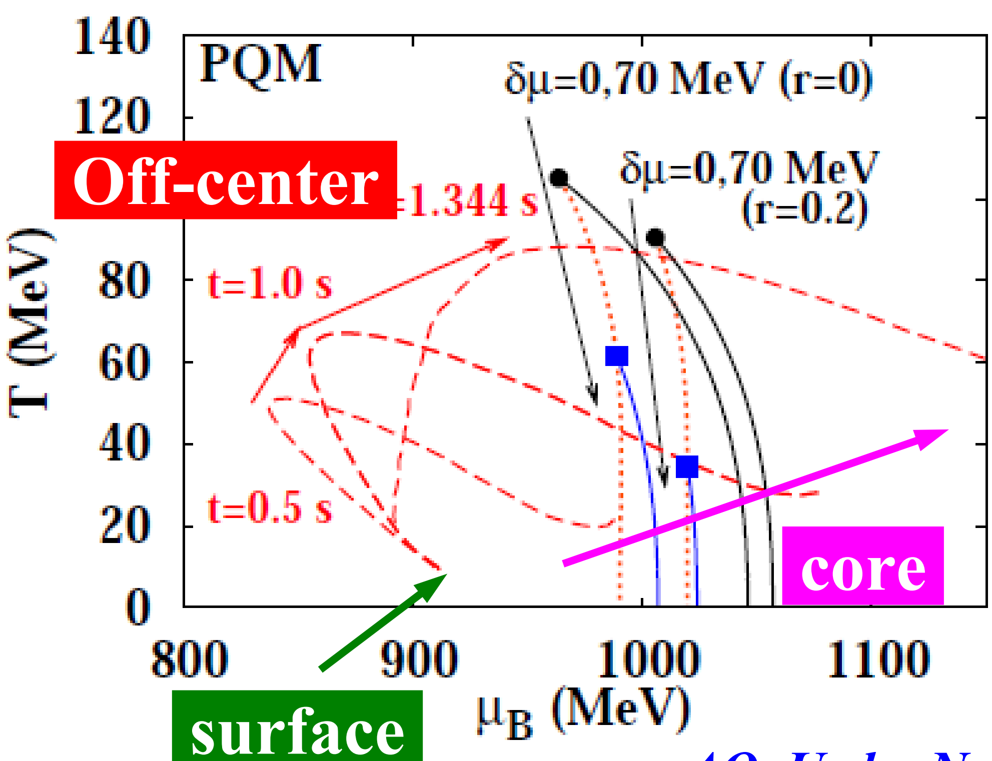


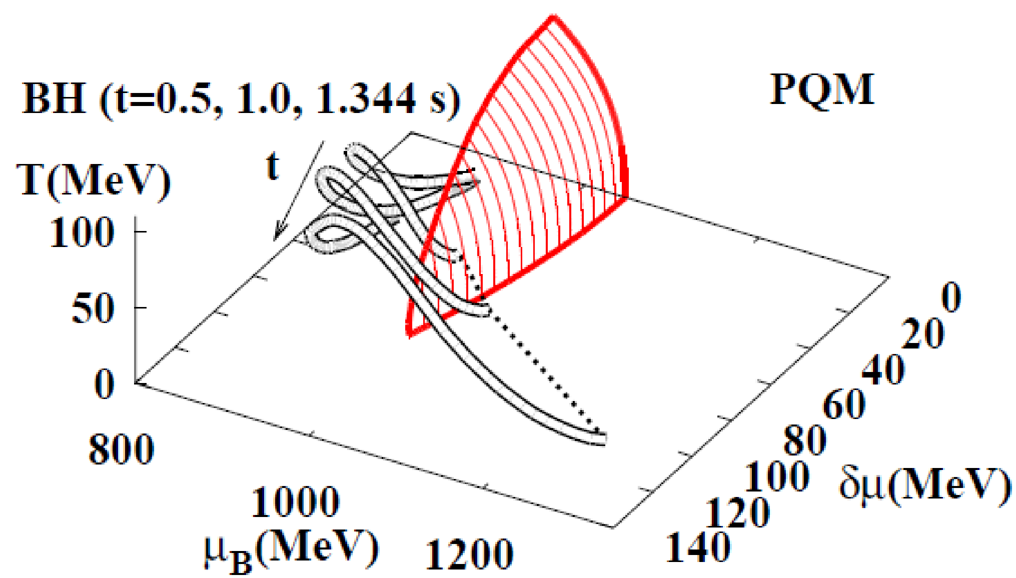


AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284
H. Ueda, T. Z. Nakano, AO, M. Ruggieri, K. Sumiyoshi, PRD88('13),074006

How is quark matter formed during BH formation ?

- Highest μ_B just before horizon formation ~ 1300 MeV
  > QCD transition μ (1000-1100 MeV)
  → *Quark matter is formed before BH formation*
- Core evolves below CP, Off-center goes above CP
  → *CP sweep*

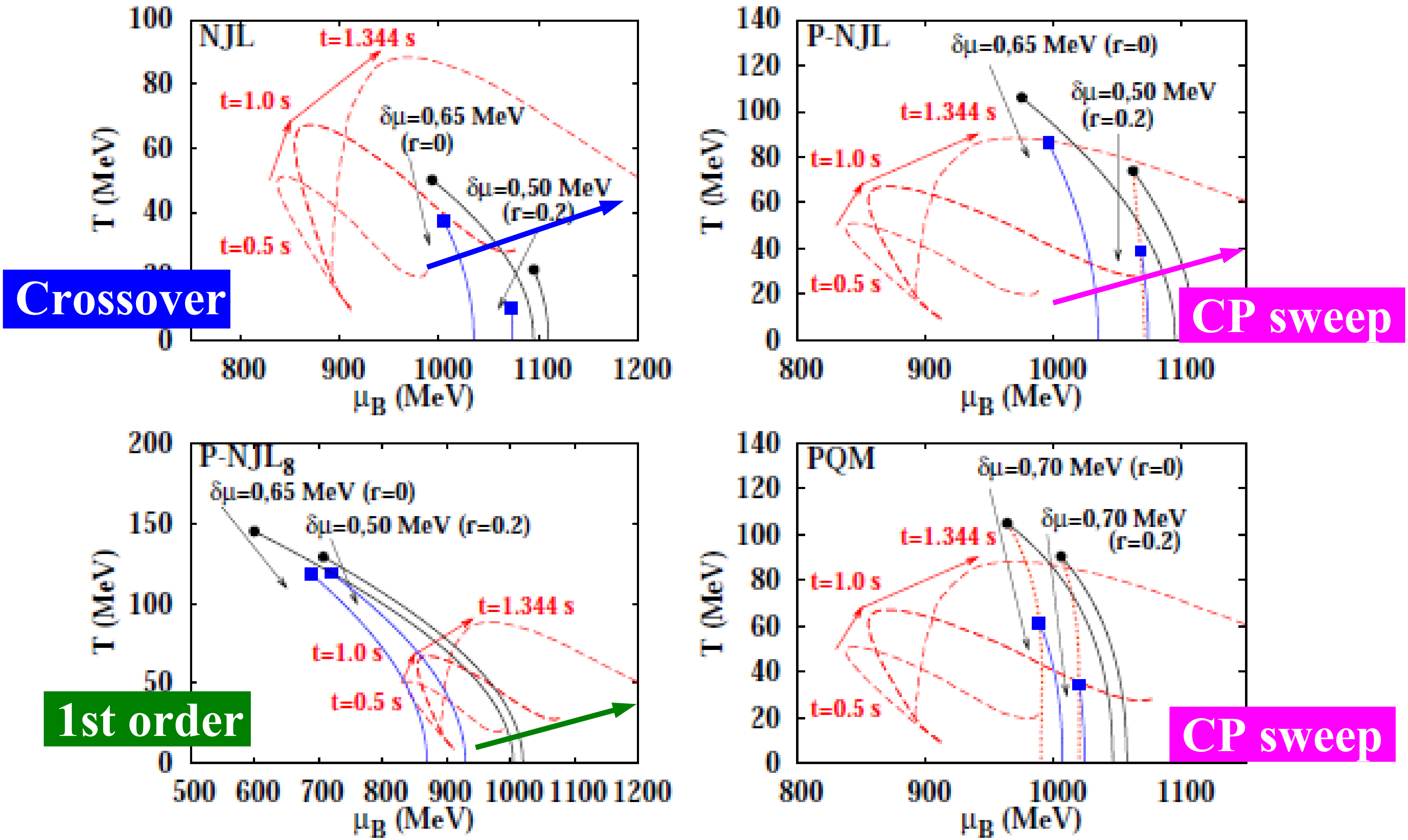




AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

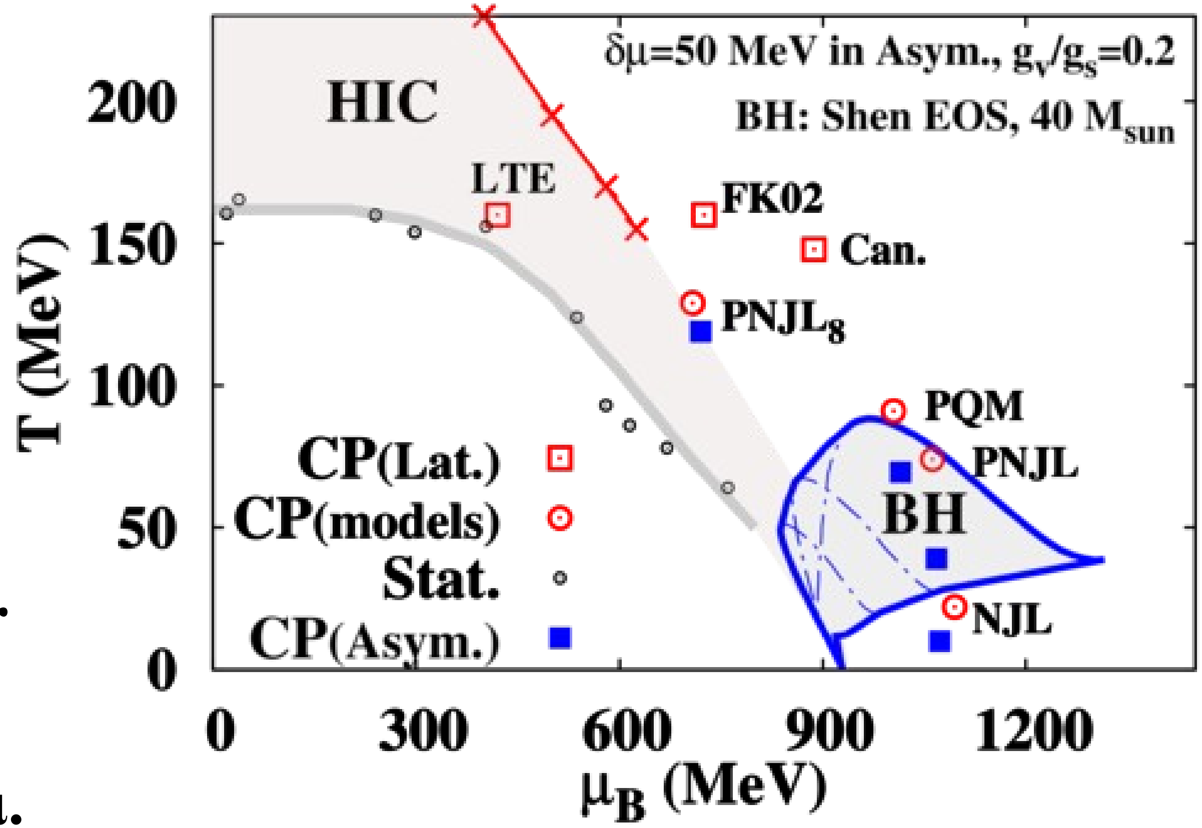
How is quark matter formed during BH formation ?

- **Model dependence to form quark matter → Three ways**



Probed Region of Phase Diagram during BH formation

- **CP location in Symmetric Matter**
  - **Lattice QCD**
    μ_{CP}=(400-900) MeV
  - **Effecitve models**
    μ_{CP}=(700-1050) MeV
- **CP in Asymmetric Matter (E.g. δμ=50 MeV)**
  - T_{CP} **decreases at finite δμ.**
    → **Accessible (T, μ_B) region during BH formation**

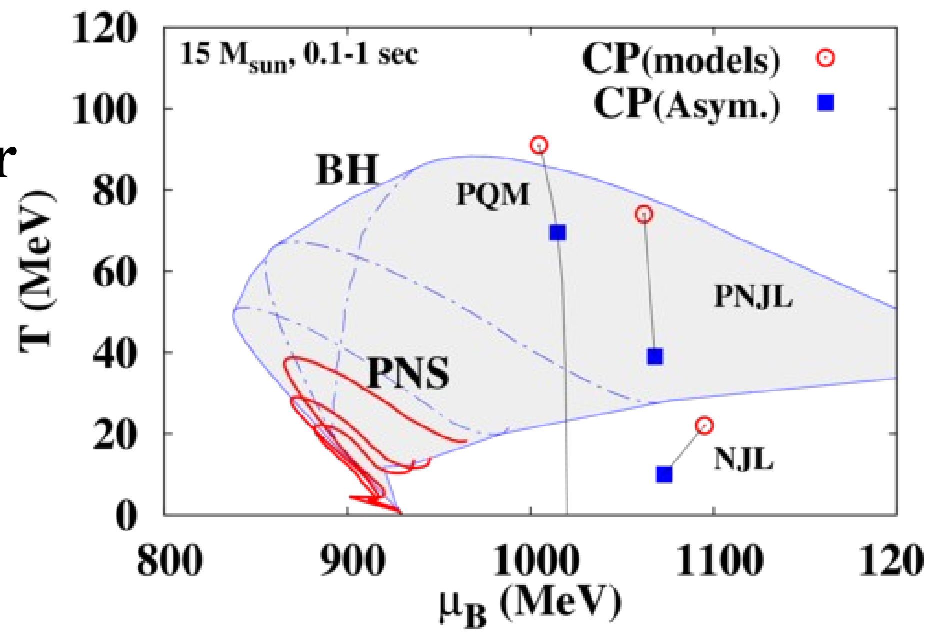


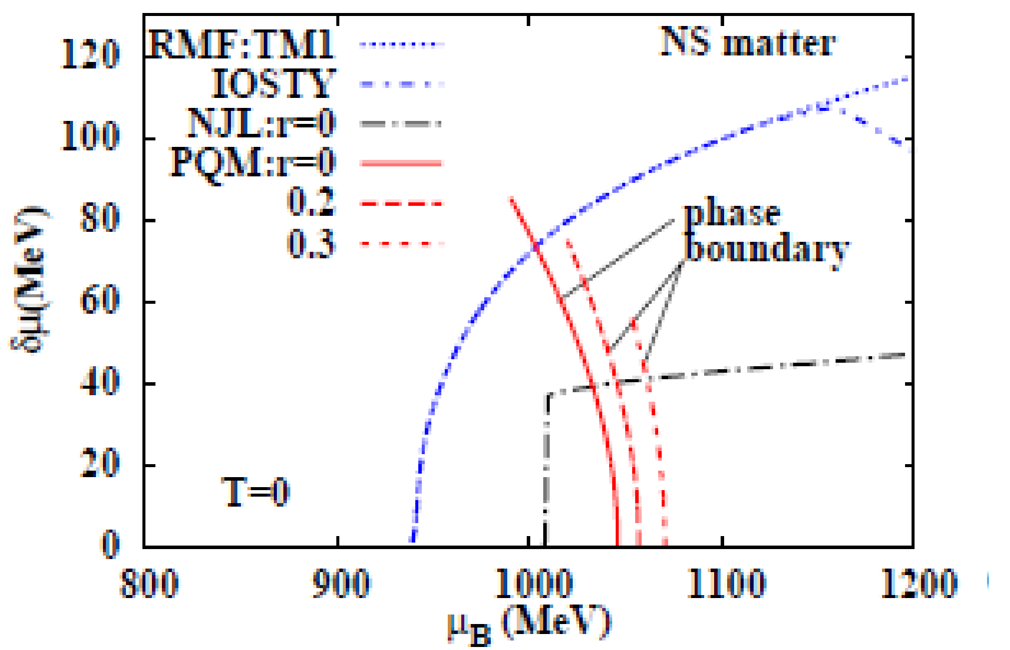
(Stephanov plot)

M.A.Stephanov, Prog.Theor.Phys.Suppl.153 ('04)139;
FK02:Z. Fodor, S.D.Katz, JHEP 0203 (2002) 014
LTE:S. Ejiri et al., Prog.Theor.Phys.Suppl. 153 (2004) 118;
Can: S. Ejiri, PRD78 (2008) 074507
Stat.:A. Andronic et al., NPA 772('06)167

How about Neutron Stars ?

- Contraction of Proto-Neutron Star
  - (T, μ_B) are not enough at 1 sec after bounce of 15 $M_\odot$ star collapse
  - Larger (T, μ_B) is expected in long time evolution (~20 sec) or heavier proto-neutron stars.
    K. Sumiyoshi et al. ApJ 629 ('05) 922;
    J. A. Pons et al., ApJ 513 ('99)780;
    J. A. Pons et al., ApJ 553 ('01) 382.
- Cold Neutron Star
  - max. δ μ~ 100 MeV
  - Possibility of cross over in NS

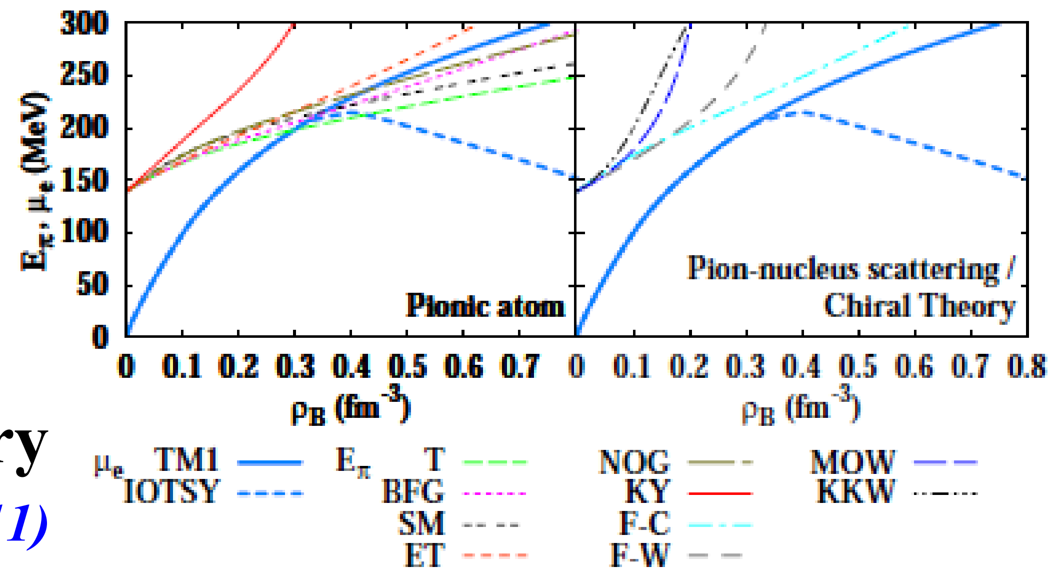




H. Ueda et al.('13)

Discussion

- How can we observe the phase transition signal ?
  - ν spectrum ? Gravitational waves ?
    Supernova: Second peak in ν & $\bar{\nu}$ emission
    Hatsuda('87), Sagert et al.('09)
- How frequent do dynamical BH formation take place ?
  - Less frequent than SN (< 20 $M_\odot$), but should be in collapse of heavy stars (>40 $M_\odot$).
    C.L.Fryer, ApJ 522('99)413; E.O'Connor, C.D.Ott, ApJ 730('11)70
- Strangeness may reduce δμ in hadronic / quark matter
  - No s-wave π cond. in NS
    AO, D. Jido, T. Sekihara, K. Tsubakihara, PRC80('09)038202.



- Hadron-Quark EOS is necessary
  E.g. Steinheimer,Schramm,Stocker('11)

Summary of Lecture 2

- **While we have the sign problem in lattice QCD at finite μ, the phase diagram study is on going using various ideas. I have shown recent results based on the strong-coupling lattice QCD.**
  - **Smaller weight cancellation allow us to study phase transition at high density.**
  - **Phase diagram in the strong coupling limit has been confirmed. (Results from MDP and AFMC methods agree.)**
  - **Cumulant ratio would be interesting !**
- **Compact stars are also good laboratories of dense matter.**
  - **NS, SN, BH, BNSM → Dense, Cold/Hot, Isospin asymmetric matter**
  - **With the first order boundary (and CP) and isospin chem. pot, there are many ways of realizing phase transition in compact star phenomena.**

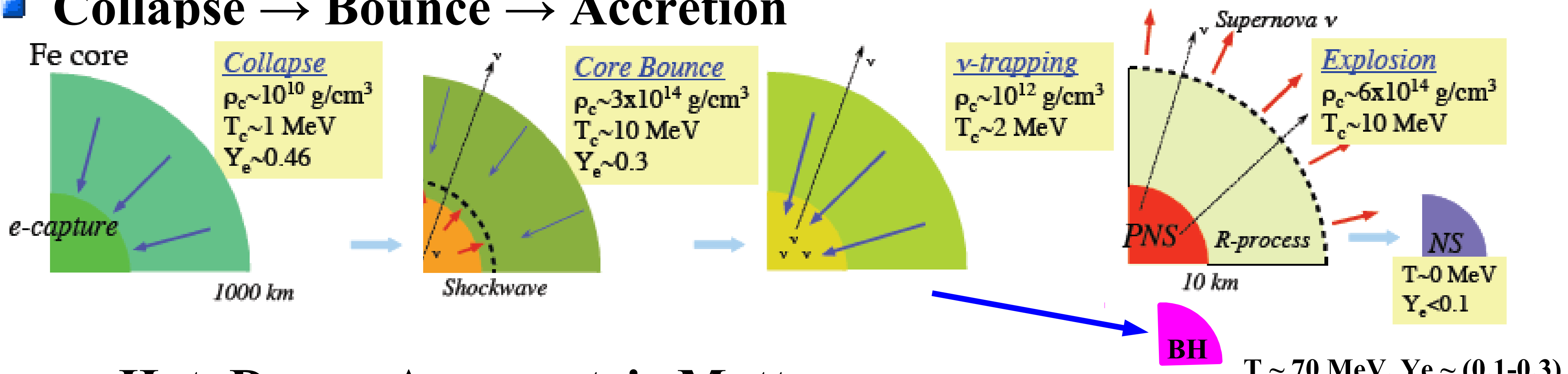
Summary

- **Dense matter is "terra incognita", and there are many unsolved problems.**
- **In heavy-ion collisions at $\sqrt{s}$ = 5-10 A GeV, we expect formation of highest baryon density matter, whose density exceeds 5 ρ_0.**
  In equilibrium, this would be above the transition density.
- **In compact star phenomena, hydro simulations with hadronic matter EOS suggest the formation of dense matter (4-5 ρ_0, $\mu_B \sim$ 1300 MeV), which is above the transition density in many effective models.**
- **We need more experimental, observational, and theoretical works to explore dense matter.**

Thank you for your attention !

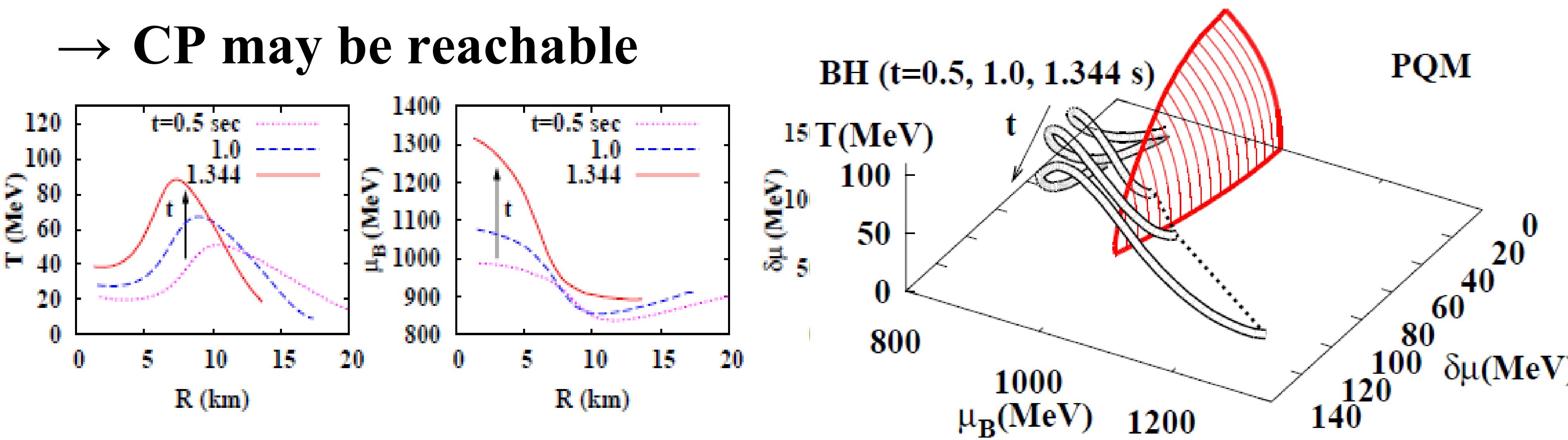
Dynamical Black Hole Formation

- **Collapse → Bounce → Accretion**



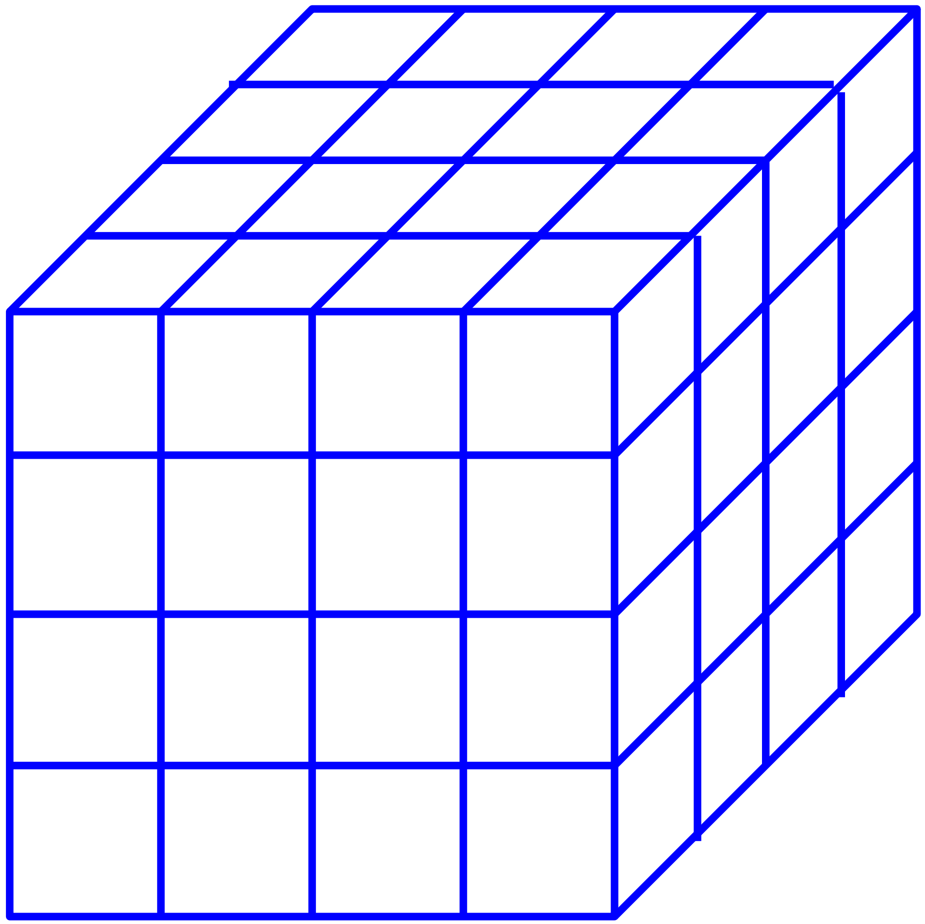
→ Hot, Dense, Asymmetric Matter
$T \sim 70$ MeV, $\mu_B \sim 1300$ MeV, $\delta\mu=\mu_e/2 \sim 130$ MeV

→ CP may be reachable

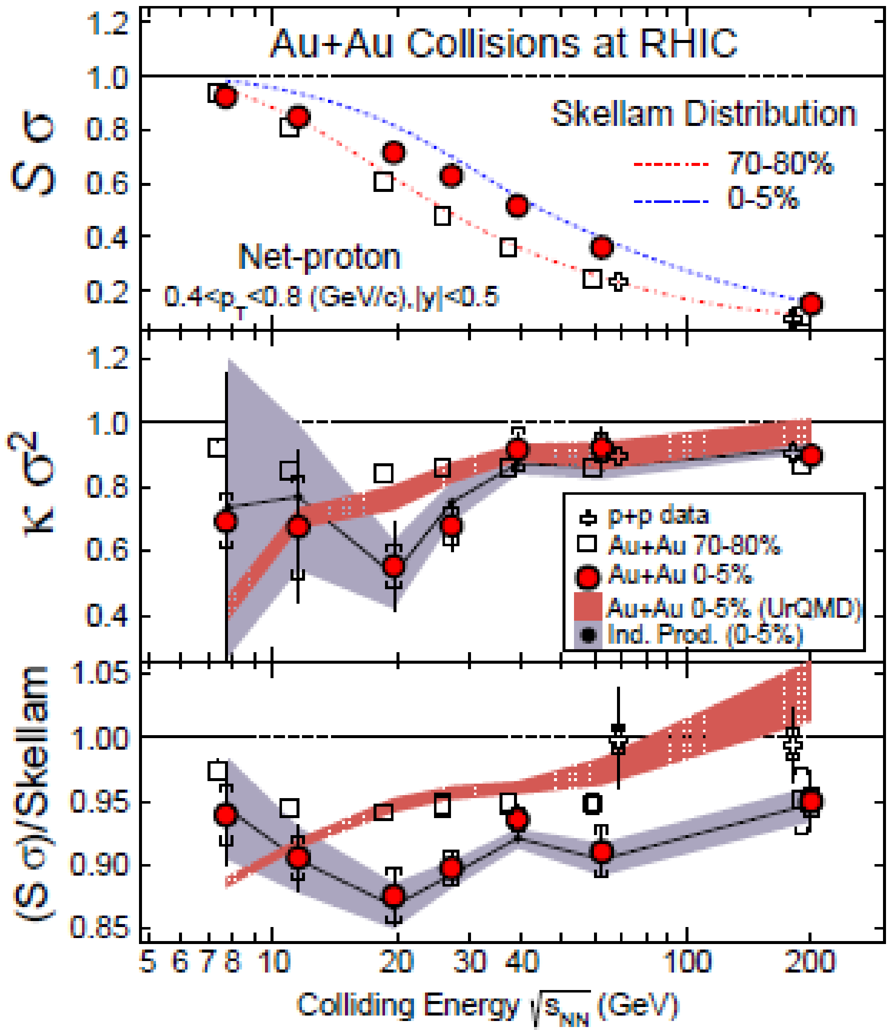


K. Sumiyoshi, et al.,('06); K.Sumiyoshi, C.Ishizuka, AO, S.Yamada, H.Suzuki ('09)
AO, H.Ueda, T.Z.Nakano, M. Ruggieri, K. Sumiyoshi ('11).

Lattice QCD



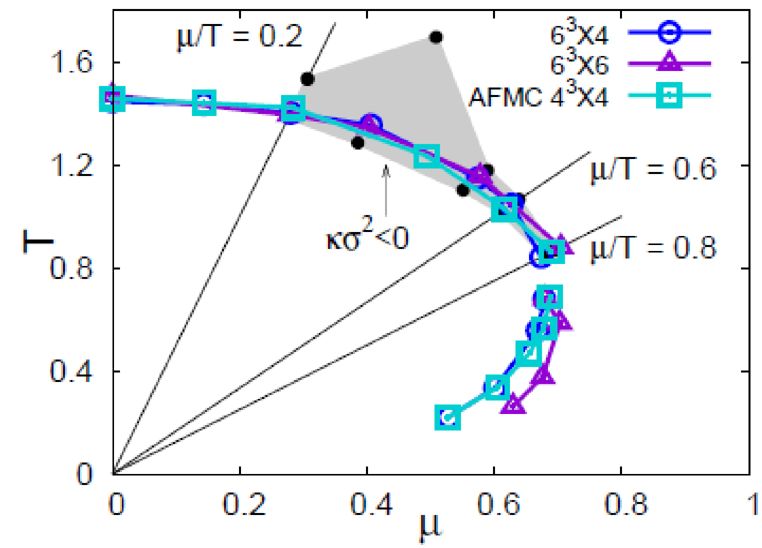
Relation to the observed data ???



STAR Collab. (PRL 112('14)032302

Need much more work !

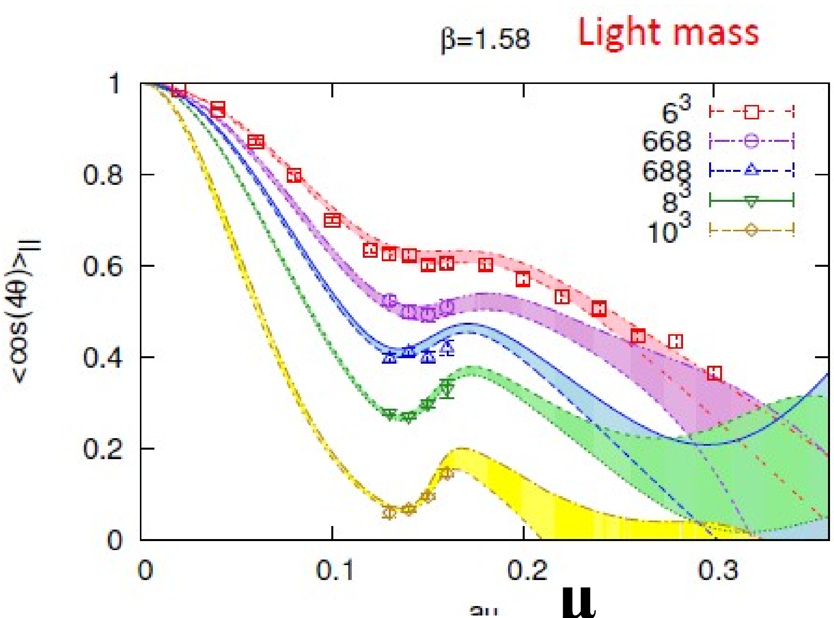
Lattice MC at μ=0
Bazarov, .., Kaczmarek, et al.('14),
Bellwied et al.('13), ….
Gavai, Gupta ('05), Allton et al. ('05),
Lattice MC at μ>0 but large mq
Jin, Kuramashi, Nakamura,
Takeda, Ukawa ('15)
Scaling function
Friman, Karsch, Redlich, Skokov ('11)

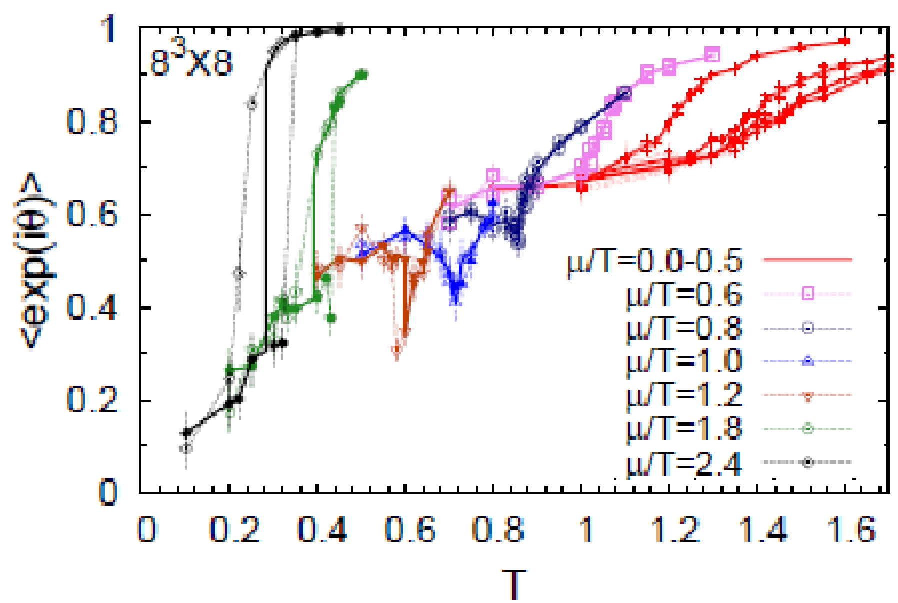


Ichihara, Morita, AO, in prep.

Comparison with Direct Simulation at finite coupling

- **Lattice MC simulation at finite μ and finite β with Nf=4**
  Takeda et al. ('13)
  - **Ave. Phase Factor ~ 0.3 at aμ ~ 0.15 (8^3 x 4, $a\mu_c = am_\pi/2 \sim 0.7$)**
- **AFMC**
  - **Ave. Phase Factor ~ 0.6 around the transition (8^4, SCL)**





Takeda, Jin, Kuramashi, Y.Nakamura, Ukawa, Lattice 2013

$a\mu_c = am_\pi/2 \sim 0.7$

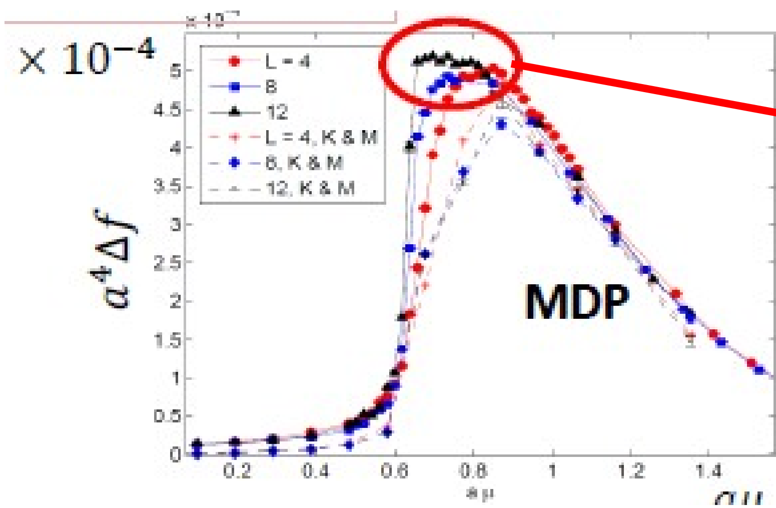
Ichihara, AO, Nakano ('14)

Discussion: Comparison with MDP

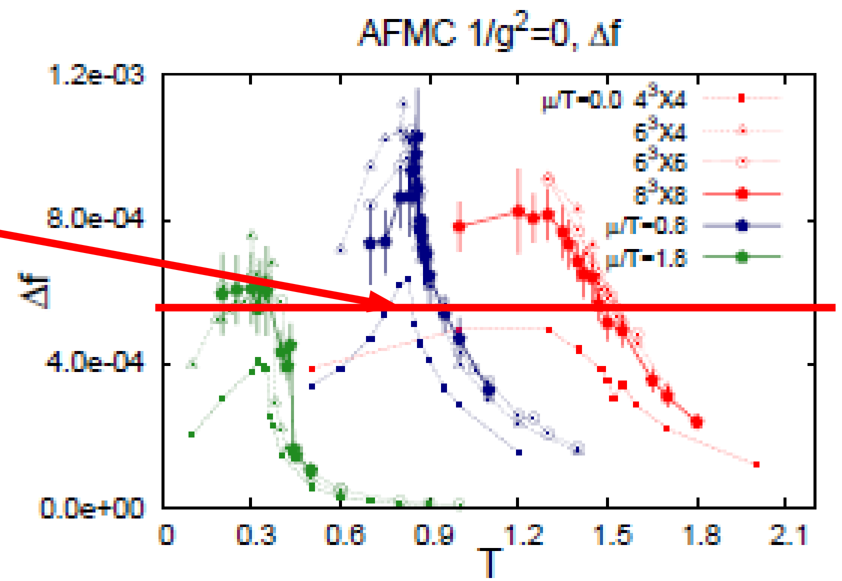
- **Free energy difference**

$$\langle \exp(i\theta) \rangle \equiv \exp(-\Omega \Delta f) \quad , \quad \Omega = \text{space-time volume}$$

- **MDP simulation on anisotropic lattice at finite T and μ**
  de Forcrand, Fromm ('10), de Forcrand, Unger ('11)
  - **Strong coupling limit.**
  - **Higher-order terms in 1/d expansion**
  - **No sign problem in the continuous time limit ($N\tau \to \infty$).**



de Forcrand, Unger ('11)



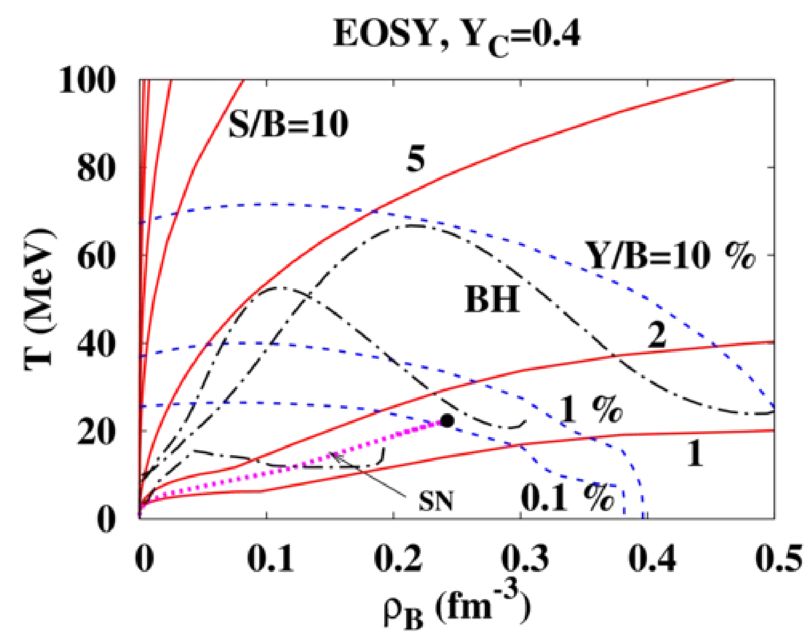
Ichihara, AO, Nakano ('14)

Gravitational Collapse of Heavy Star

- **Core collapse supernova (type II)**
  Fe core collapse → Core bounce
  → ν trapping
  → proto-neutron star
  + explosion of envelope
- **or → Black Hole formation**
  + Failed Supernova

  M. Liebendorfer et al., ApJS 150('04)263
  K. Sumiyoshi et al., PRL 97('06)091101

  - **Dynamical collapse with accretion.**
  - **Hot and Dense nuclear matter is formed.**

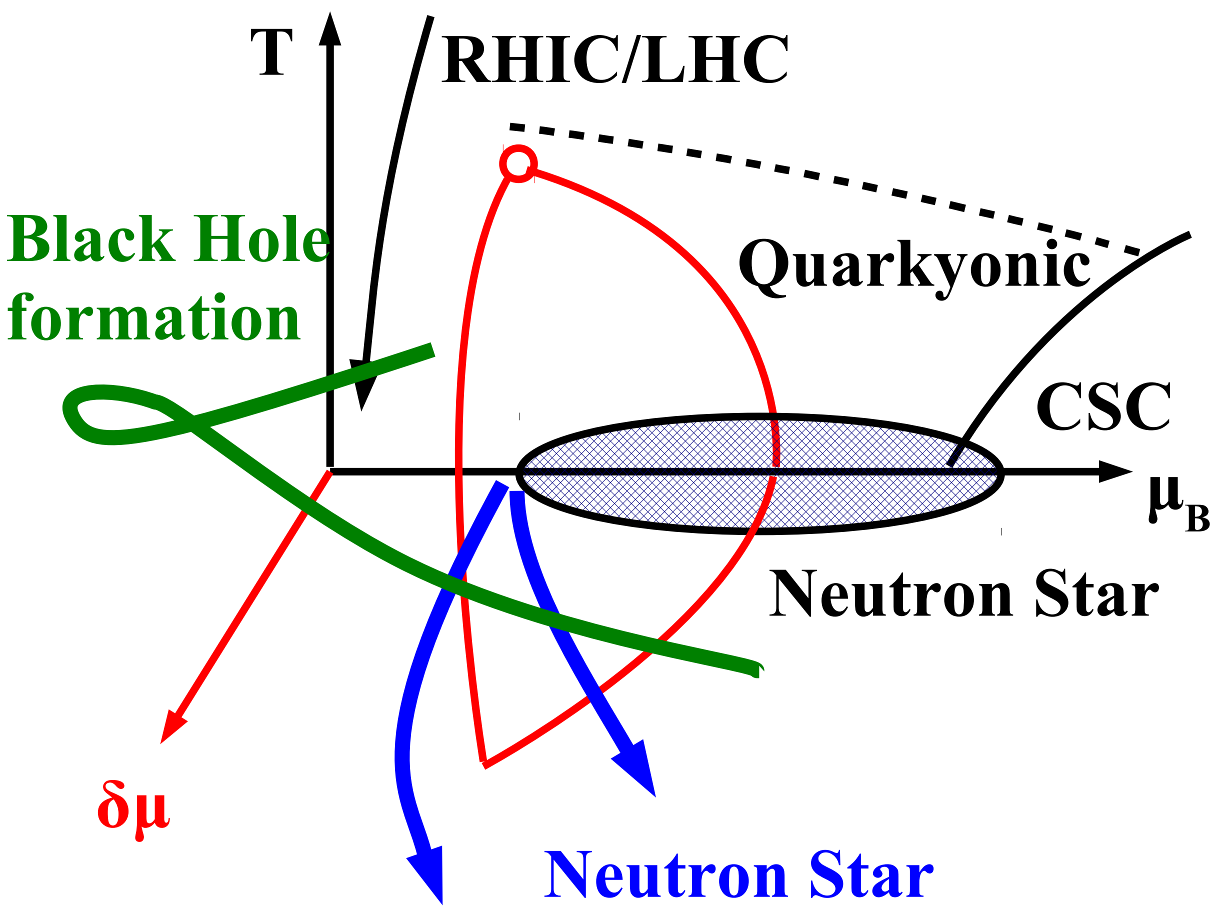


C.Ishizuka,AO,K.Tsubakihara, K. Sumiyoshi, S.Yamada, JPG 35('08) 085201;AO et al., NPA 835('10) 374.

QCD phase diagram in Compact Astrophys. Phen.

- **Phase diagram probed in High-Energy Heavy-Ion Collisions**
  - **Hot & Dense *Symmetric* matter**
- **Phase diagram probed in Compact Astrophysical Phenomena**
  - **Hot and/or Dense *Asymmetric* matter**

→ 3D phase diagram must be considered !
(c.f. P. Zhuang)



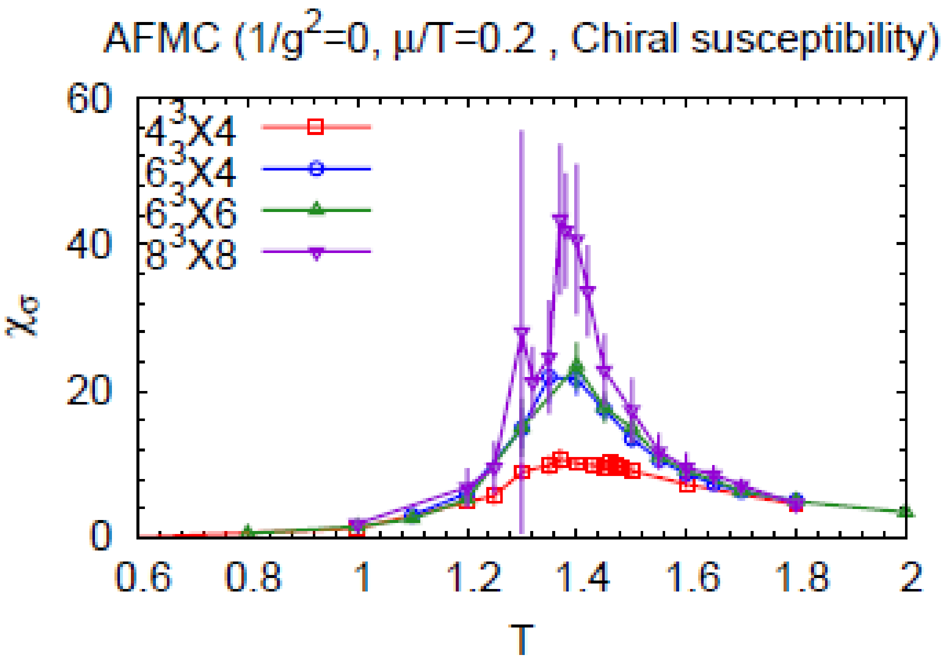
We compare the 3D phase diagram in effective models and thermal condition in Compact Stars

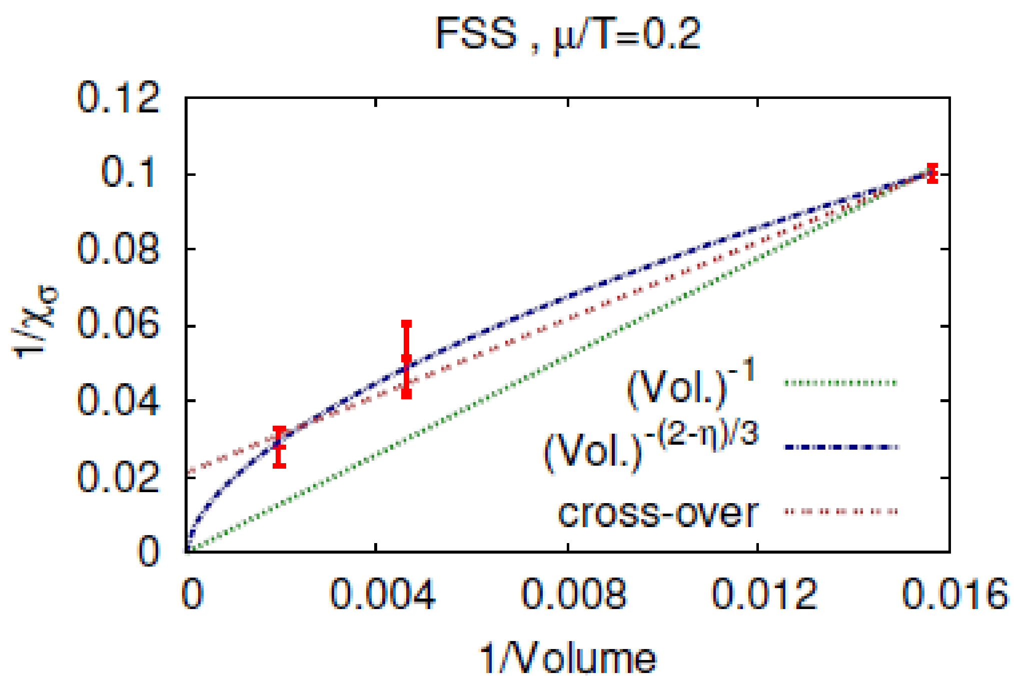
Critical Point sweep during black hole formation

Phase diagram in (Isospin) Asymmetric Matter

Finite Size Scaling of Chiral Susceptibility

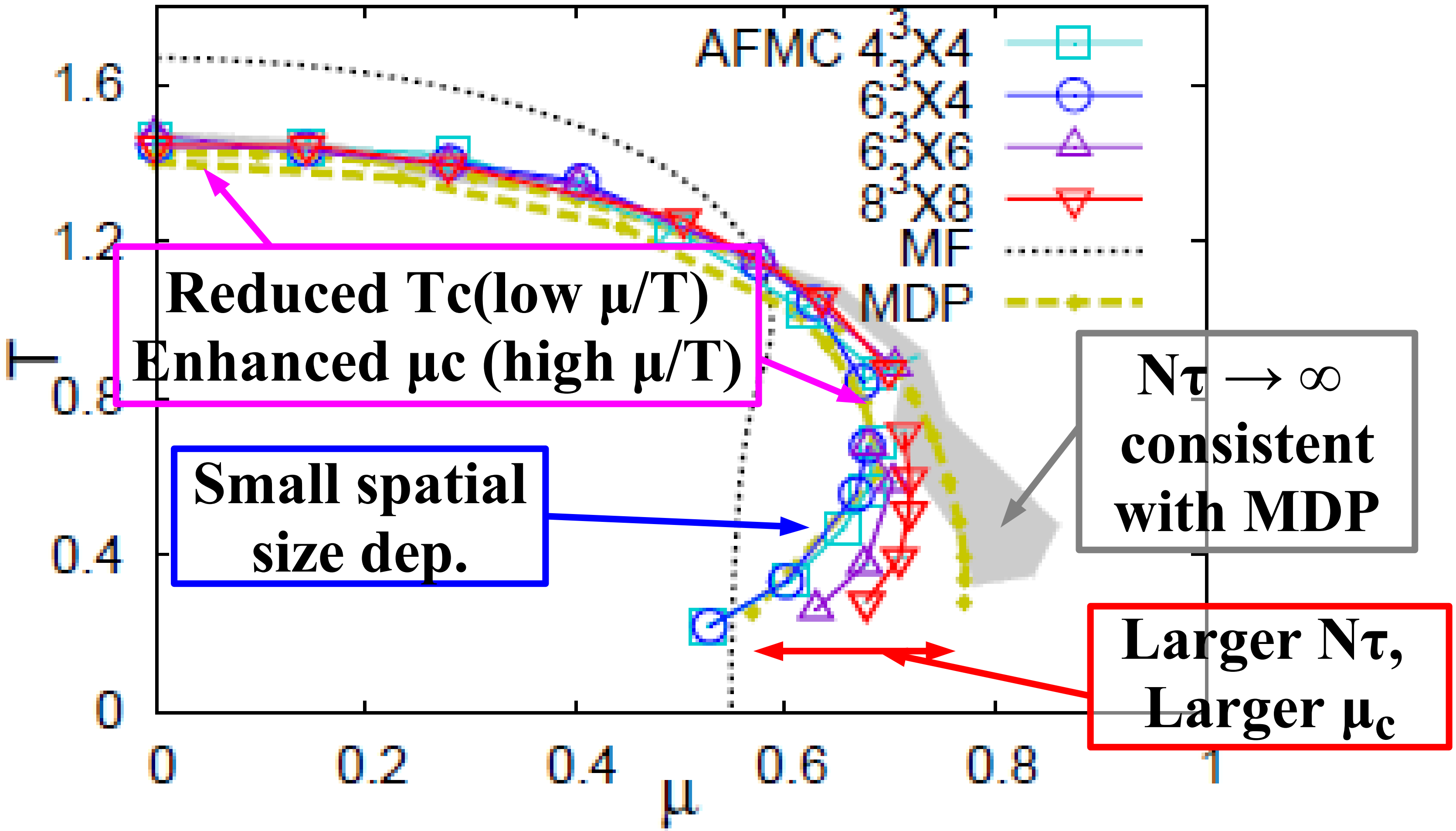
- Finite size scaling of χ_σ in the V (spatial vol.) $\to \infty$ limit
  - Crossover: Finite
  - Second order: $\chi_\sigma \propto V^{(2-\eta)/3}$, η=0.0380(4) in 3d O(2) spin
    Campostrini et al. ('01)
  - First order: $\chi_\sigma \propto V$
- AFMC results : Not First order at low μ/T.





Ichihara, AO, Nakano ('14)

Phase diagram



Ichihara, AO, Nakano ('14)

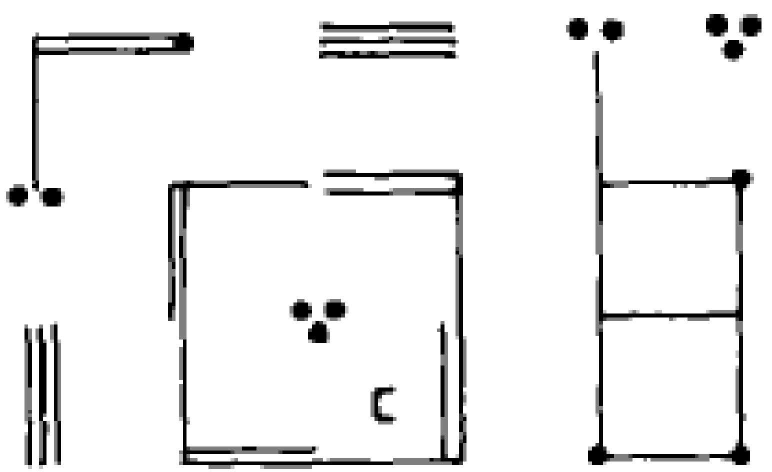
Monomer-Dimer-Polymer simulation

- The partition function of LQCD can be given as the sum of monomer-dimer-polymer (MDP) configuration weight.
  The sign problem is mild. *Karsch, Mutter ('89)*

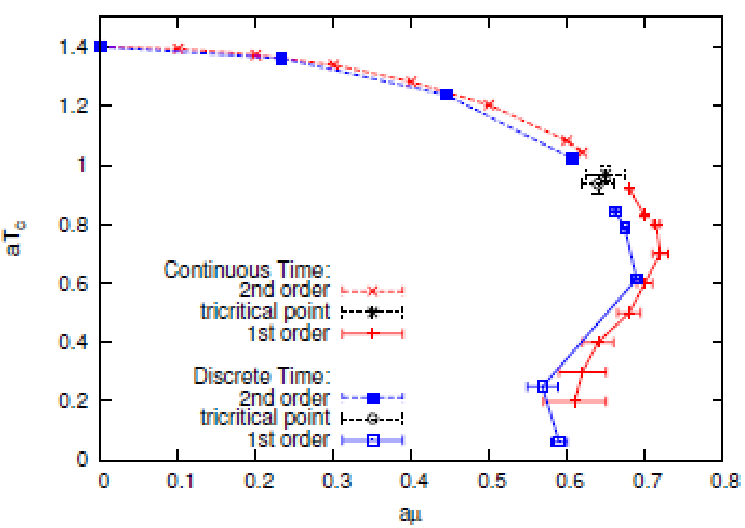
$$Z(2ma, \mu, r) = \sum_K w_K$$

$$w_K = (2ma)^{N_M} r^{2N_t} (1/3)^{N_1 N_2} \prod_x w(x) \prod_C w(C)$$

- MDP with worm algorithm is applied to study the phase diagram
  de Forcrand, Fromm ('10), de Forcrand, Unger ('11)



Karsch, Mutter ('89)



de Forcrand, Unger ('11)

Origin of the sign problem in AFMC

- **Extended Hubbard-Stratonovich transformation**

Miura, Nakano, AO ('09), Miura, Nakano, AO, Kawamoto ('09)

$$e^{\alpha AB} = \int d\varphi\, d\phi\, e^{-\alpha\left[(\varphi+(A+B)/2)^2+(\phi+i(A-B)/2)^2-AB\right]}$$

$$= \int d\varphi\, d\phi\, e^{-\alpha\left[\varphi^2+\phi^2+\varphi(A+B)+i\phi(A-B)\right]}$$

Complex

We need "i" to bosonize product of different kind.
→ Fermion determinant becomes complex.

- **Bosonization in AFMC in the strong coupling limit**

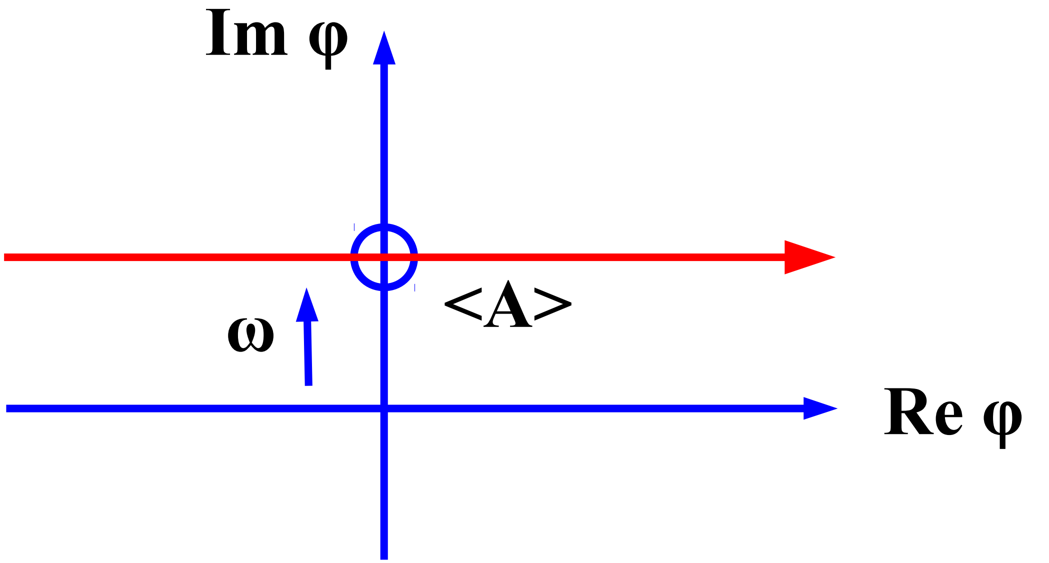
$$\exp\left\{\alpha f(\mathrm{k})\left[M_{-\mathbf{k},\tau}M_{\mathbf{k},\tau} - M_{-\bar{\mathbf{k}},\tau}M_{\bar{\mathbf{k}},\tau}\right]\right\}$$

$$= \int d\sigma_{\mathbf{k},\tau}\, d\sigma^*_{\mathbf{k},\tau}\, d\pi_{\mathbf{k},\tau}\, d\pi^*_{\mathbf{k},\tau} \exp\left\{-\alpha f(\mathrm{k})\left[|\sigma_{\mathbf{k},\tau}|^2 + |\pi_{\mathbf{k},\tau}|^2\right.\right.$$

$$\left.\left. +\sigma^*_{\mathbf{k},\tau}M_{\mathbf{k},\tau} + M_{-\mathbf{k},\tau}\sigma_{\mathbf{k},\tau} - i\pi^*_{\mathbf{k},\tau}M_{\bar{\mathbf{k}},\tau} - iM_{-\bar{\mathbf{k}},\tau}\pi_{\mathbf{k},\tau}\right]\right\}$$

- **Mean field treatment of repulsive interaction**

$$
\begin{aligned}
e^{-\alpha A^2} &= \int d\phi \exp\left(-\alpha\left[\phi^2 - 2i\phi A\right]\right) \\
&= \int d\phi \exp\left(-\alpha\left[(\phi + i\omega)^2 - 2i(\phi + i\omega)A\right]\right) \\
&= \int d\phi \exp\left(-\alpha\left[\phi^2 + 2i\phi(\omega - A) - \omega^2 + 2\omega A\right]\right) \\
&\simeq \exp\left(\alpha\left[\omega^2 - 2\omega A\right]\right) \quad \left(\phi = i\omega,\quad \omega = \langle A \rangle\right)
\end{aligned}
$$



Auxiliary Field Effective Action

- **Fermion det. + U0 integral can be done analytically.**
  → Auxiliary field effective action

$$S_{\rm eff}^{\rm AF} = \sum_{\boldsymbol{k},\tau,f(\boldsymbol{k})>0} \frac{L^3 f(\boldsymbol{k})}{4N_c}\left[|\sigma_{\boldsymbol{k},\tau}|^2 + |\pi_{\boldsymbol{k},\tau}|^2\right]$$

$$-\sum_{\boldsymbol{x}} \log\left[X_N(\boldsymbol{x})^3 - 2X_N(\boldsymbol{x}) + 2\cosh(3\mu/T)\right]$$

$$X_N(\boldsymbol{x}) = X_N[m(\boldsymbol{x},\tau)] \qquad (\text{known func.})$$

$$m_x = m_0 + \frac{1}{4N_c}\sum_j \left((\sigma + i\varepsilon\pi)_{x+\hat{j}} + (\sigma + i\varepsilon\pi)_{x-\hat{j}}\right)$$

- **X_N = Known function of m(x, τ)** *Faldt, Petersson ('86)*

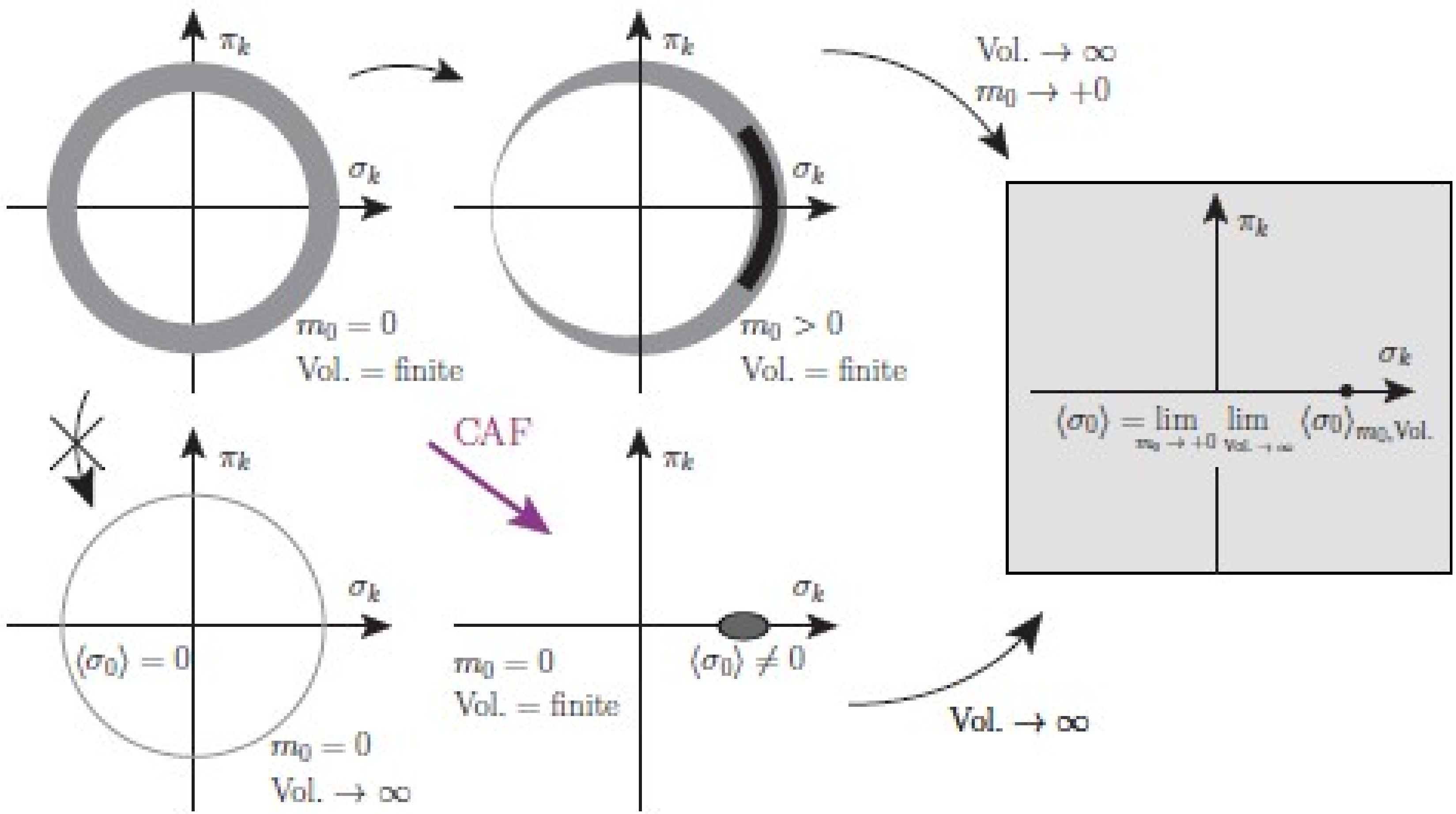
  For constant m, X_N= 2 cosh (N_τ arcsinh (m/γ))

- **Imag. part from X_N → Relatively smaller at large μ/T**

- **Imag. part from low momentum AF cancels due to iε factor.**

Chiral Angle Fixing

How can we simulate correct thermodynamic chiral limit using finite volume simulations ?

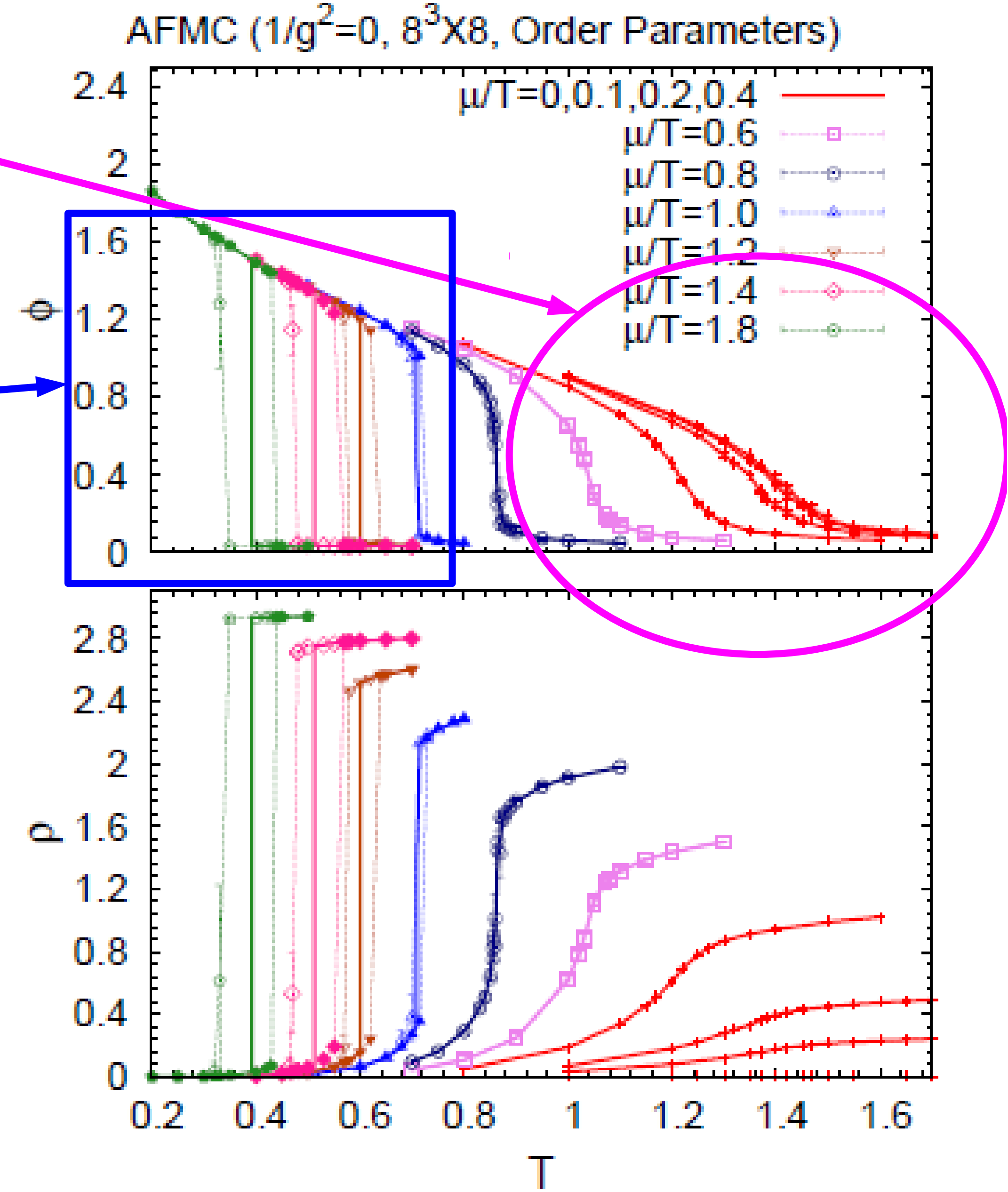


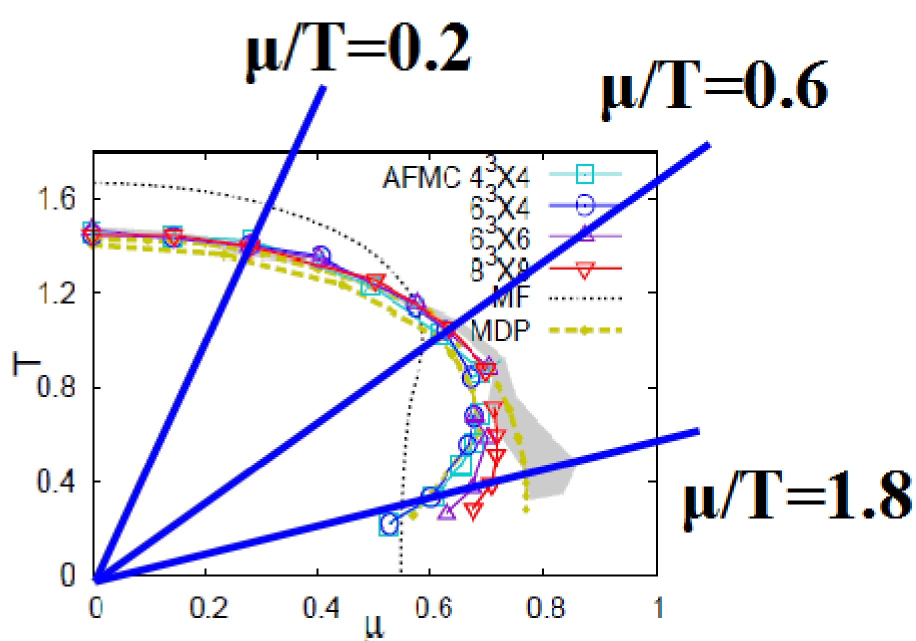
Ichihara, AO, Nakano ('14)

c.f. rms spin is adopted in spin systems *Kurt, Dieter ('10)*

Order Parameters

- **Low μ/T region**
  → 2nd order or crossover
  (would-be second)
- **High μ/T region**
  → sudden change
  & hysteresis
  (would-be first)





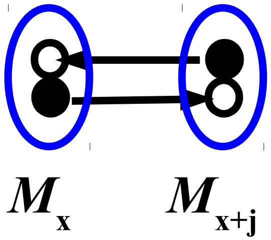
Ichihara, AO, Nakano ('14)

Effective action

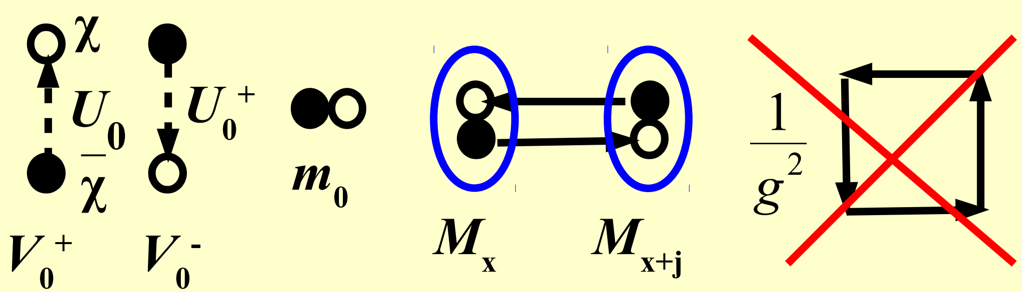
- **Effective action in the strong coupling limit (SCL)**
  - **Ignore plaquette action ($1/g^2$)**
    → We can integrate each link independently !
  - **Integrate out *spatial* link variables of min. quark number diagrams (1/d expansion)**

$$S_{\mathrm{eff}} = S_F^{(t)} - \frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}} + m_0 \sum_x M_x \quad \left(M_x = \bar{\chi}_x \chi_x\right)$$

Damgaard, Kawamoto, Shigemoto ('84)



$$\int dU\, U_{ab} U^+_{cd} = \delta_{ad}\delta_{bc}/N_c$$



$$\int dU\, U_{ab} U^+_{cd} = \delta_{ad}\delta_{bc}/N_c$$

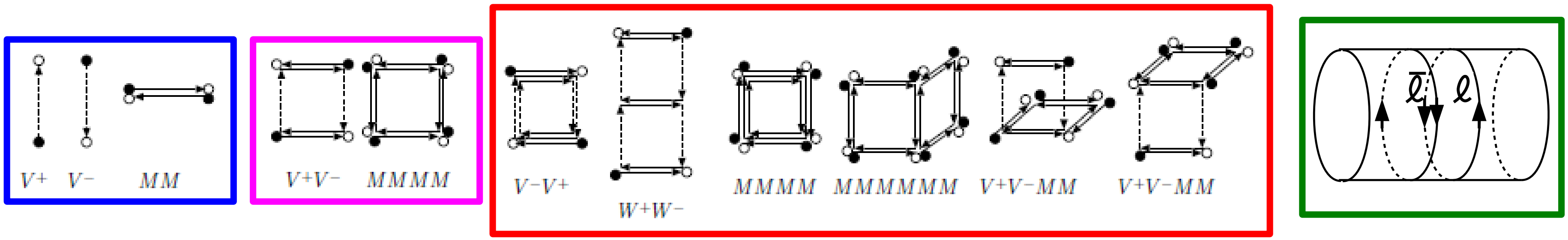
Lattice QCD in SCL
→ Fermion action with nearest neighbor four Fermi interaction

Finite Coupling Effects

- **Effective Action with finite coupling corrections**
  Integral of $\exp(-S_G)$ over spatial links with $\exp(-S_F)$ weight → $S_{\rm eff}$

$$S_{\rm eff} = S_{\rm SCL} - \log\langle \exp(-S_G)\rangle = S_{\rm SCL} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$\langle S_G^n\rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\rm eff} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

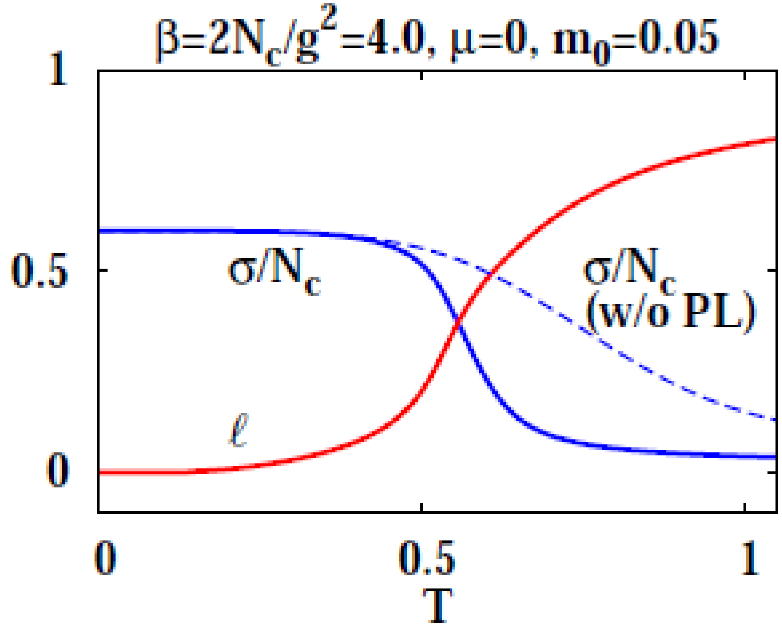
$$-\left(\frac{1}{g^2N_c}\right)^{N_\tau} N_c^2 \sum_{\mathbf{x},j>0}\left(\bar{P}_{\mathbf{x}}P_{\mathbf{x}+\hat{j}} + h.c.\right)$$ *Polyakov loop (Gocksch, Ogilvie ('85), Fukushima ('04) Nakano, Miua, AO ('11))*

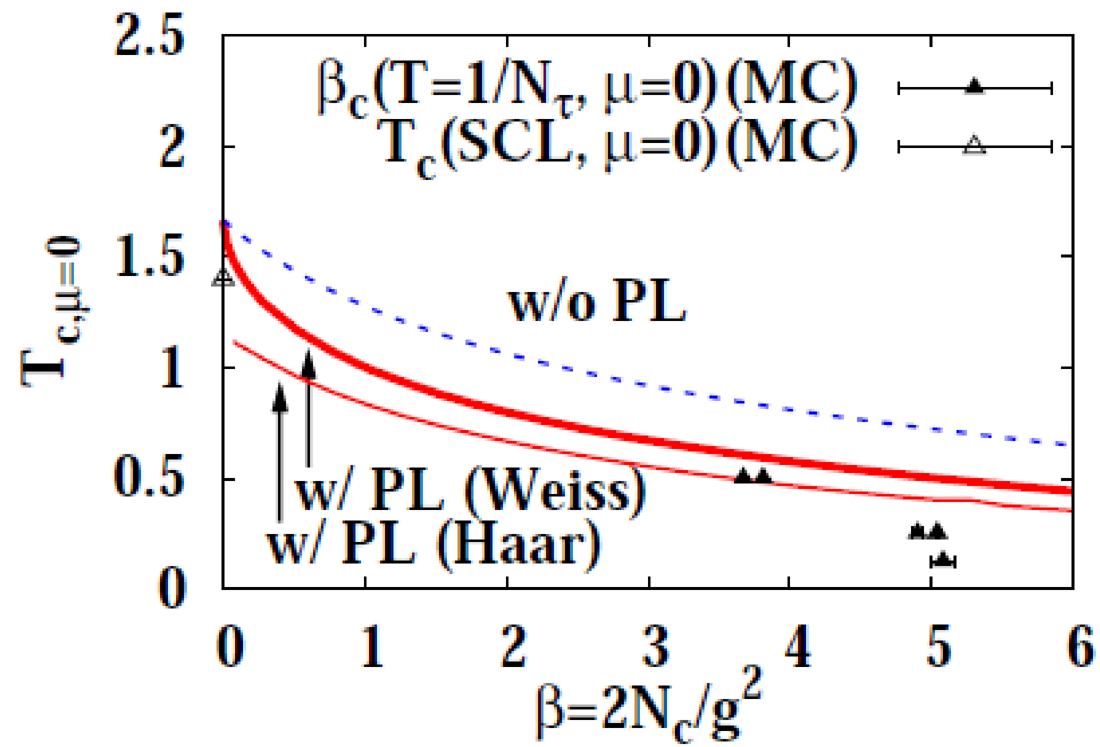
SC-LQCD with Polyakov Loop Effects at $\mu=0$

T. Z. Nakano, K. Miura, AO, PRD 83 (2011), 016014 [arXiv:1009.1518 [hep-lat]]

- **P-SC-LQCD reproduces MC results of $T_c(\mu=0)$ ($\beta = 2N_c/g^2 \leq 4$)**

  MC data: SCL (Karsch et al. (MDP), de Forcrand, Fromm (MDP)), $N_\tau=2$ (de Forcrand, private), $N_\tau=4$ (Gottlieb et al.('87), Fodor-Katz ('02)), $N_\tau=8$ (Gavai et al.('90))





Lattice Unit

SC-LQCD: Setups & Disclaimer

- **Effective action in SCL ($1/g^0$), NLO ($1/g^2$), NNLO ($1/g^4$) terms and Polyakov loop.**

  NLO: Faldt-Petersson ('86), Bilic-Karsch-Redlich ('92)
  Conversion radius > 6 in pure YM ? Osterwalder-Seiler ('78)

- **One species of unrooted staggered fermion (N_f=4 @ cont.)**

  Moderate N_f deps. of phase boundary: BKR92, Nishida('04), D'Elia-Lombardo ('03)

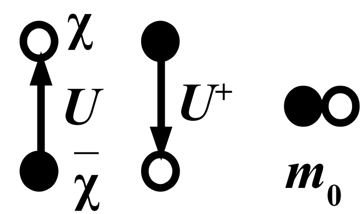
- **Leading order in 1/d expansion (d=3=space dim.)**
  → Min. # of quarks for a given plaquette configurations, no spatial B hopping term.
- **Different from "strong couling" in "large N_c"**

> ***Still far from "Realistic", but SC-LQCD would tell us useful qualitative features of the phase diagram and EOS.***

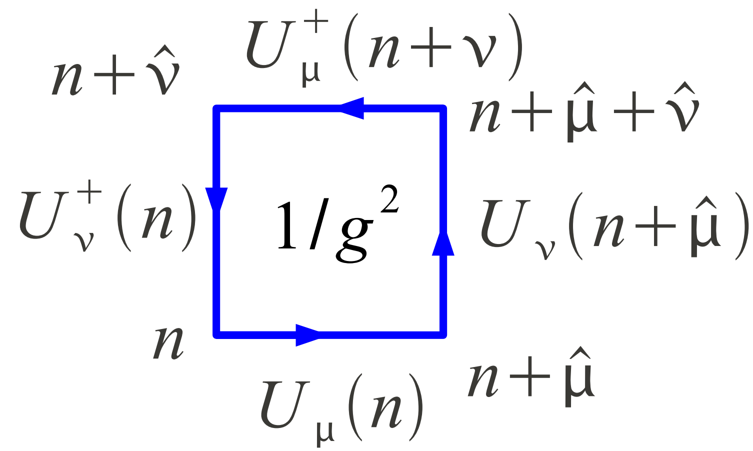
Strong Coupling Lattice QCD

- **Lattice QCD action (unrooted staggered fermion)**

$$L=\frac{1}{2}\sum_{x,\mu}\eta_\mu(x)\left[\bar{\chi}_x U_\mu(x) e^{\mu\delta_{\mu,0}}\chi_{x+\hat{\mu}}-\bar{\chi}_{x+\hat{\mu}} U^+_\mu(x) e^{-\mu\delta_{\mu,0}}\chi_x\right]+m_0\sum_x \bar{\chi}_x\chi_x$$
$$+\frac{2N_c}{g^2}\sum_{plaq.}\left[1-\frac{1}{N_c}\mathrm{Re\,tr}\,U_{\mu\nu}(x)\right]$$



- **Strong coupling limit**
  - **Plaquette terms vanish, and each linsk**



Strong-coupling lattice QCD
Integrate out spatial links first
→ Many-body problem of quarks with color singlet interactions

Sign problem in lattice QCD

- **Fermion determinant (= stat. weight of MC integral) becomes complex at finite μ in LQCD.**
  - **γ_5 Hermiticity**

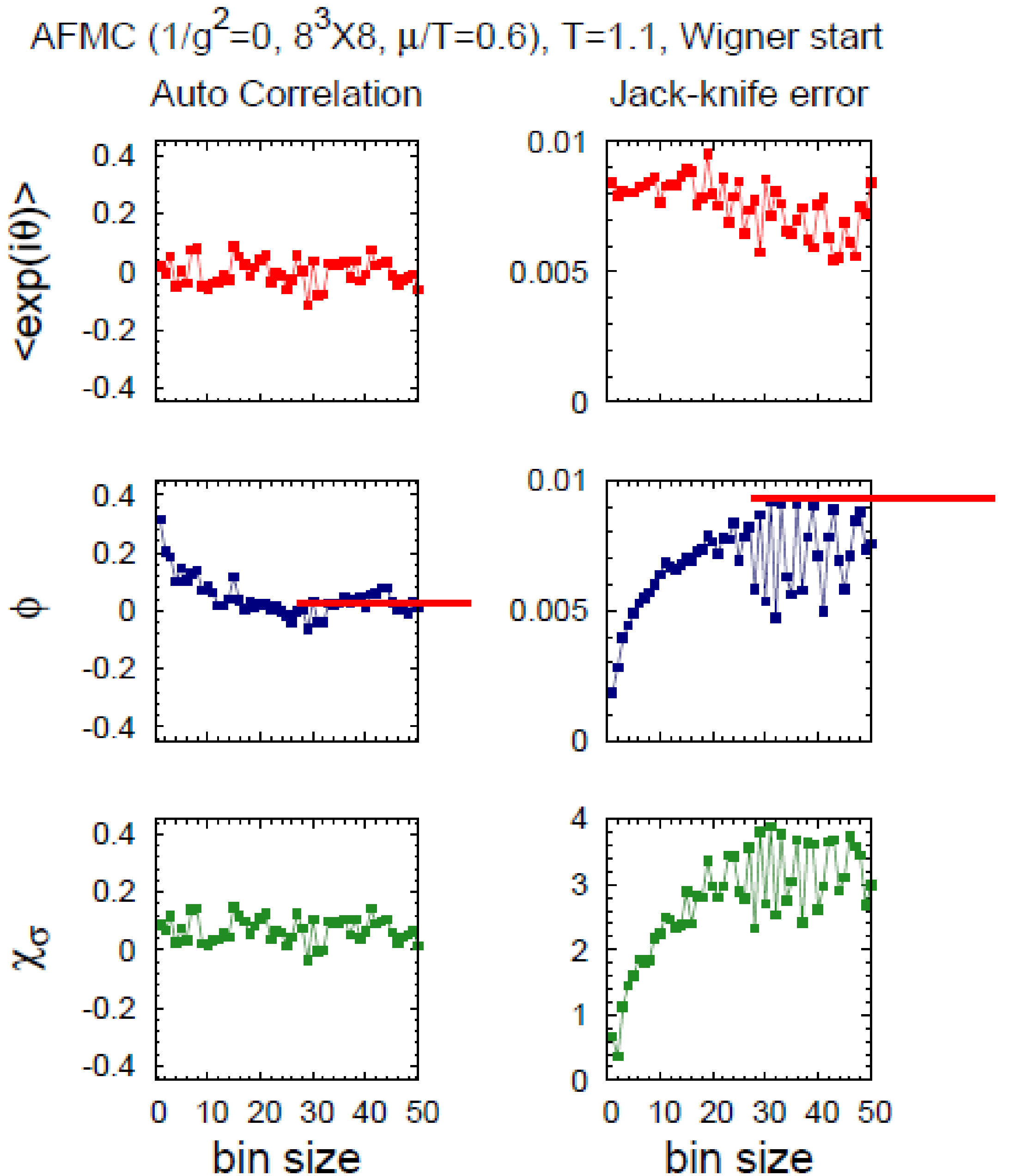
$$Z=\int D[U,q,\bar{q}]\ \exp\left(-\bar{q}\,D(\mu,U)\,q-S_G(U)\right)$$
$$=\int D[U]\ \mathrm{Det}\left(D(\mu,U)\right)\ \exp\left(-S_G(U)\right)$$

$$\gamma_5 D(\mu,U)\gamma_5=\left[D(-\mu^*,U^+)\right]^+$$
$$\rightarrow \mathrm{Det}\left(D(\mu,U)\right)=\left[\mathrm{Det}\left(D(-\mu^*,U^+)\right)\right]^*$$

  - **Fermion det. (Det D) is real for zero μ (and pure imag. μ)**
  - **Fermion det. is complex for finite real μ.**
  - **Phase quenched weight = Weight at isospin chem. pot.**

$$Z_{\mathrm{pq}}(T,\mu_u=\mu_d=\mu)=Z_{\mathrm{full}}(T,\mu_u=-\mu_d=\mu)$$

Error estimate by Jack-knife method



Error
= Jack-knife error
after autocorrelation disappears

Ichihara, AO, Nakano ('14)

Abstract

QCD phase diagram is attracting much attention in these years. It is now extensively studied in the Beam Energy Scan (BES) program at RHIC, and is closely related to the beginning of our universe (big bang) and the final form of matter (neutron stars).

The robust mechanism for the QCD phase transition is the spontaneous chiral symmetry breaking and restoration. The spontaneous symmetry breaking is well understood in chiral effective models of QCD such as the Nambu-Jona-Lasinio model; zero-point energy of quarks favors finite chiral condensate. By using the high-temperature expansion, the transition temperature is found to decrease with increasing chemical potential.

In the first lecture (during the dense matter school), I explain the basic mechanism of the chiral phase transition in effective models, and the expected shape of the phase boundary using the high-temperature expansion. I also introduce the strong coupling lattice QCD, in which we can examine that the same mechanism applies to QCD at strong coupling.

Discovery of the QCD critical point and the first order phase transition at high density is one of the ultimate goals in the BES program and forthcoming FAIR and NICA fascilities, and heavy-ion programs at J-PARC. We also expect formation of dense matter in compact astrophysical phenomena, such as the neutron star core, supernova explosion, dynamical collapse to black holes, and binary neutron star mergers. From the theoretical side, it is desirable to draw the QCD phase diagram using the lattice QCD Monte-Carlo simulations. However, the sign problem in lattice QCD at finite chemical potential causes difficulty in performing precise Monte-Carlo simulations of finite density matter. There are some exceptions such as the color SU(2) QCD, finite isospin chemical potential, or imaginary chemical potential. The strong coupling lattice QCD can be one of these exceptions; while the sign problem exists, it is milder and two independent methods predict the same QCD phase boundary.

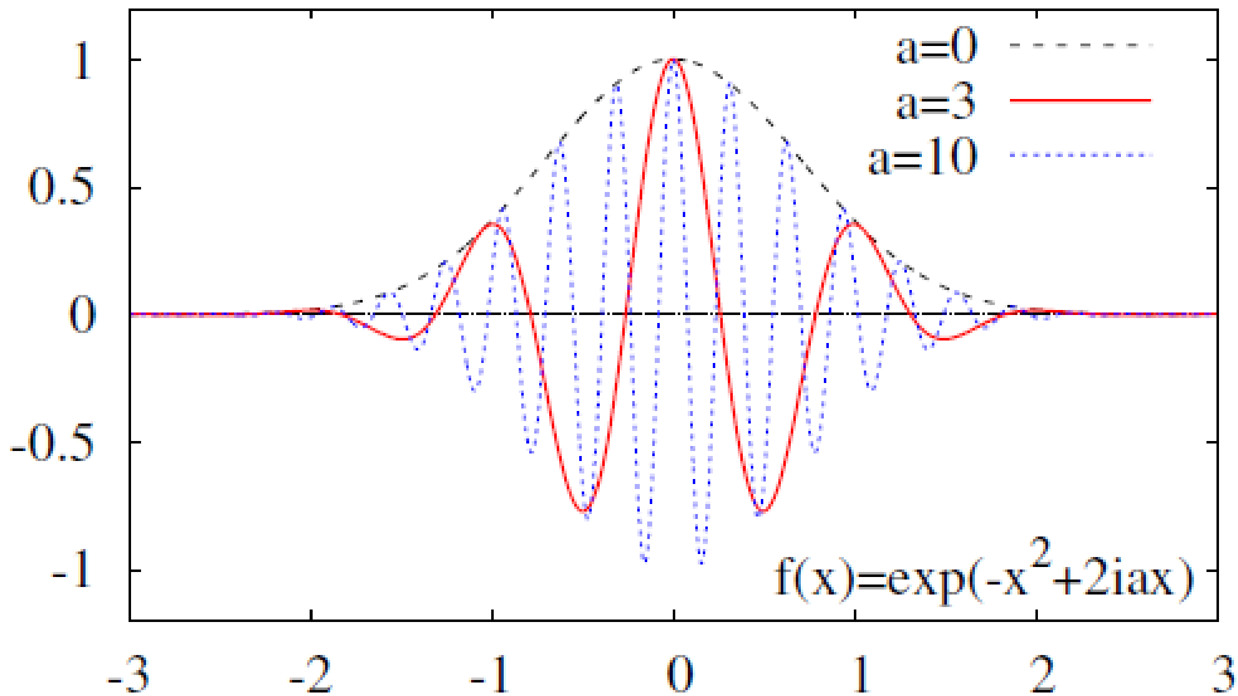
In the second lecture (during SQM), we discuss the phase diagram in effective models and the strong coupling lattice QCD, and some observables expected to appear around the critical point. We also discuss the thermodynamic conditions realized during the failed supernova, where the black hole is formed dynamically. We find that cold, dense, and isospin asymmetric matter is formed during the black hole formation, and it may be possible to sweep the QCD critical point in compact star phenomena.

Sign Problem

- **Monte-Carlo integral of oscillating function**

$$Z=\int dx \exp(-x^2+2iax)=\sqrt{\pi}\exp(-a^2)$$

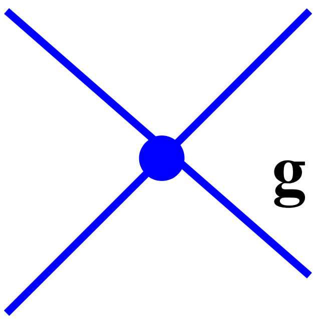
$$\langle O\rangle=\frac{1}{Z}\int dx\, O(x)\, e^{-x^2+2iax}$$



Easy problem for human is not necessarily easy for computers.

Sign Problem (cont.)

- **Generic problem in quantum many-body problems**
  - **Example: Euclid action of interacting Fermions**



$$S = \sum_{x,y} \bar{\psi}_x D_{x,y} \psi_y + g \sum_x (\bar{\psi}\psi)_x (\bar{\psi}\psi)_x$$

  - **Bosonization and MC integral (g>0 → repulsive)**

$$\exp(-g M_x M_x) = \int d\sigma_x \exp(-g\sigma_x^2 - 2ig\sigma_x M_x) \quad (M_x = (\bar{\psi}\psi)_x)$$

$$Z = \int D[\psi, \bar{\psi}, \sigma] \exp\left[-\bar{\psi}(D + 2ig\sigma)\psi - g\sum_x \sigma_x^2\right]$$

$$= \int D[\sigma] \ \underline{\mathrm{Det}(D + 2ig\sigma)} \exp\left[-g \sum_x \sigma_x^2\right]$$

complex Fermion det.
→ complex stat. weight
→ sign problem

How can we investigate QCD phase diagram ?

- **Non-pert. & ab initio approach**
  = Monte-Carlo simulation of lattice QCD
  but lattice QCD at finite μ has the sign problem.

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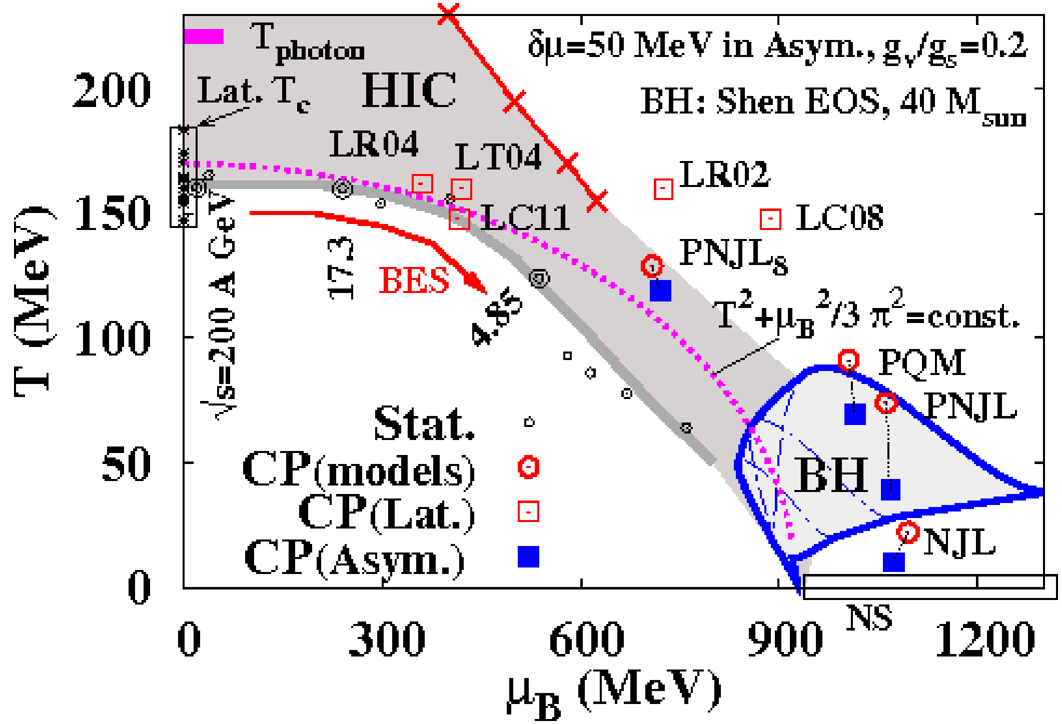
  but lattice QCD at finite μ has the sign problem.
- Effective model and/or Approximations are necessary.
  - Effective models:
    NJL, PNJL, PQM, ...
    Model dependence is large.
  - Approximation / Truncation
    Taylor expansion,
    Imag. μ , Canonical,
    Re-weighting,
    Strong coupling LQCD
  - Alternative method
    Fugacity expansion,
    Histogram method,
    Complex Langevin

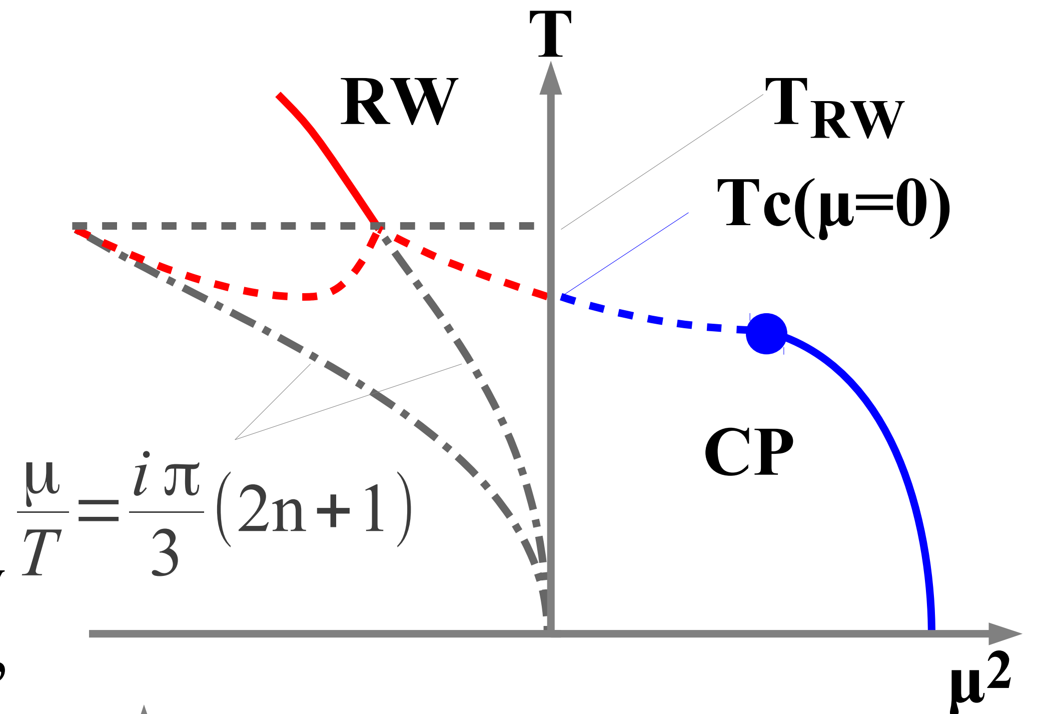


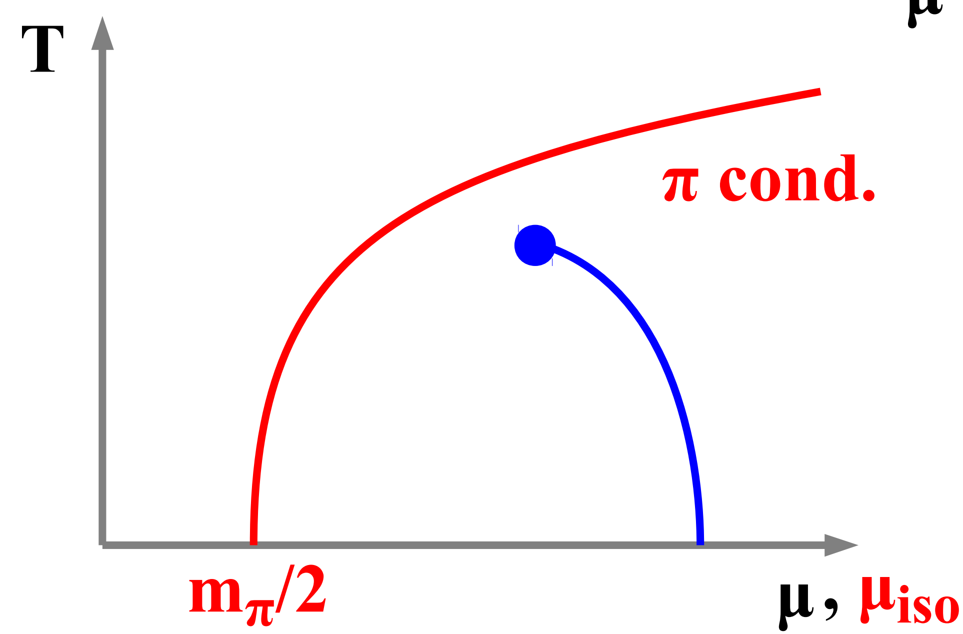
Lattice QCD at fnite μ

- **Various method work at small μ (μ/T < 1).**
- **Large μ**
  - **Roberge-Weiss transition**
    → Conv. μ/T < π/3 at T>TRW
  - **No go theorem**
    Splittorff ('06), Han, Stephanov ('08), Hanada, Yamamoto ('11), Hidaka, Yamamoto ('11)
  - **Phase quenched sim.**
    ~ Isospin chem. pot.

$$Z_{\text{phase quench}}(T,\mu_u=\mu_d=\mu) = Z_{\text{full}}(T,\mu_u=-\mu_d=\mu)$$

  - **→ CP in π cond. phase**
    (Silver Blaze)

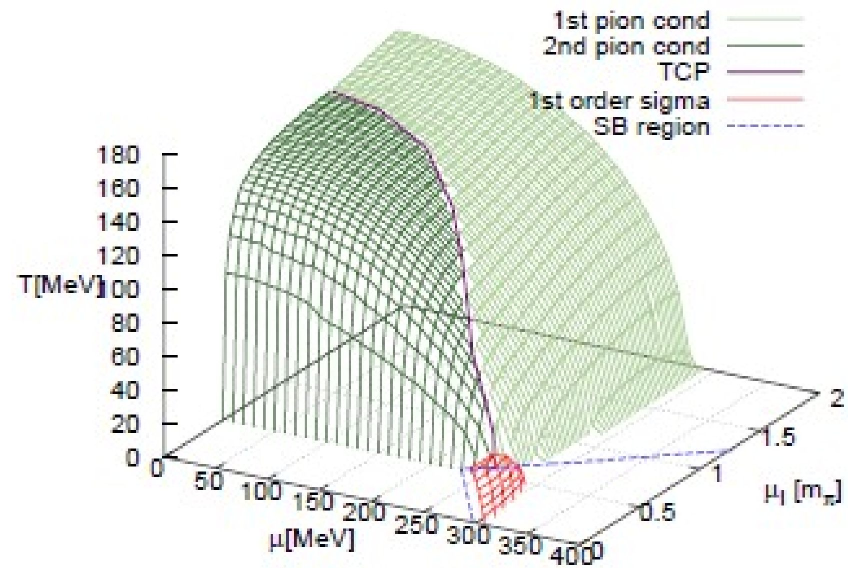


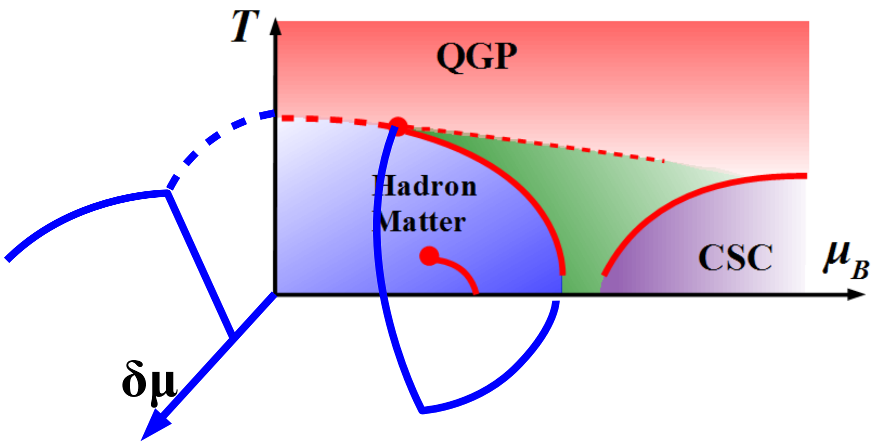


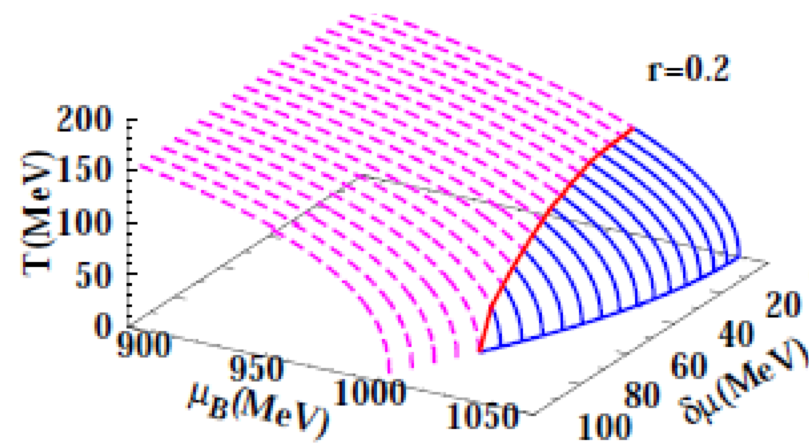
Phase diagram in isospin chemical potential space

- **Important in Compact Astrophys. phen.**
- **At vanishing quark chem. pot., there is no sign problem.**
- **Interesting 3D phase structure.**

FRG: Kamikado, Strodthoff, von Smekal, Wambach ('13)





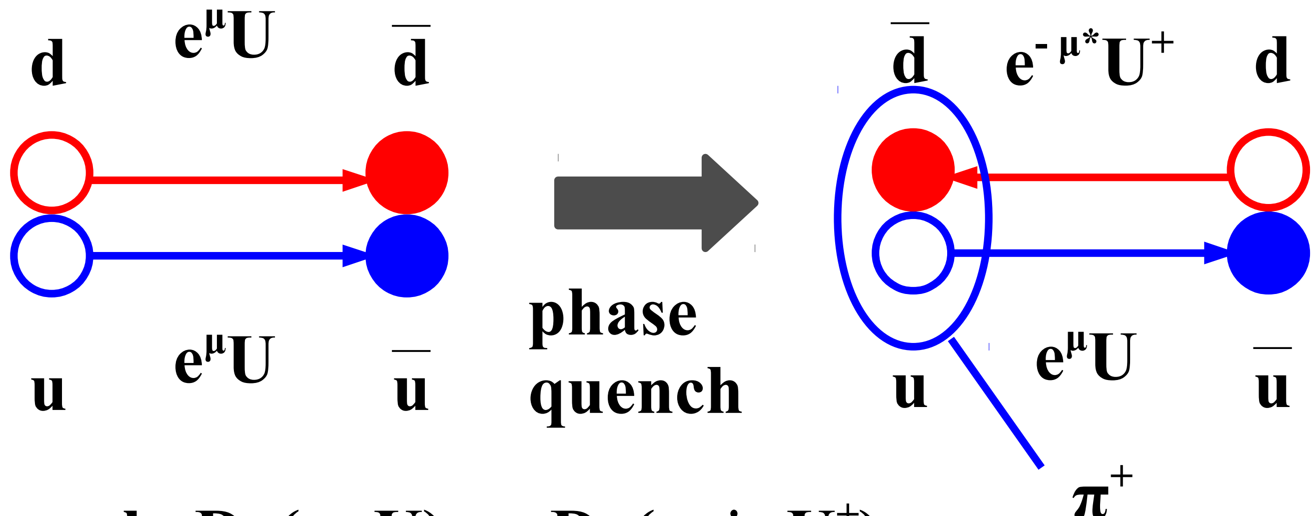


Kogut, Sinclair ('04); Sakai et al.('10);
AO, Ueda, Nakano, Ruggieri, Sumiyoshi ('11)

PQM: Ueda, Nakano, AO, Ruggieri, Sumiyoshi ('13)

Silver Blaze

- "Watson, the dog did not bark at night. This is the evidence that he is the criminal who stole Silver Blaze."
- In physics,
  "If $\delta\mu > m_\pi/2$ at low T and you do not have pion condensation, that theory should be wrong."



- Phase quench $D_d(\mu, U) \rightarrow D_d(-\mu^*, U^+)$
  $\rightarrow$ We can compose pions from original di-quark configuration.
- To do: Directly sample with complex S (CLE), Integrate U first (SC-LQCD), and some other method....

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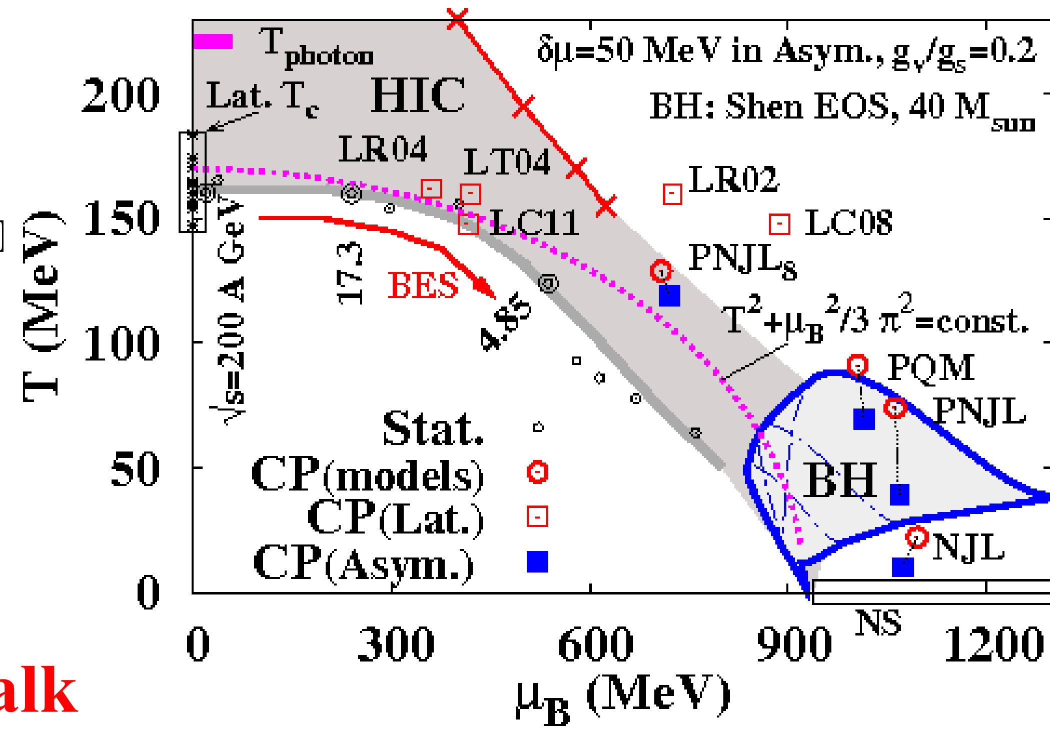
    Taylor expansion,
    Imag. μ , Canonical,
    Re-weighting,
    Strong coupling LQCD
  - **Alternative method**
    Fugacity expansion, **This talk**
    Histogram method, *Nakamura, Nagata*
    Complex Langevin *Ejiri*
    Stamatescu

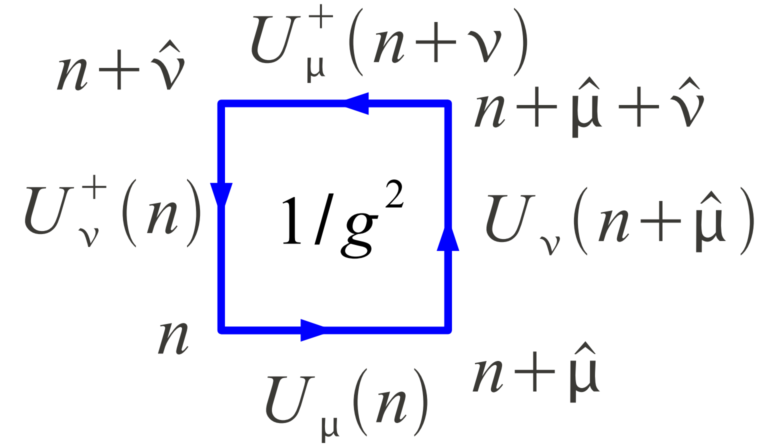


Strong coupling lattice QCD

Lattice QCD action

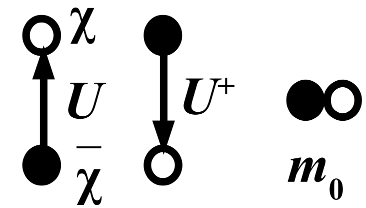
- **Gluon field → Link variables** $U_\mu(x) \simeq \exp(i g A_\mu)$
- **Gluon action → Plaquette action**

$$S_G = \frac{2N_c}{g^2} \sum_{plaq.} \left[1 - \frac{1}{N_c} \mathrm{Re\,tr}\, U_{\mu\nu}(n) \right]$$



  - **Loop → surface integral of "rotation" $F_{\mu\nu}$ in the U(1) case.**

- **Quark action (staggered fermion)**



$$S_F = \frac{1}{2} \sum_x \left[\bar{\chi}_x e^{\mu} U_{0,x} \chi_{x+\hat{0}} - \bar{\chi}_{x+\hat{0}} e^{-\mu} U_{0,x}^+ \chi_x \right]$$

$$+ \frac{1}{2} \sum_{x,j} \eta_j(x) \left[\bar{\chi}_x U_{x,j} \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_{x,j}^+ \chi_x \right] + \sum_x m_0 M_x$$

$$= S_F^{(t)} + S_F^{(s)} + \sum_x m_0 M_x$$