Hydrodynamic helicity and strange hyperons polarization in Heavy Ion Collisions

SQM-2015

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Main Topics

- Vorticity and helicity
- Vortical structures
- Polarization of hyperons
- Handedness

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength) – "small Bang"

- Differential rotation – vorticity
- P-odd
- May lead to various P-odd effects
- Calculation in quark - gluon string model (Baznat,Gudima,Sorin,OT,PRC'13 arXiv:1301.7003 and work in preparation)

Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \Sigma\, r \times p$
- Differential pseudovector characteristics – vorticity
- ω = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows
  v || curl v
- Investigation in QGSM

Simulation in QGSM (Kinetics -> HD)

$50 \times 50 \times 100$ cells $dx = dy = 0.6$ fm, $dz = 0.6/\gamma$ fm

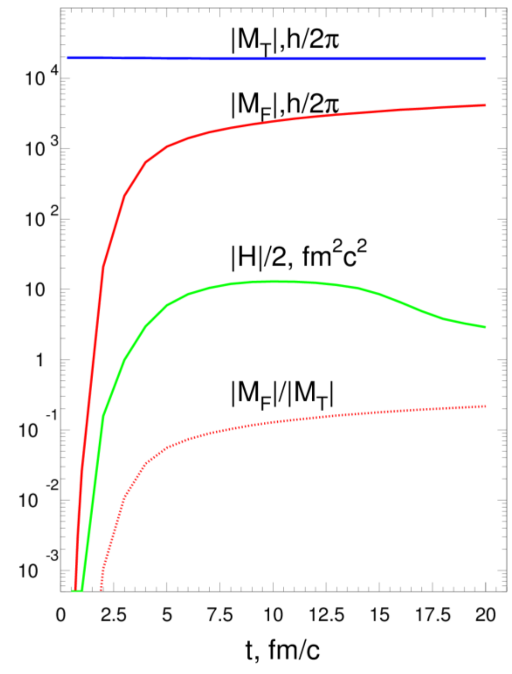
- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

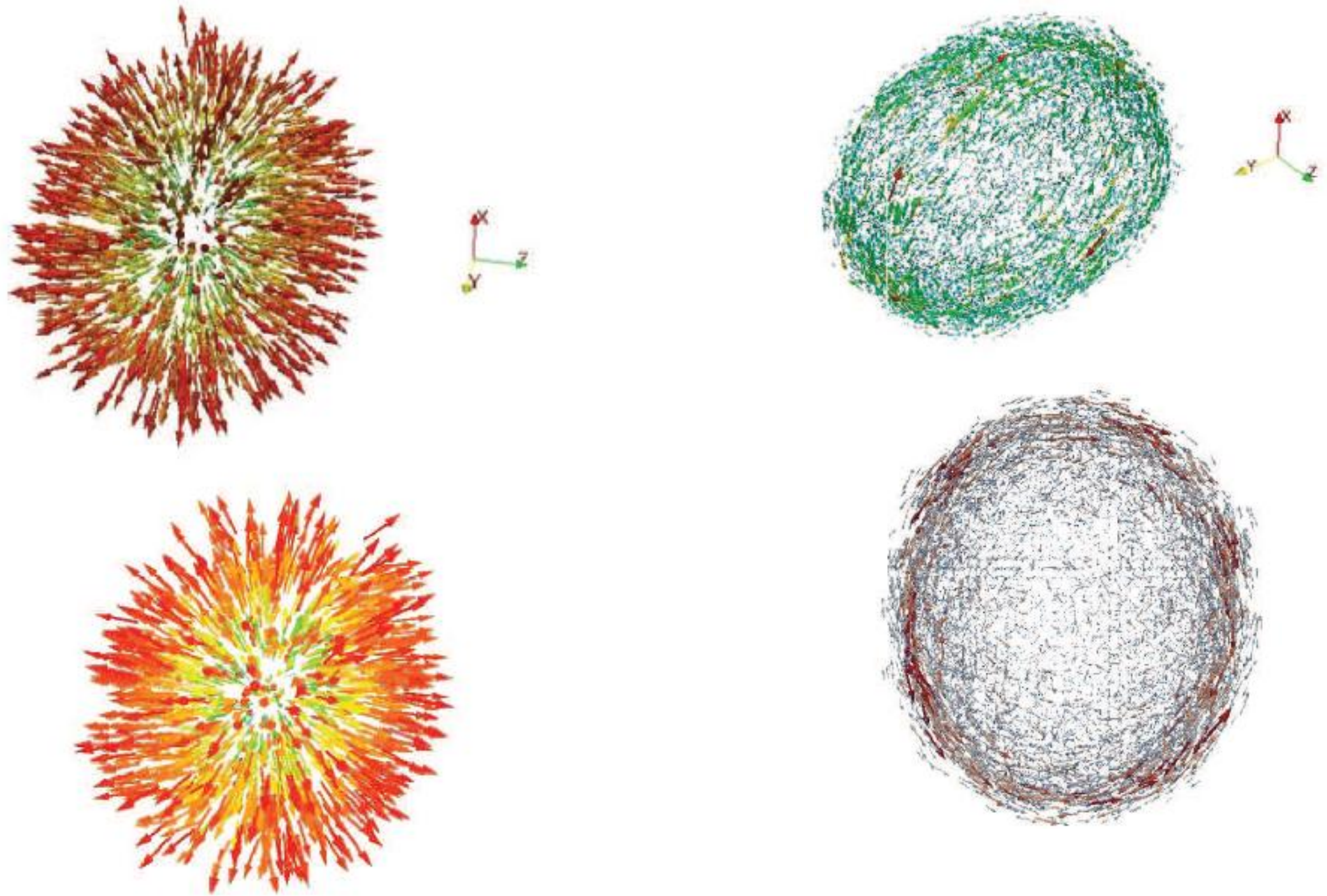
- Vorticity – from discrete partial derivatives

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)
- Conservation of OAM with a good accuracy!

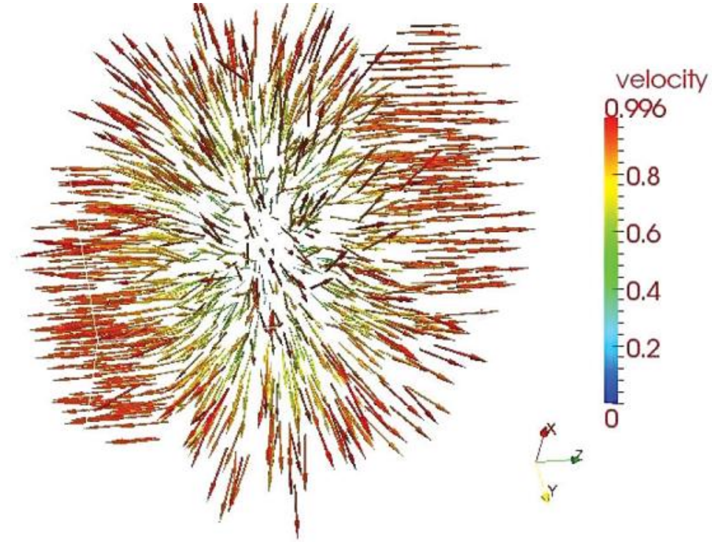


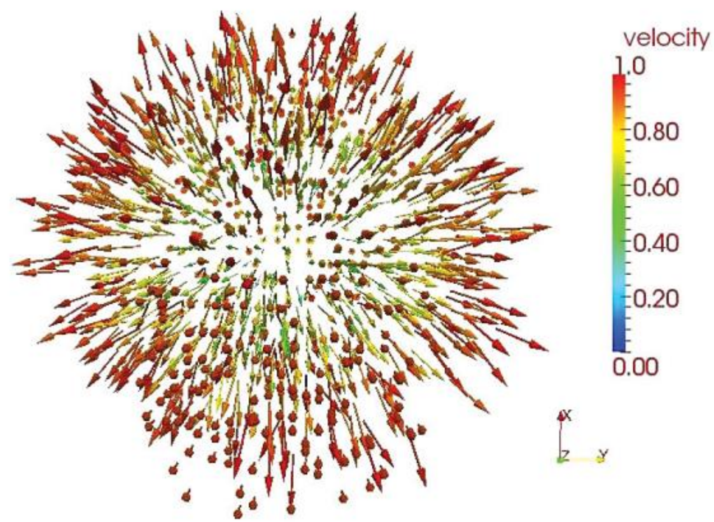
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



Distribution of velocity ("Small Bang")

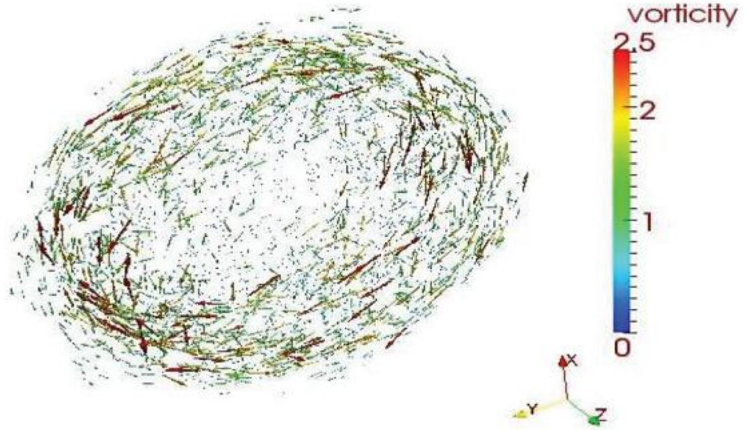
- 3D/2D projection
- z-beams direction
- x-impact paramater

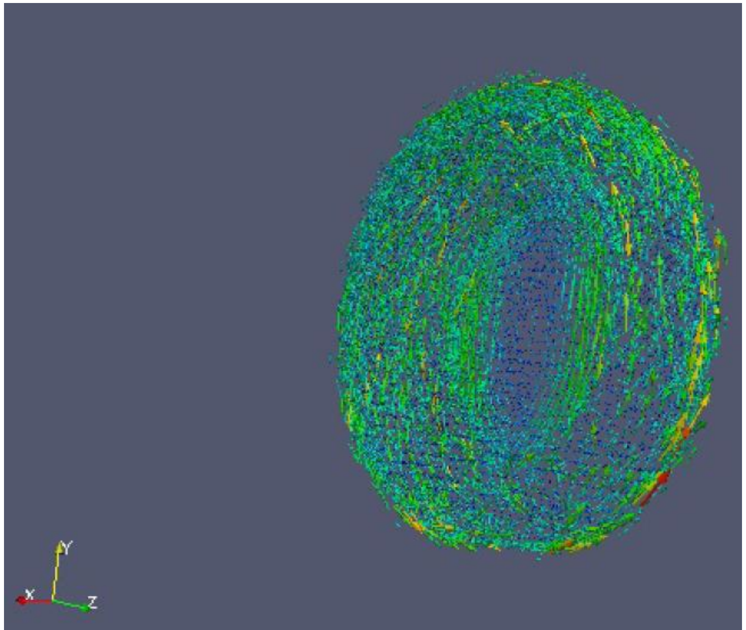


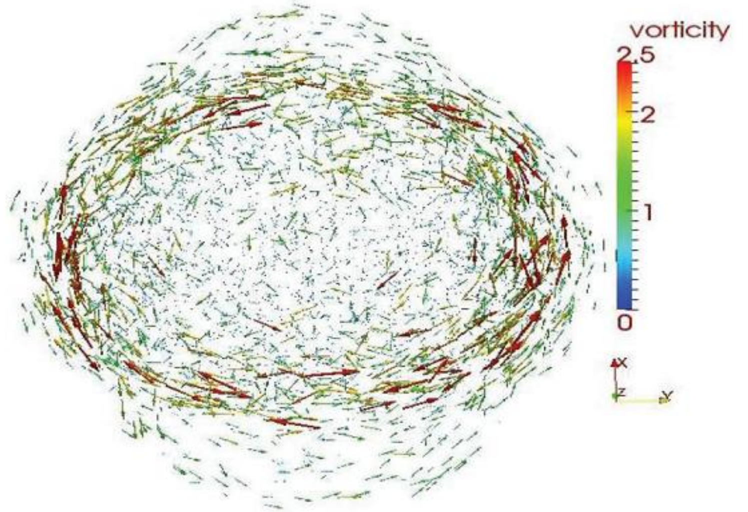


Distribution of vorticity ("small galaxies")

- Layer (on core - corona borderline) patterns

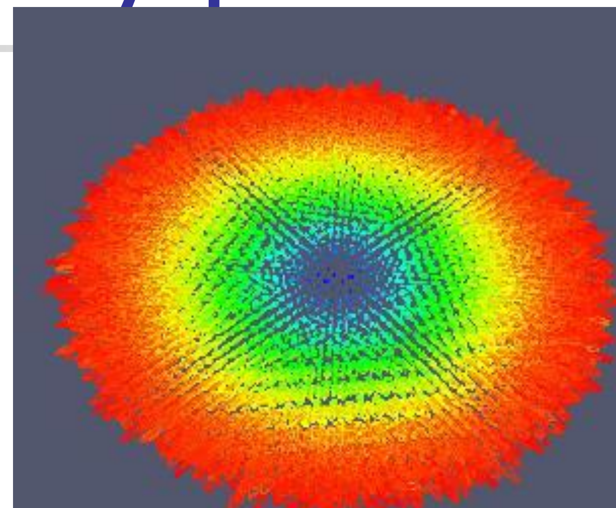




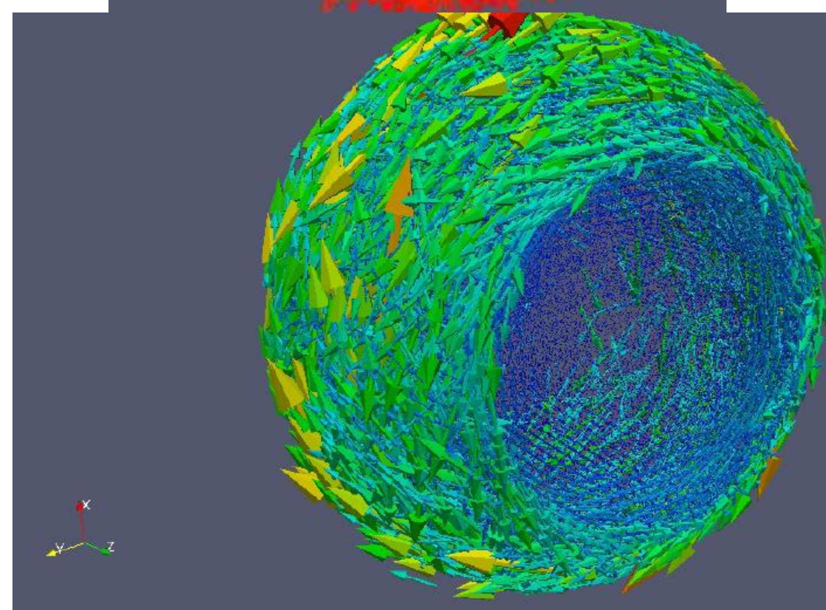


Velocity and vorticity patterns

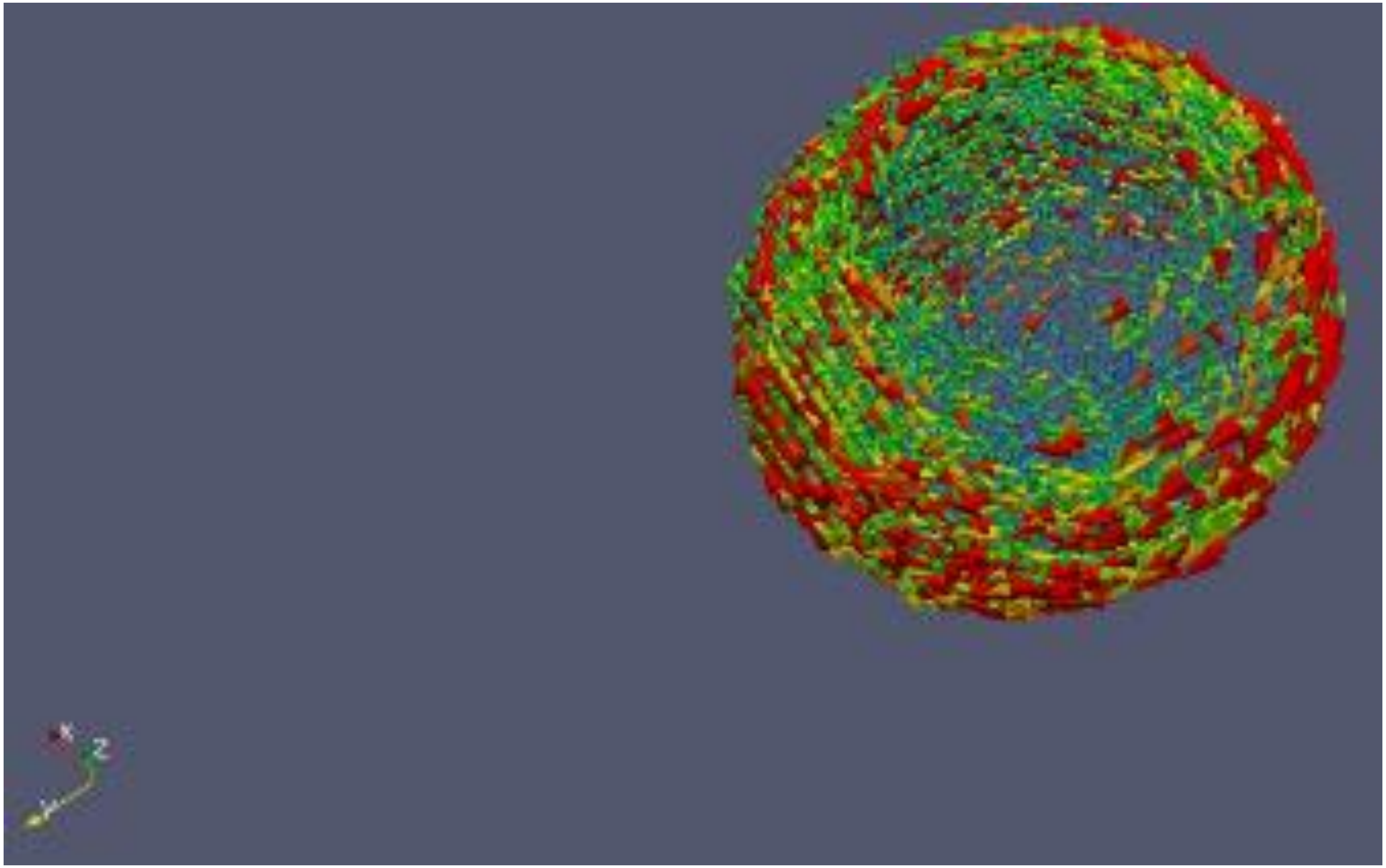
- Velocity



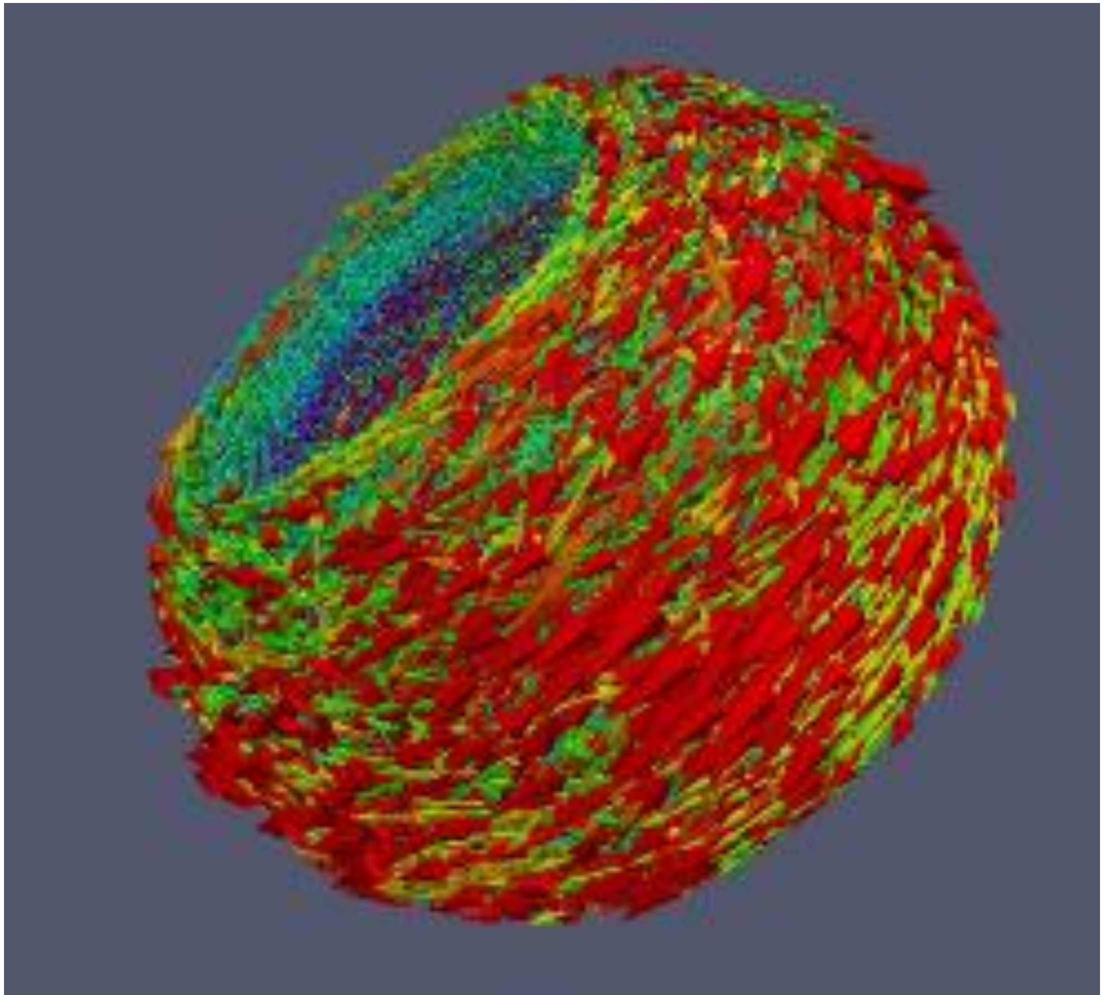
- Vorticity pattern – vortex sheets - due to L BUT cylinder symmetry!



Vortex sheet (fixed direction of L)

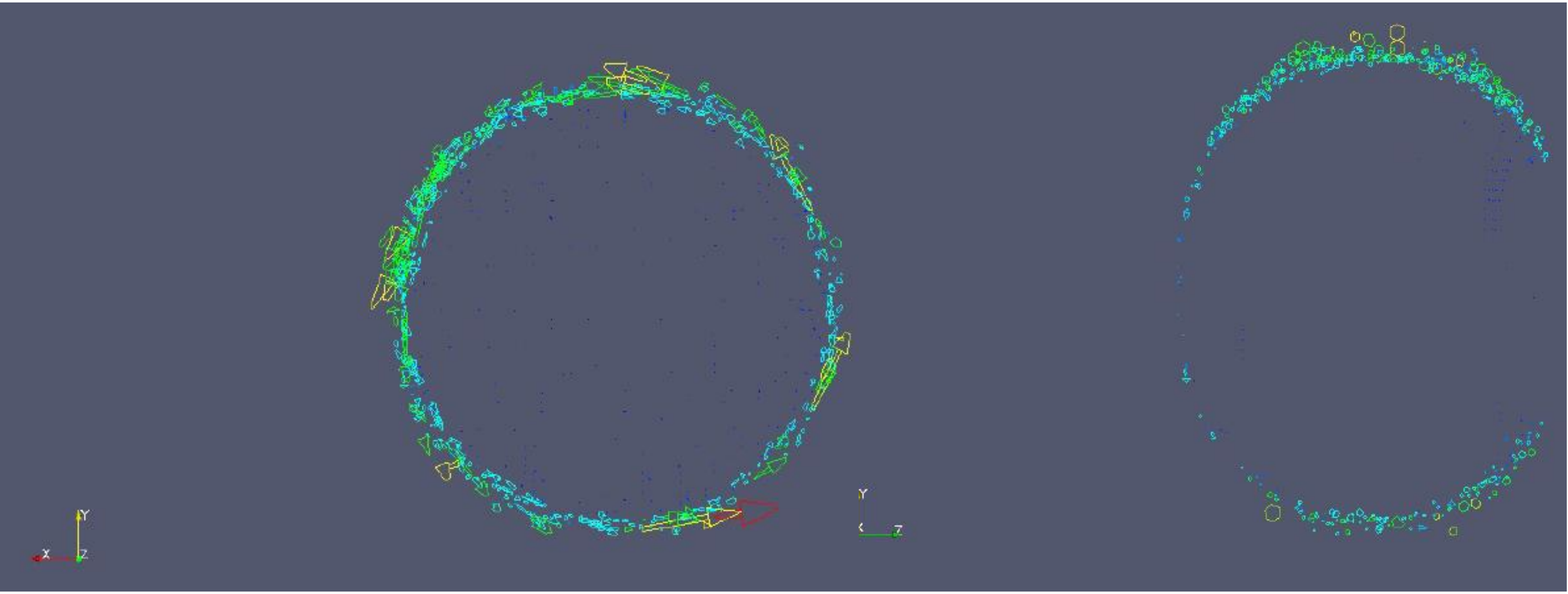


Vortex sheet （Average over L directions）



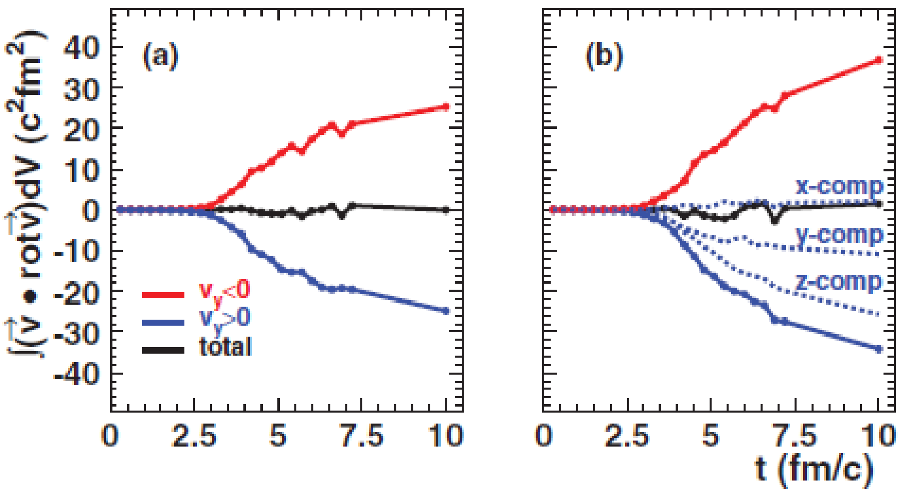
Sections of vorticity patterns

- Front and side views



Helicity separation

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (OT,Usubov - poster)



What is the relative orientation of velocity and vorticity?

- Measure – Cauchy-Schwarz inequality
- Small but non-negligible correlation
- Maximal correlation - Beltrami flows

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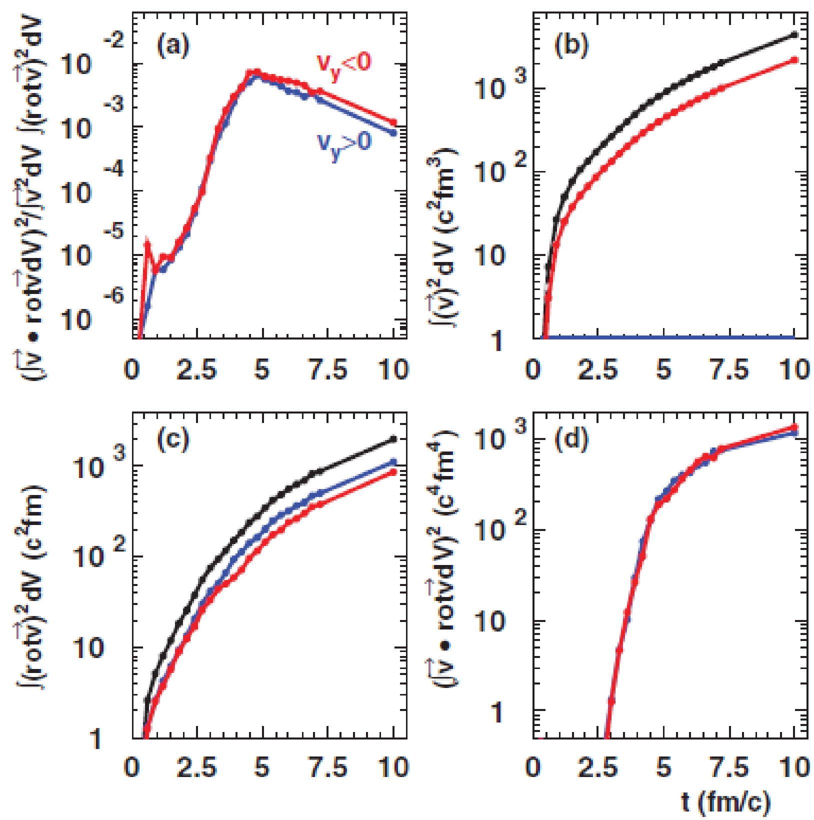
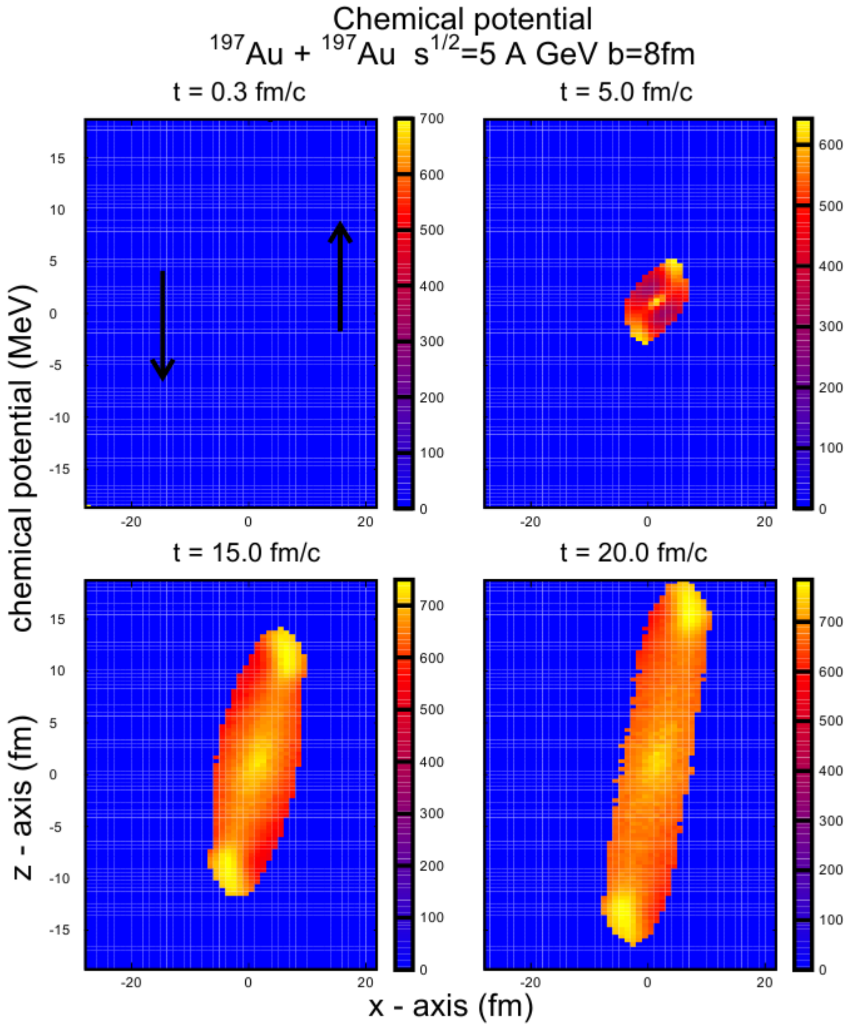


How to observe this effect?

- Coupling of HD helicity to quark axial current via axial VVA anomaly
- V: $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- AH -> H(elicity) density
- VVA: QUARK polarization~helicity
- Strange quarks: May lead to POLARIZATION of (Lambda) hyperons (cf other mechanisms)
- Large chemical potential: appropriate for NICA/FAIR energies

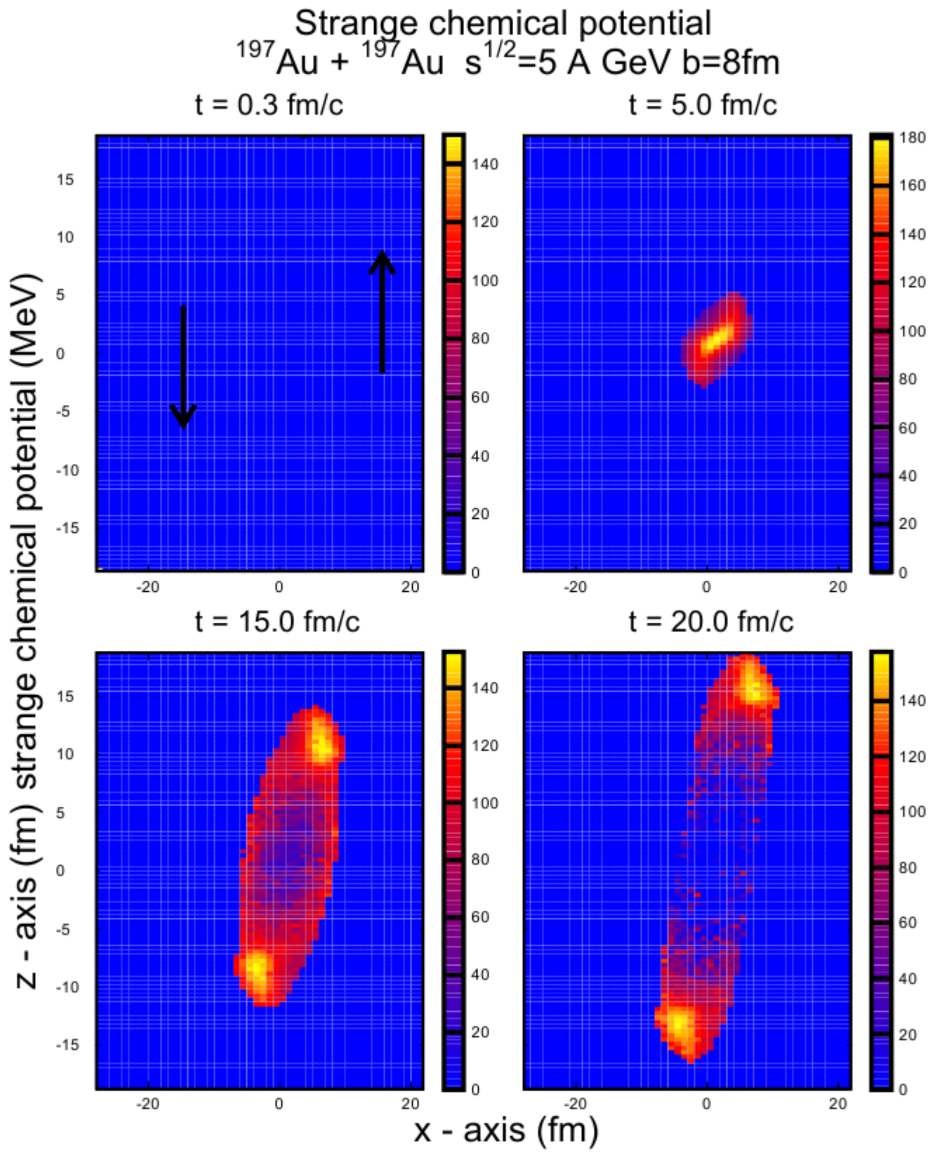
Chemical potential : Kinetics -> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: y=0

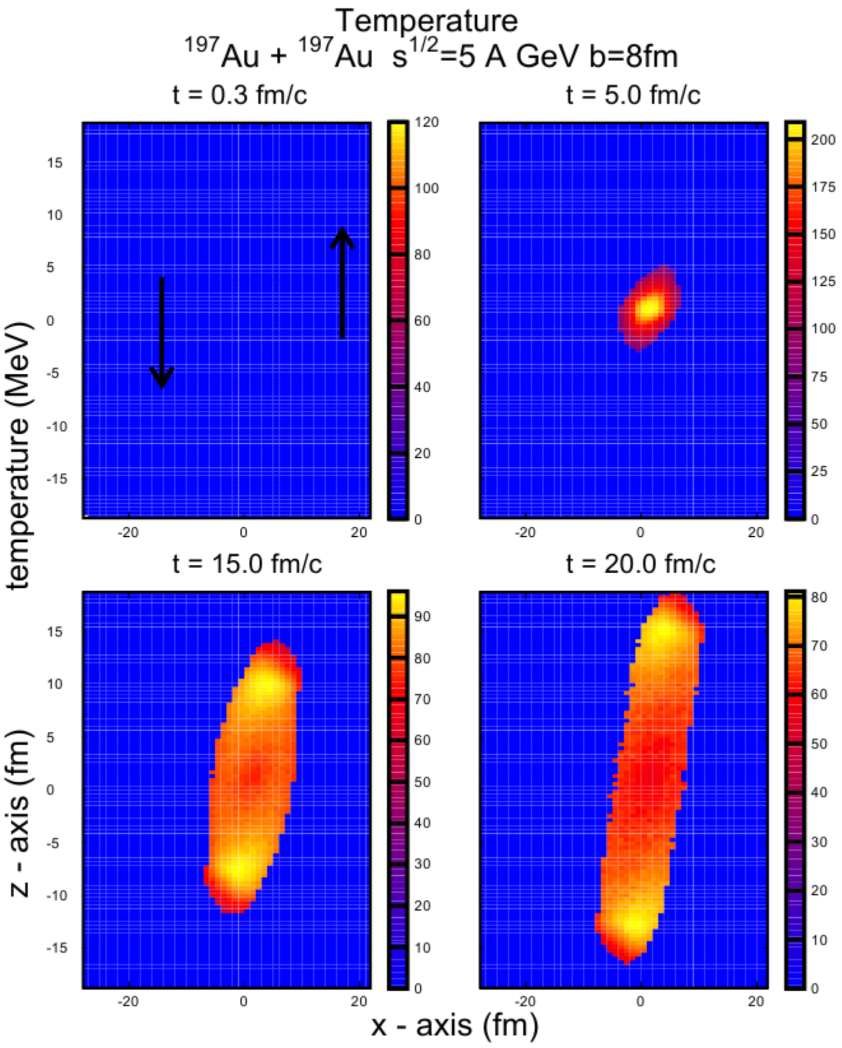


Strange chemical potential (polarization of Lambda is carried by strange quark!)

- Emergent effect



Temperature

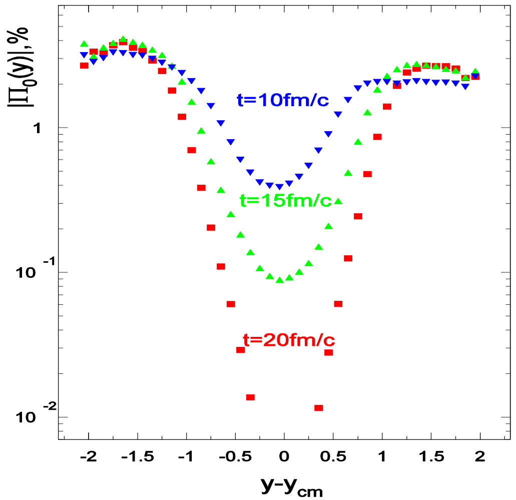


Helicity -> rest frame polarization

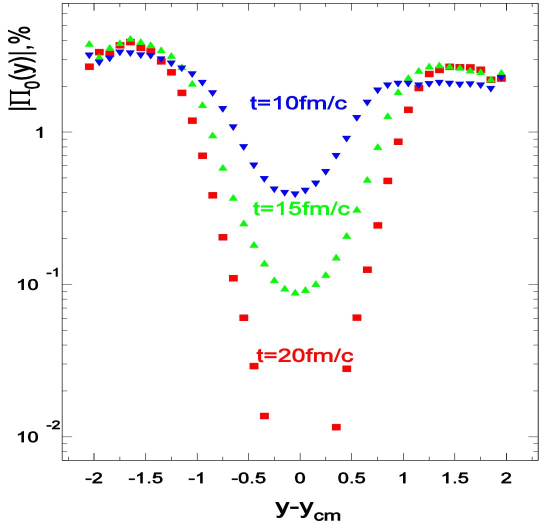
- Helicity ~ 0th component of polarization in lab. frame – effect of boost to Lambda rest frame – various options

$$\Pi_0(y)=1/(4\pi^2)\int \gamma^2(x)\mu_s^2(x)|v\cdot rot(v)|n_\Lambda(y,x)w_1 d^3x / \int n_\Lambda(y,x)w_2 d^3x$$

$w_1=1$, $w2= 1$

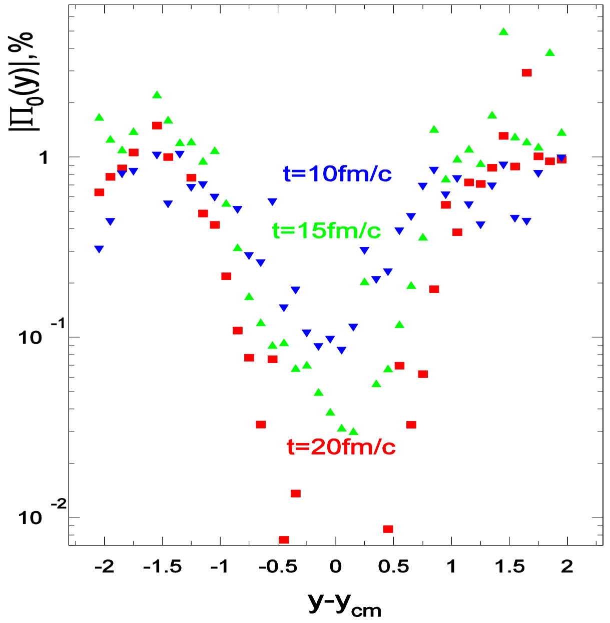


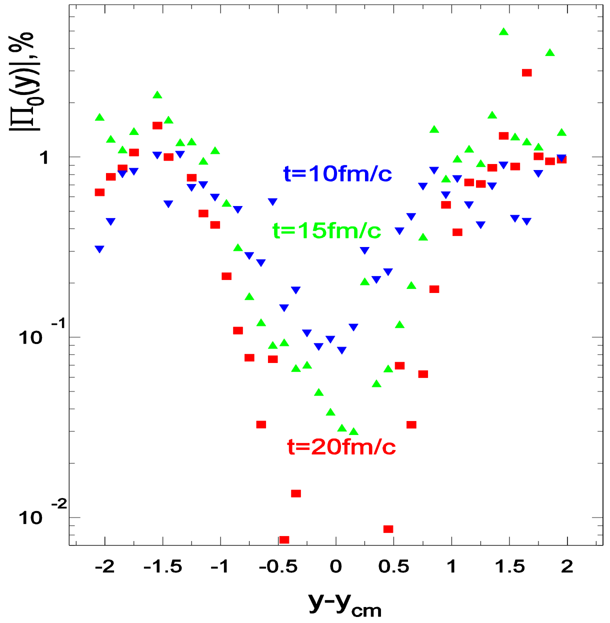
$w_1=1$, $w_2=p_y/m$



Various methods of boost implementation

- $w_1 = m/p_y$, $w_2 = 1$ $w_1 = m/p_y$, $w_2 = p_y/m$





Handedness: directly observable P-odd momentum correlations

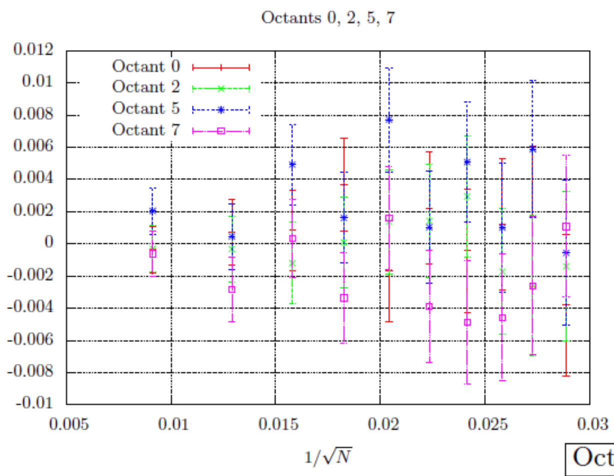
- Found in jets in e^+e^- annihilation (LEP, BELLE)
- First attempt in HIC: OT, Usubov, **arXiv:1406.4451** (to appear in PRC, poster by Rahim Usubov)
- Average =0: Phase space – 8 octants

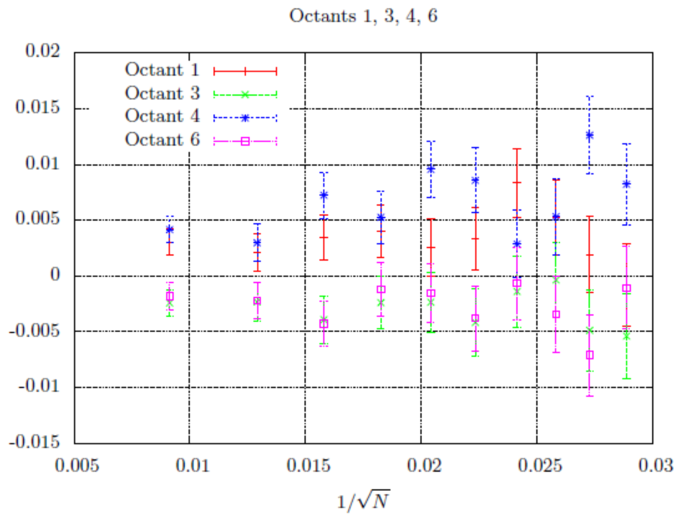
Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \le 0$
2	$p_x > 0, p_y \le 0, p_z > 0$
3	$p_x > 0, p_y \le 0, p_z \le 0$
4	$p_x \le 0, p_y > 0, p_z > 0$
5	$p_x \le 0, p_y > 0, p_z \le 0$
6	$p_x \le 0, p_y \le 0, p_z > 0$
7	$p_x \le 0, p_y \le 0, p_z \le 0$

$$\eta = \frac{\sum(\vec{p_3}, \vec{p_2}, \vec{p_1})}{\sum |(\vec{p_3}, \vec{p_2}, \vec{p_1})|}$$

Handedness separation

- Indication for small separation effect in some of the octants





Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \leq 0$
2	$p_x > 0, p_y \leq 0, p_z > 0$
3	$p_x > 0, p_y \leq 0, p_z \leq 0$
4	$p_x \leq 0, p_y > 0, p_z > 0$
5	$p_x \leq 0, p_y > 0, p_z \leq 0$
6	$p_x \leq 0, p_y \leq 0, p_z > 0$
7	$p_x \leq 0, p_y \leq 0, p_z \leq 0$

CONCLUSIONS

- Vortical structures (vortex sheets)
- Helicity separation effect
- Lambda polarization of % order predominantly in forward/backward regions
- Result is surprisingly similar to thermal vorticity calculation (Becattini, Csernai, Wang)

Spin-gravity interactions

- How to describe hadron spin/gravity(inertia) couplings?
- Matrix elements of Energy- Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

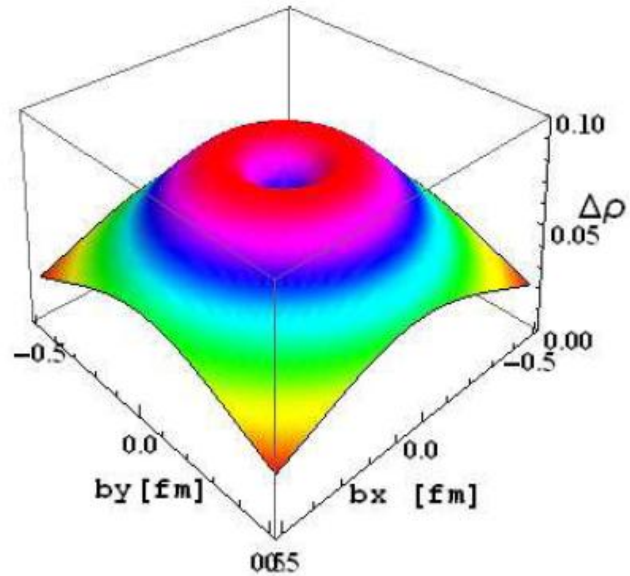


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu \qquad \sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 13 JANUARY 1992 NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction (~$\mathbf{P}^2$) is compensated by 1/ $\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

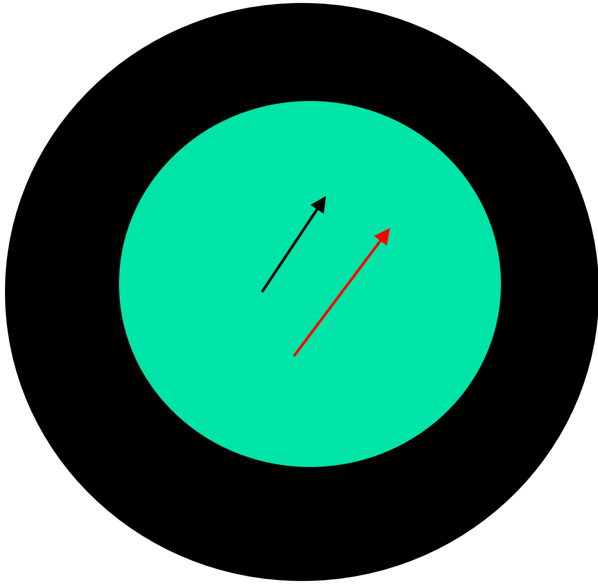
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Special role of g=2 for any spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Torsion – acts only on spin (violates EP)

Dirac eq+FW transformation-Obukhov,Silenko,OT, **arXiv:1410.6197**

- Hermitian Dirac Hamiltonian

$$e_i^{\widehat{0}} = V\,\delta_i^0, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$ds^2 = V^2c^2dt^2 - \delta_{\widehat{a}\widehat{b}}W^{\widehat{a}}{}_cW^{\widehat{b}}{}_d\,(dx^c - K^c cdt)\,(dx^d - K^d cdt)$$

$$\mathcal{H} = \beta mc^2V + q\Phi + \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right),$$

$$\mathcal{F}^b{}_a = VW^b{}_{\widehat{a}}, \qquad \Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\left(\Gamma_{\widehat{0}\widehat{b}\widehat{c}} + \Gamma_{\widehat{b}\widehat{c}\widehat{0}} + \Gamma_{\widehat{c}\widehat{0}\widehat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\hat{0}}\right)$$

$$\check{T}^{\alpha} = -\frac{1}{2}\,\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\boldsymbol{\Omega}^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'},\left\{\boldsymbol{p},\check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)},\left(\{\boldsymbol{p}^2,\check{\boldsymbol{T}}\} - \{\boldsymbol{p},(\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}$$

Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s}.$$

- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

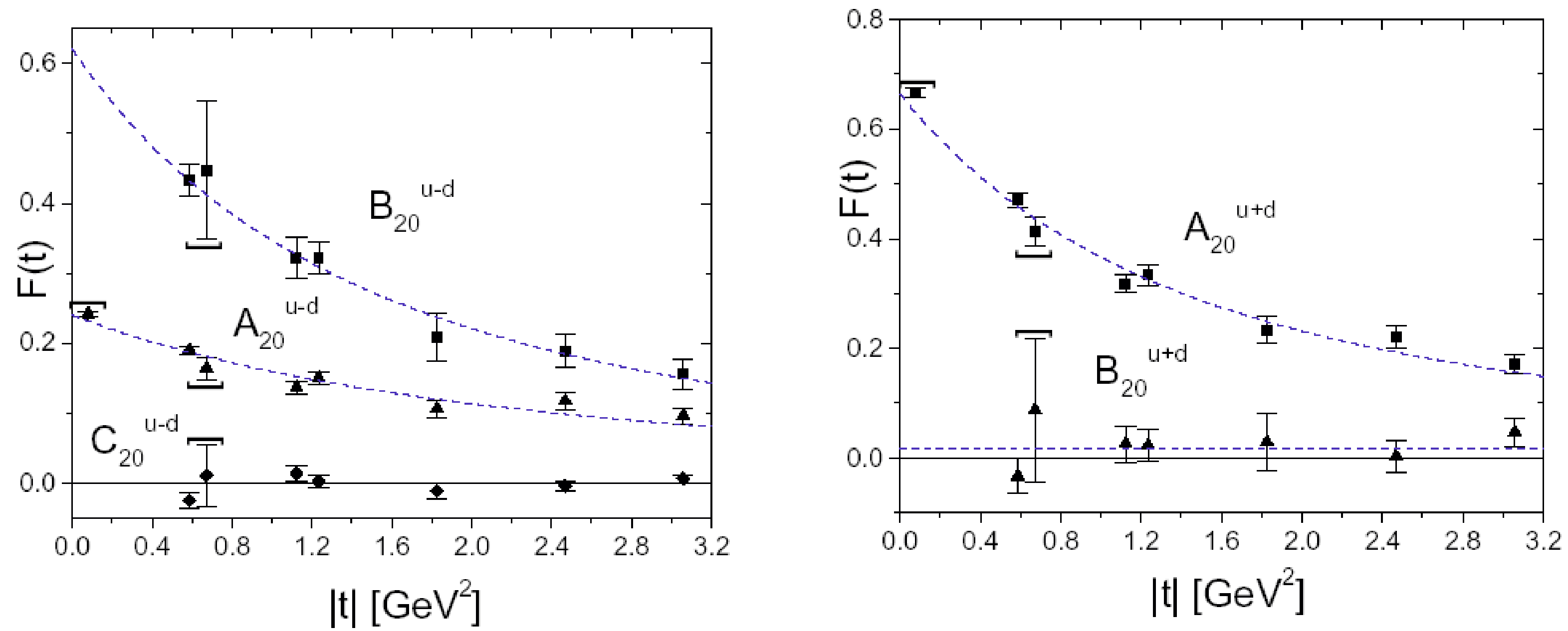
- New(based on Gemmel et al '10)

$$\frac{\hbar c}{2} |\check{\boldsymbol{T}}| \cdot |(1-\mathcal{G})\cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}.$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

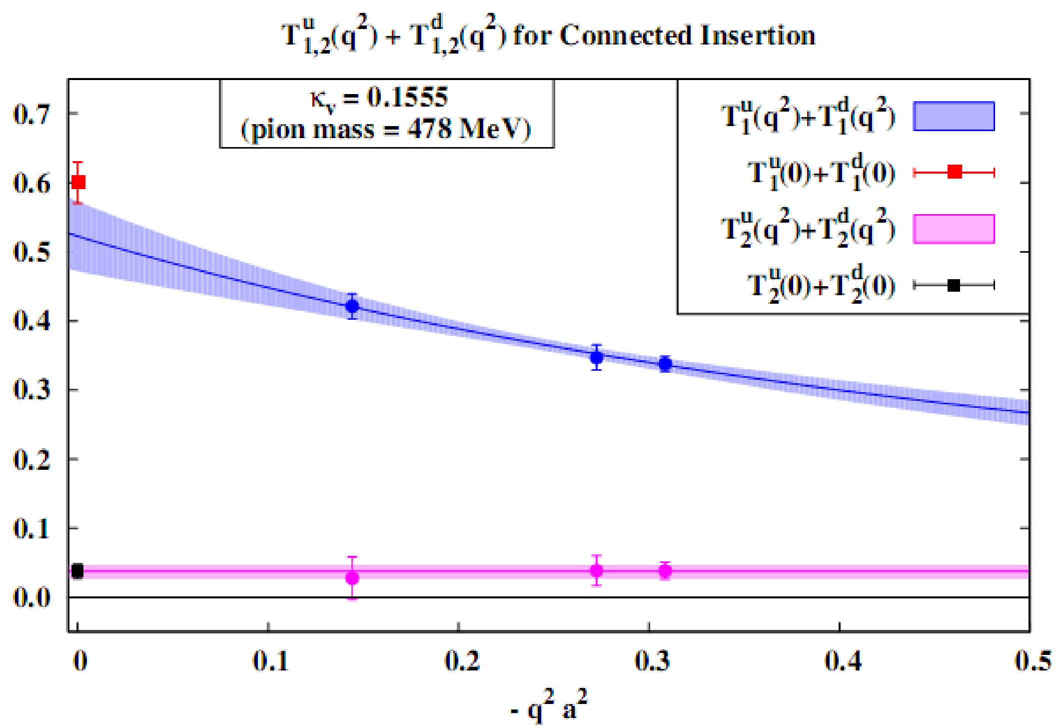
Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. **arXiv:1312.4816**)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!

Conclusionss

- Rotation in heavy-ion collisions – essentially non-inertial frame
- Related P-odd effects are not numerically large (smearing) but may be observable

- BACKUP SLIDES

Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji's sum rule ('96). Gravity counterpart – OT'99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT'14)

Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x,x) = 0$ (but relation of gluon Sivers to twist 3 still not founs – prediction!)
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

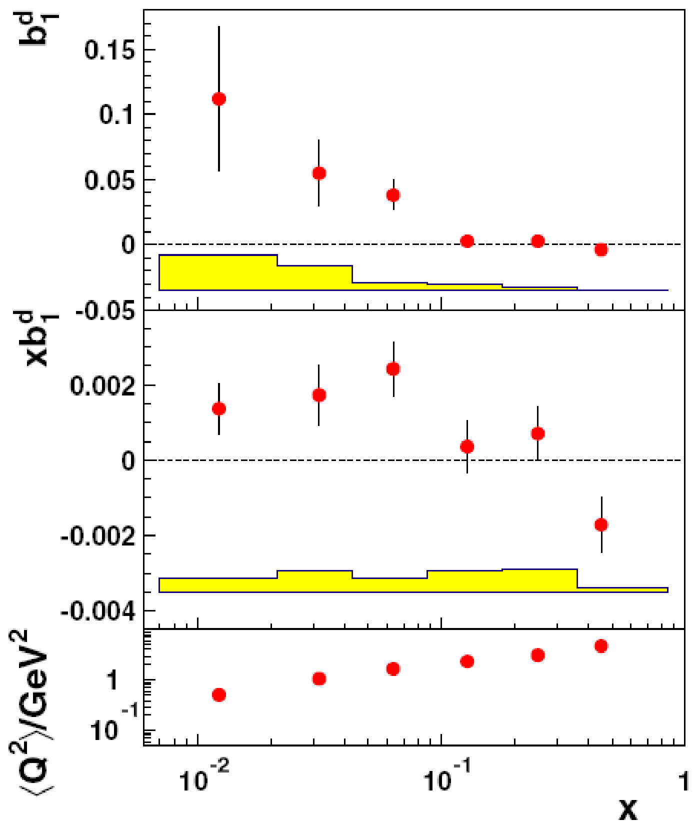
$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:

$$\int_0^1 C_i^T(x)dx = 0$$



Are more accurate data possible?

- HERMES – unlikely

- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle

CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR’s for deuteron tensor polarization- indirectly probe EP and its extension separately for quarks and gluons

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible