



# Thermal and non-thermal charmed meson production in heavy ion collisions at LHC



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*in collaboration with*

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Within such paradigm, a number of questions on heavy flavours arise:

- Are heavy quarks thermalized in quark-gluon plasma?
- What is the mass dependence of medium-induced quark energy loss?
- Are charmed hadrons ( $D$ ,  $J/\psi$ ) in a kinetic equilibrium with the medium?
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In this talk, I'll present some results of the phenomenological analysis of LHC data on  $p_T$ -spectra and elliptic flow of charmed hadrons ( $D$ ,  $J/\psi$ ) in PbPb collisions at  $\sqrt{s_{NN}}=2.76$  TeV in the frameworks of two-component HYDJET++ model. The comparison with RHIC results is also discussed.

# HYDJET and HYDJET++

## relativistic heavy ion event generators

**HYDJET (HYDroynamics + JETs)** - event generator to simulate heavy ion event as merging of two independent components (soft hydro-type part + hard multi-partonic state, the latter is based on **PYQUEN - PYthia QUENched**).

<http://cern.ch/lokhtin/hydro/hydjet.html>

*(latest version 1.9)*

Original paper: I.Lokhtin, A.Snigirev, Eur. Phys. J. C 46 (2006) 2011

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**HYDJET++ (HYDJET v.2.\*)** – continuation of HYDJET (identical hard component + improved soft component including full set of thermal resonance production).

<http://cern.ch/lokhtin/hydjet++>

*(latest version 2.2)*

Original paper: I.Lokhtin, L.Malinina, S.Petrushanko, A.Snigirev, I.Arsene, K.Tywoniuk, Comp. Phys. Comm. 180 (2009) 779

# HYDJET++ (soft component): physics frames

Soft (hydro) part of HYDJET++ is based on the adapted FAST MC model:

**Part I:** *N.S.Amelin, R.Lednisky, T.A.Pocheptsov, I.P.Lokhtin, L.V.Malinina, A.M.Snigirev, Yu.A.Karpenko, Yu.M.Sinyukov, Phys. Rev. C 74 (2006) 064901*

**Part II:** *N.S.Amelin, R.Lednisky, I.P.Lokhtin, L.V.Malinina, A.M.Snigirev, Yu.A.Karpenko, Yu.M.Sinyukov, I.C.Arsene, L.Bravina, Phys. Rev. C 77 (2008) 014903*

- ✓ **fast** HYDJET-inspired MC procedure for soft hadron generation
- ✓ multiplicities are determined assuming **thermal equilibrium**
- ✓ hadrons are produced on the hypersurface represented by **a parameterization** of relativistic hydrodynamics with given **freeze-out conditions**
- ✓ **chemical and kinetic freeze-outs** are separated
- ✓ decays of **hadronic resonances** are taken into account (360 particles from SHARE data table) with “home-made” decayer
- ✓ written within **ROOT** framework (C++)
- ✓ contains **16 free parameters** (but this number may be reduced to 9)

# HYDJET++ (soft): input parameters

- 1-5. Thermodynamic parameters at chemical freeze-out:  $T^{\text{ch}}$ ,  $\{\mu_B, \mu_S, \mu_C, \mu_Q\}$  (option to calculate  $T^{\text{ch}}$ ,  $\mu_B$  and  $\mu_S$  using phenomenological parameterization  $\mu_B(\sqrt{s})$ ,  $T^{\text{ch}}(\mu_B)$  is foreseen).
- 6-7. Strangeness suppression factor  $\gamma_S \leq 1$  and charm enhancement factor  $\gamma_C \geq 1$  (options to use phenomenological parameterization  $\gamma_S(T^{\text{ch}}, \mu_B)$  and to calculate  $\gamma_C$  are foreseen).
- 8-9. Thermodynamical parameters at thermal freeze-out:  $T^{\text{th}}$ , and  $\mu_\pi$  - effective chemical potential of positively charged pions.
- 10-12. Volume parameters at thermal freeze-out: proper time  $\tau_f$ , its standard deviation (emission duration)  $\Delta\tau_f$ , maximal transverse radius  $R_f$ .
13. Maximal transverse flow rapidity at thermal freeze-out  $\rho_u^{\text{max}}$ .
14. Maximal longitudinal flow rapidity at thermal freeze-out  $\eta^{\text{max}}$ .
15. Flow anisotropy parameter:  $\delta(\mathbf{b}) \rightarrow u^\mu = u^\mu(\delta(\mathbf{b}), \varphi)$
16. Coordinate anisotropy:  $\varepsilon(\mathbf{b}) \rightarrow R_f(\mathbf{b}) = R_f(0) [V_{\text{eff}}(\varepsilon(0), \delta(0)) / V_{\text{eff}}(\varepsilon(\mathbf{b}), \delta(\mathbf{b}))]^{1/2} [N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0)]^{1/3}$

**For impact parameter range bmin-bmax:**

$$V_{\text{eff}}(\mathbf{b}) = V_{\text{eff}}(0) N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0), \quad \tau_f(\mathbf{b}) = \tau_f(0) [N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0)]^{1/3}$$



# HYDJET++ (hard component): PYQUEN (PYthia QUENched)

Initial parton configuration

PYTHIA6.4 w/o hadronization: `mstp(111)=0`



Parton rescattering & energy loss (collisional, radiative) + emitted g

PYQUEN rearranges partons to update ns strings



Parton hadronization and final particle formation

PYTHIA6.4 with hadronization: call PYEXEC

Three model parameters: initial maximal QGP temperature  $T_0$ , QGP formation time

$\tau_0$  and number of active quark flavors in QGP  $N_f$

(+ minimal  $p_T$  of hard process  $P_{T\min}$  to specify the number of hard NN collisions)

*I.P.Lokhtin, A.M.Snigirev, Eur. Phys. J. 45 (2006) 211 (latest version 1.5.1)*

## 1) Thermal charm production in HYDJET++ (soft component)

Thermal charmed hadrons  $J/\psi$ ,  $D^0$ ,  $\bar{D}^0$ ,  $D^+$ ,  $D^-$ ,  $D_s^+$ ,  $D_s^-$ ,  $\Lambda_c^+$ ,  $\Lambda_c^-$  are generated within the statistical hadronization model

*(A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel,*

*Phys.Lett. B 571 (2003) 36; Nucl. Phys. A 789 (2007) 334)*

$$N_D = \gamma_c N_D^{\text{th}} (I_1(\gamma_c N_D^{\text{th}}) / I_0(\gamma_c N_D^{\text{th}})), \quad N_{J/\psi} = \gamma_c^2 N_{J/\psi}^{\text{th}}$$

$\gamma_c$  - charm enhancement factor may be obtained from the equation:

$$N_{cc} = 0.5 \gamma_c N_D^{\text{th}} (I_1(\gamma_c N_D^{\text{th}}) / I_0(\gamma_c N_D^{\text{th}})) + \gamma_c^2 N_{J/\psi}^{\text{th}}$$

where number of c-quark pairs  $N_{cc}$  is calculated with PYTHIA

(the factor  $K \sim 2$  is applied to take into account NLO pQCD corrections)

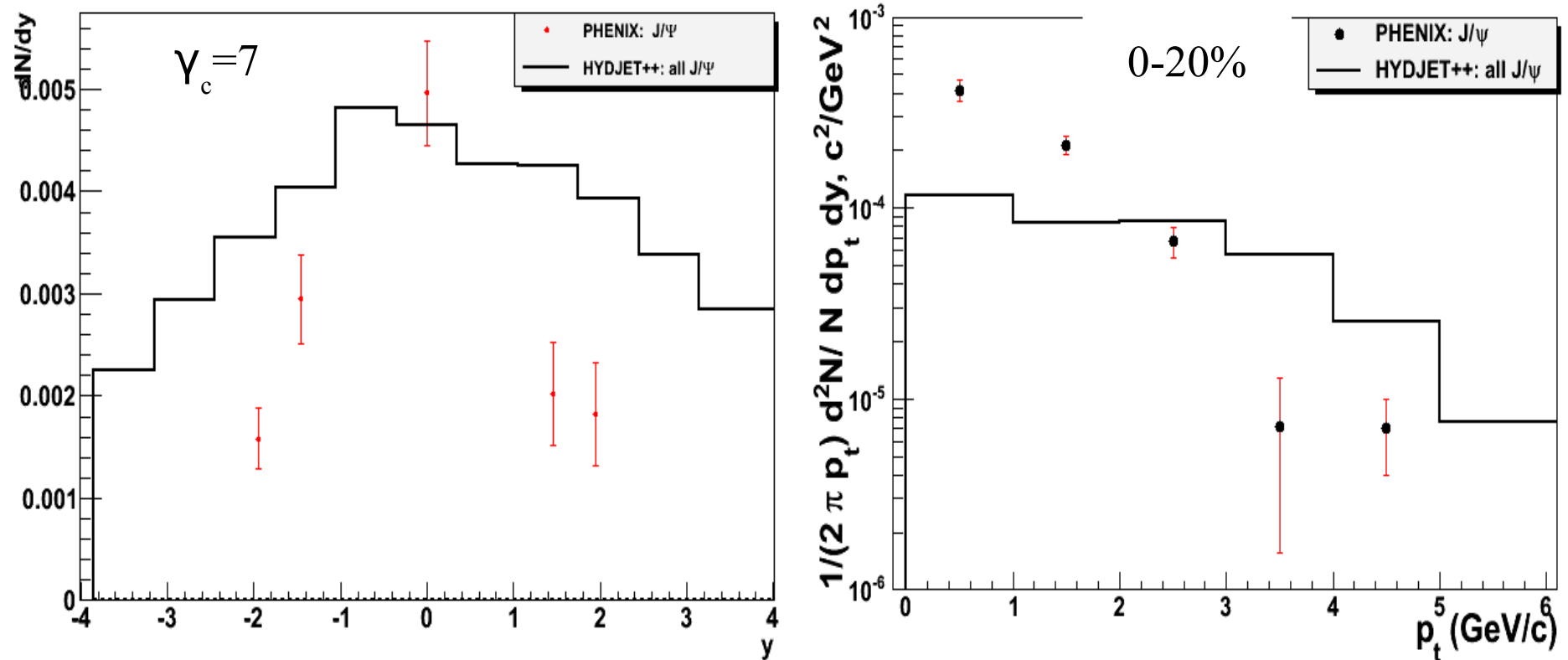
and multiplied by the number of NN sub-collisions for given centrality

## 2) Non-thermal charm production in HYDJET++ (hard component)

Non-thermal charmed hadrons are generated within PYTHIA/PYQUEN taking into account medium-induced rescattering and radiative and collisional energy loss of heavy quarks (b, c)

# Charmed mesons at RHIC ( $J/\psi$ )

I.P. Lokhtin et al., J.Phys.Conf.Ser. 270 (2011) 012060

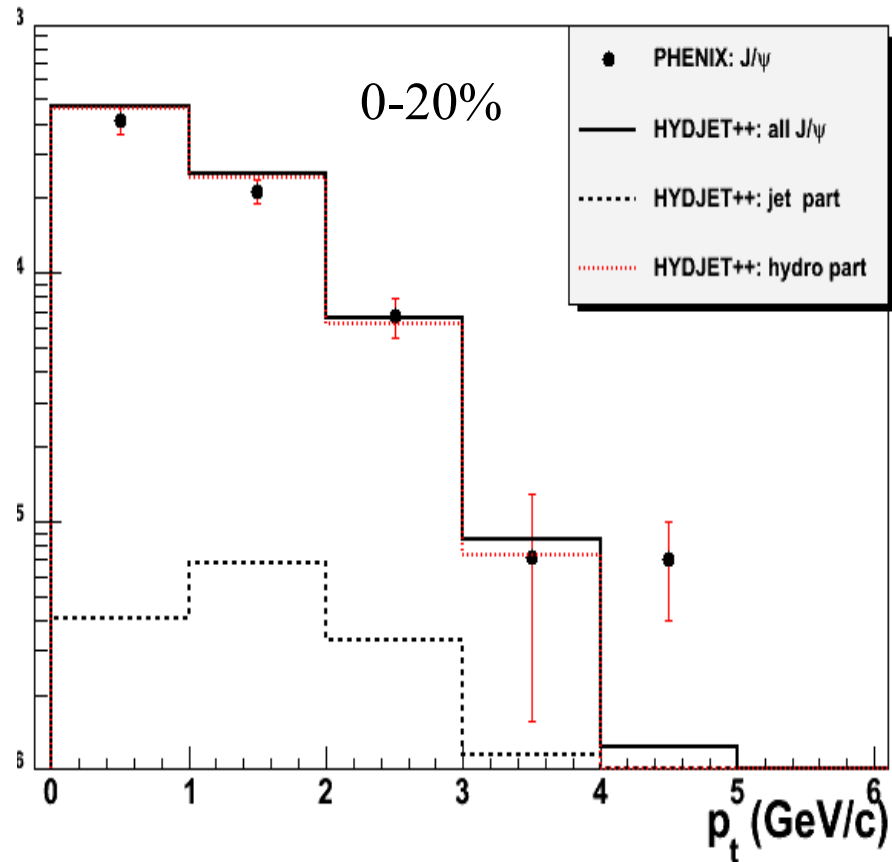
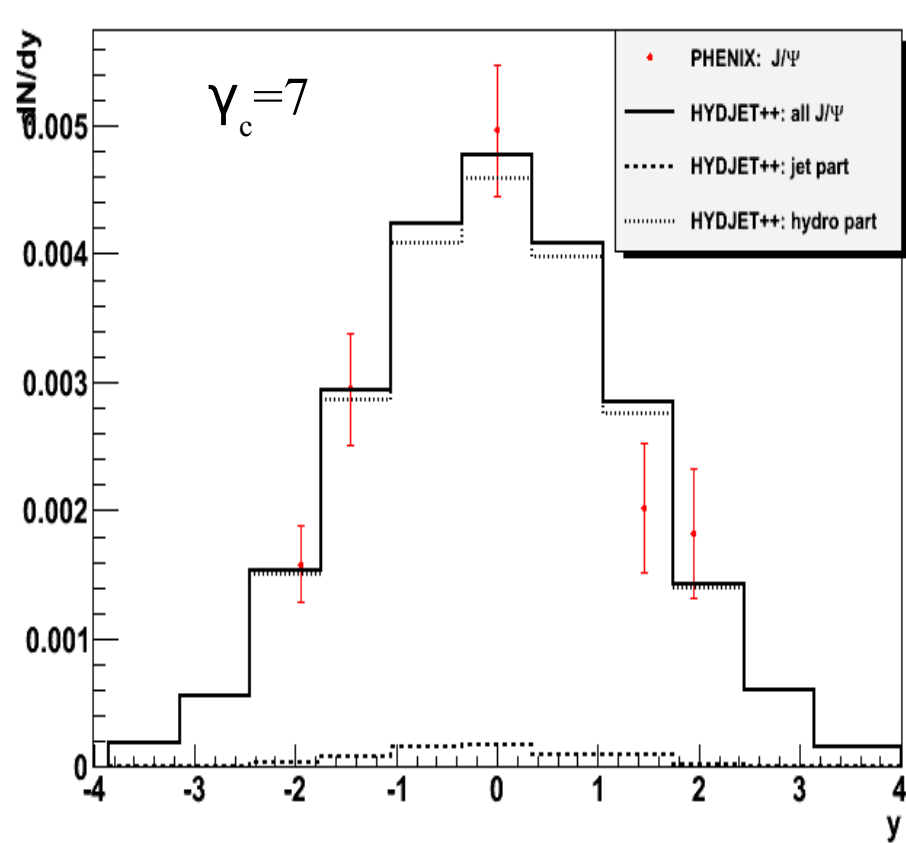


Points: PHENIX data *PRL* 98 (2007) 232301); histograms: HYDJET++

If thermal freeze-out for  $J/\psi$  happens at the same temperature as for inclusive hadrons,  $T_{th} = 100$  MeV ( $\eta^{max} = 3.3$ ,  $\rho_u^{max} = 1.1$ ) then simulated spectra are much wider than the data

# Charmed mesons at RHIC ( $J/\psi$ )

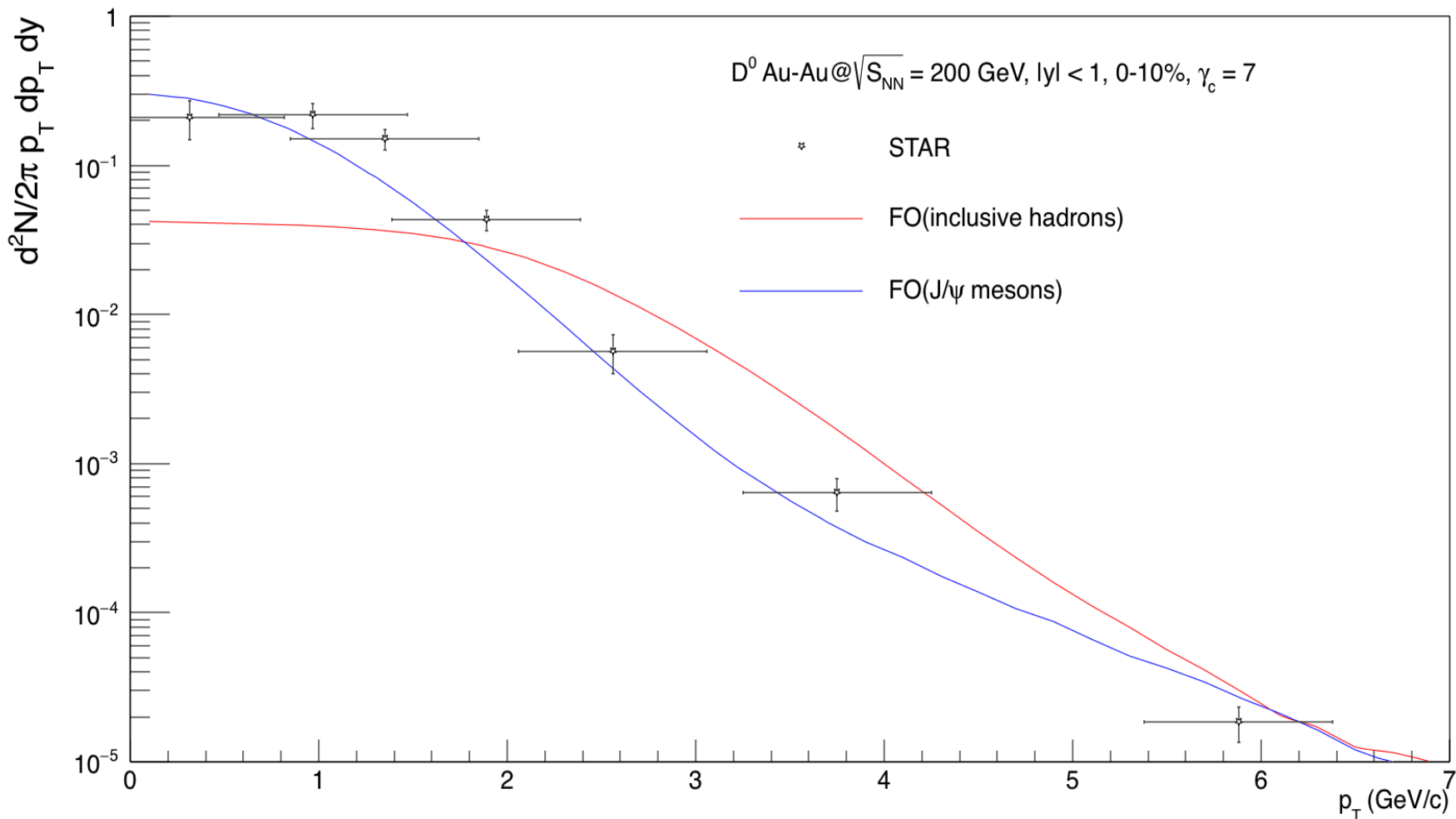
I.P. Lokhtin et al., J.Phys.Conf.Ser. 270 (2011) 012060



Points: PHENIX data *PRL* 98 (2007) 232301); histograms: HYDJET++

But if thermal freeze-out for  $J/\psi$  happens at the same temperature as chemical freeze-out,  $T_{th}(J/\psi) = T_{ch} = 165$  MeV ( $\eta^{max} = 1.1$ ,  $\rho_u^{max} = 0.5$ ), then simulated spectra match the data

# Charmed mesons at RHIC (D)



Points: STAR data *PRL 113 (2014) 142301*; histograms: HYDJET++

Simulated  $p_T$ -spectrum match the data if freeze-out parameters for D are the same as for J/ $\psi$ :  $T_{th} = T_{ch} = 165$  MeV ( $\eta^{\max} = 1.1$ ,  $\rho_u^{\max} = 0.5$ )

# Charmed mesons at RHIC (summary)

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- Momentum spectra of D and  $J/\psi$  mesons in most central AuAu collisions may be reproduced by two-component model including thermal (soft) and non-thermal (hard) components with the same freeze-out parameters

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# Charmed mesons at RHIC (summary)

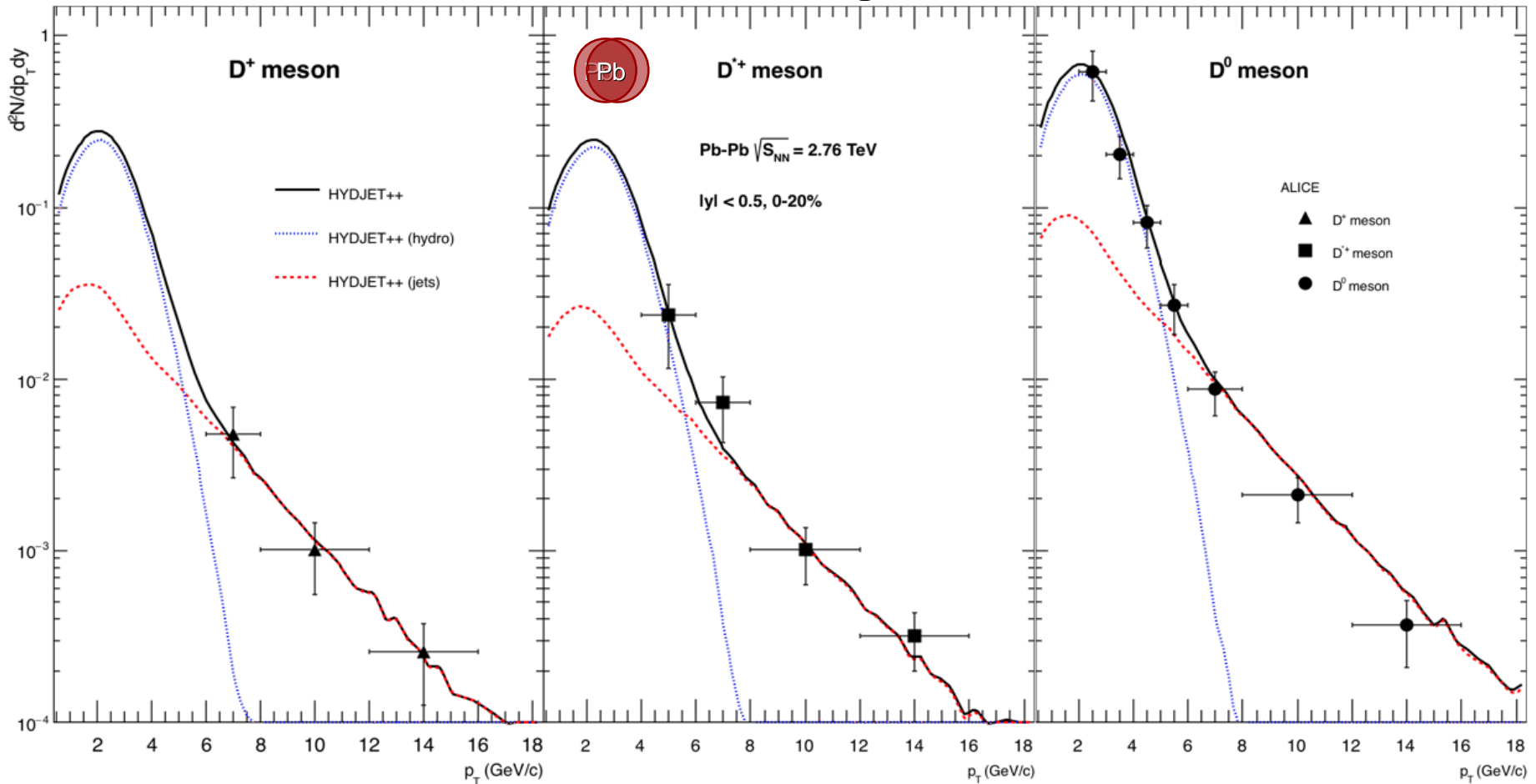
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What about charmed mesons at the LHC?

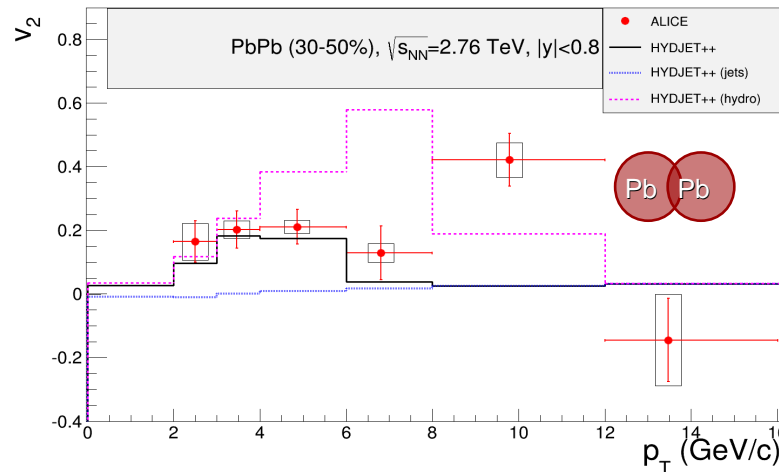
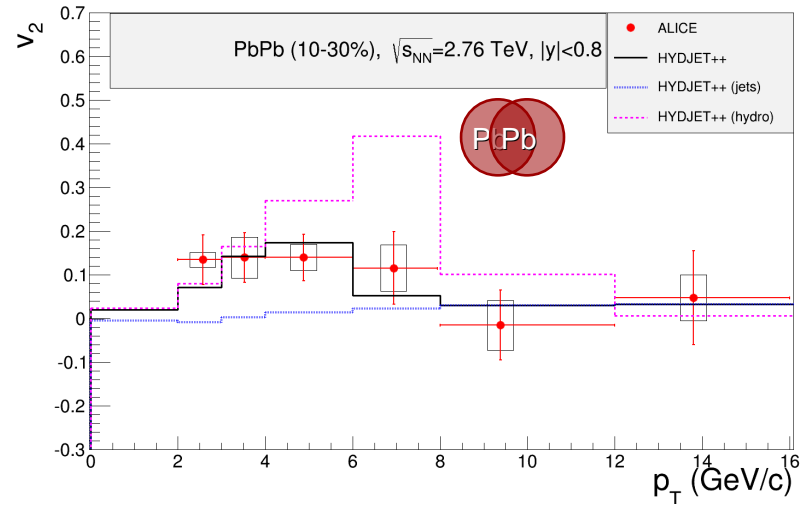
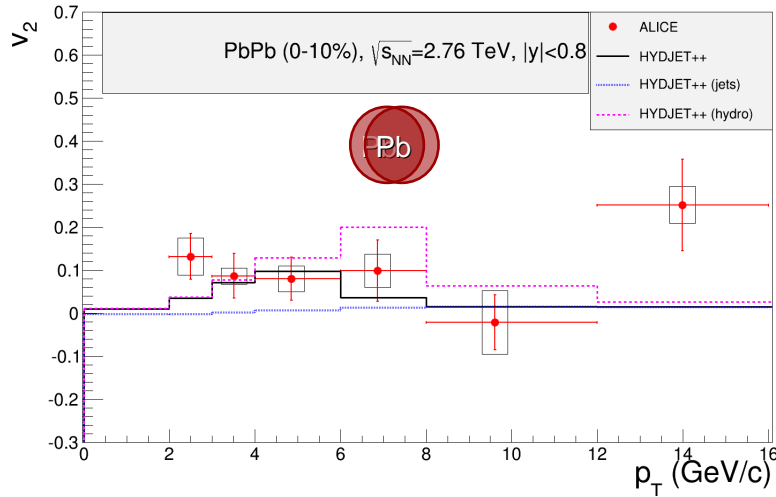
# D mesons at LHC ( $p_T$ -spectrum)



Points: ALICE data (*JHEP 1209 (2012) 112*); histograms: HYDJET++  
 ( $T_{th} = 105$  MeV,  $T_{ch} = 165$  MeV,  $\eta^{max} = 4.5$ ,  $\rho_u^{max} = 1.265$ ,  $v_c = 11.5$ ,  $P_{Tmin} = 8.2$  GeV/c)

HYDJET++ reproduces  $p_T$ -spectrum of D-mesons with the *same freeze-out parameters* as for inclusive hadrons  $\Rightarrow$  significant part of D-mesons (*thermal component*) is in the kinetic equilibrium with the medium; *non-thermal component* is important at high  $p_T$  19

# D mesons at LHC (elliptic flow $v_2 = \langle \cos(2\varphi - \psi_R) \rangle$ )

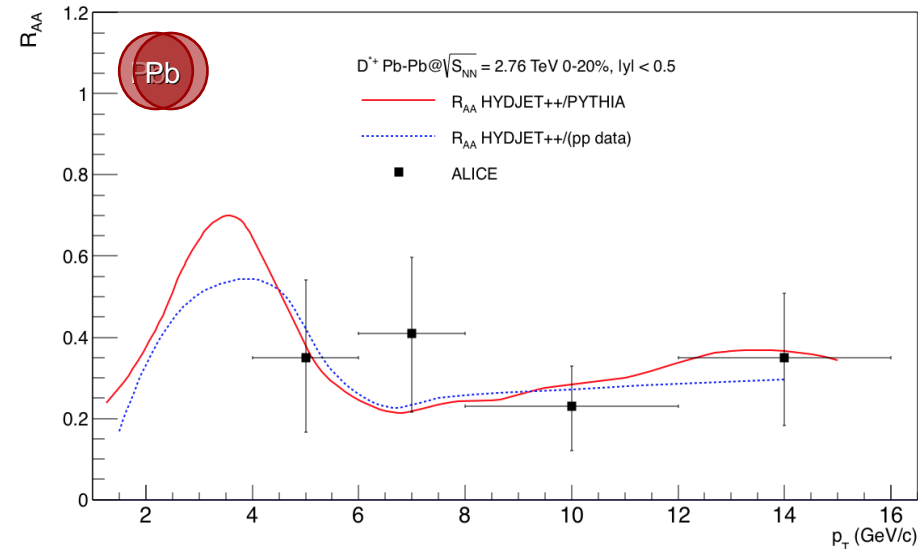
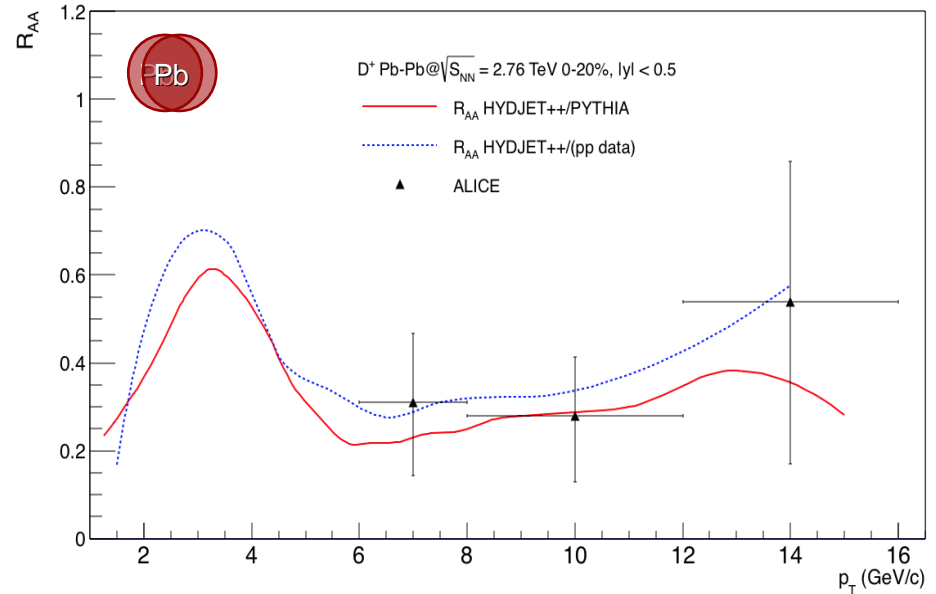
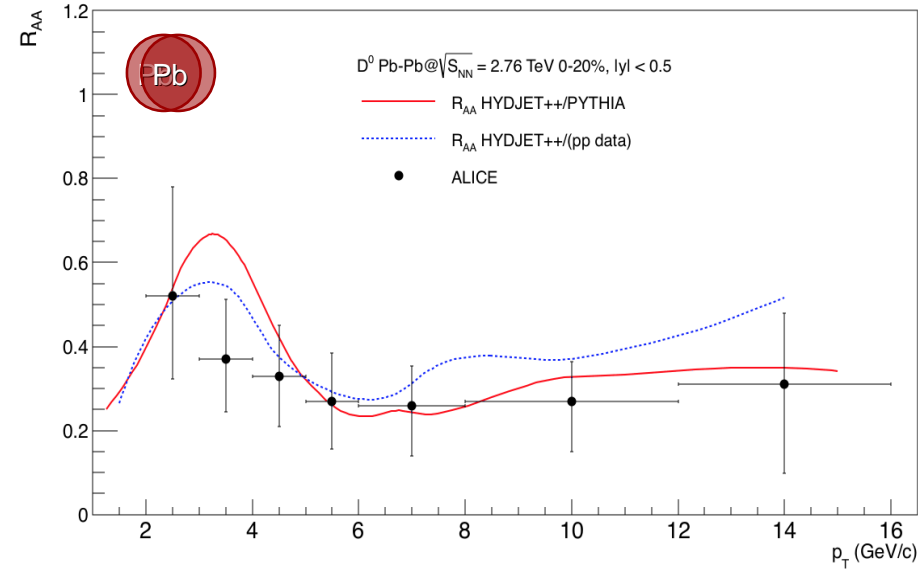


Points: ALICE data (PRC 90 (2014) 034904); histograms: HYDJET++

HYDJET++ reproduces  $v_2(p_T)$  of D-mesons with the *same freeze-out parameters* as for inclusive hadrons  $\Rightarrow$  significant part of D-mesons (*thermal component*) is in the kinetic equilibrium with the medium; *non-thermal component* is important at high  $p_T$

# D mesons at LHC (nuclear modification factor $R_{AA}$ )

$$R_{AA} = \frac{\sigma_{pp}^{inel}}{\langle N_{coll} \rangle} \frac{d^2 N_{AA} / dp_T d\eta}{d^2 \sigma_{pp} / dp_T d\eta} \sim \frac{\text{“QCD Medium”}}{\text{“QCD Vacuum”}} \left\{ \begin{array}{l} R_{AA} > 1: \text{enhancement} \\ R_{AA} = 1: \text{no medium effect} \\ R_{AA} < 1: \text{suppression} \end{array} \right.$$

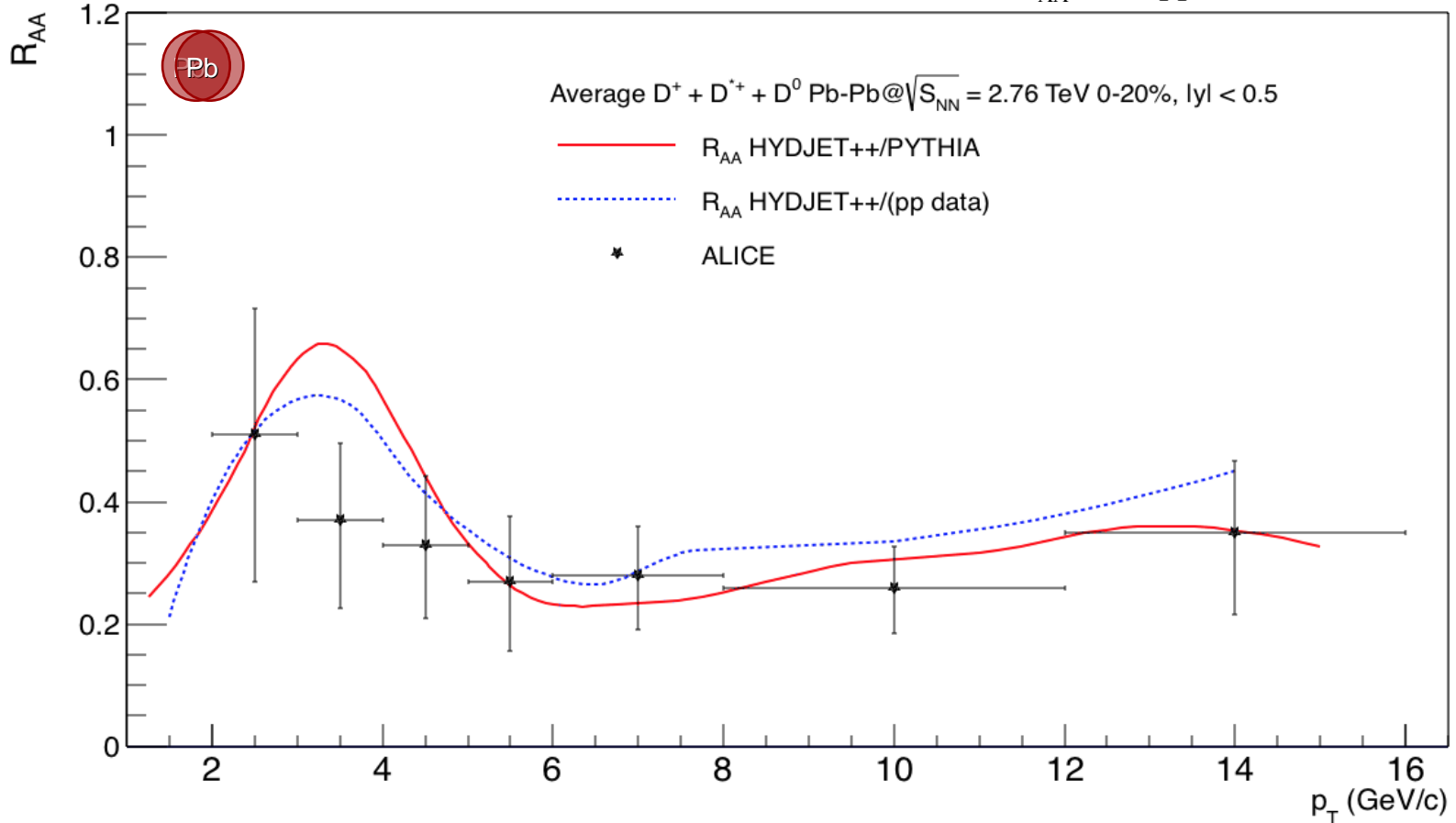


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HYDJET++ reproduces  $R_{AA}(p_T)$  of D-mesons up to very high  $p_T \Rightarrow$  treatment of heavy quark energy loss in hard component of HYDJET++ (PYQUEN) seems quite successful

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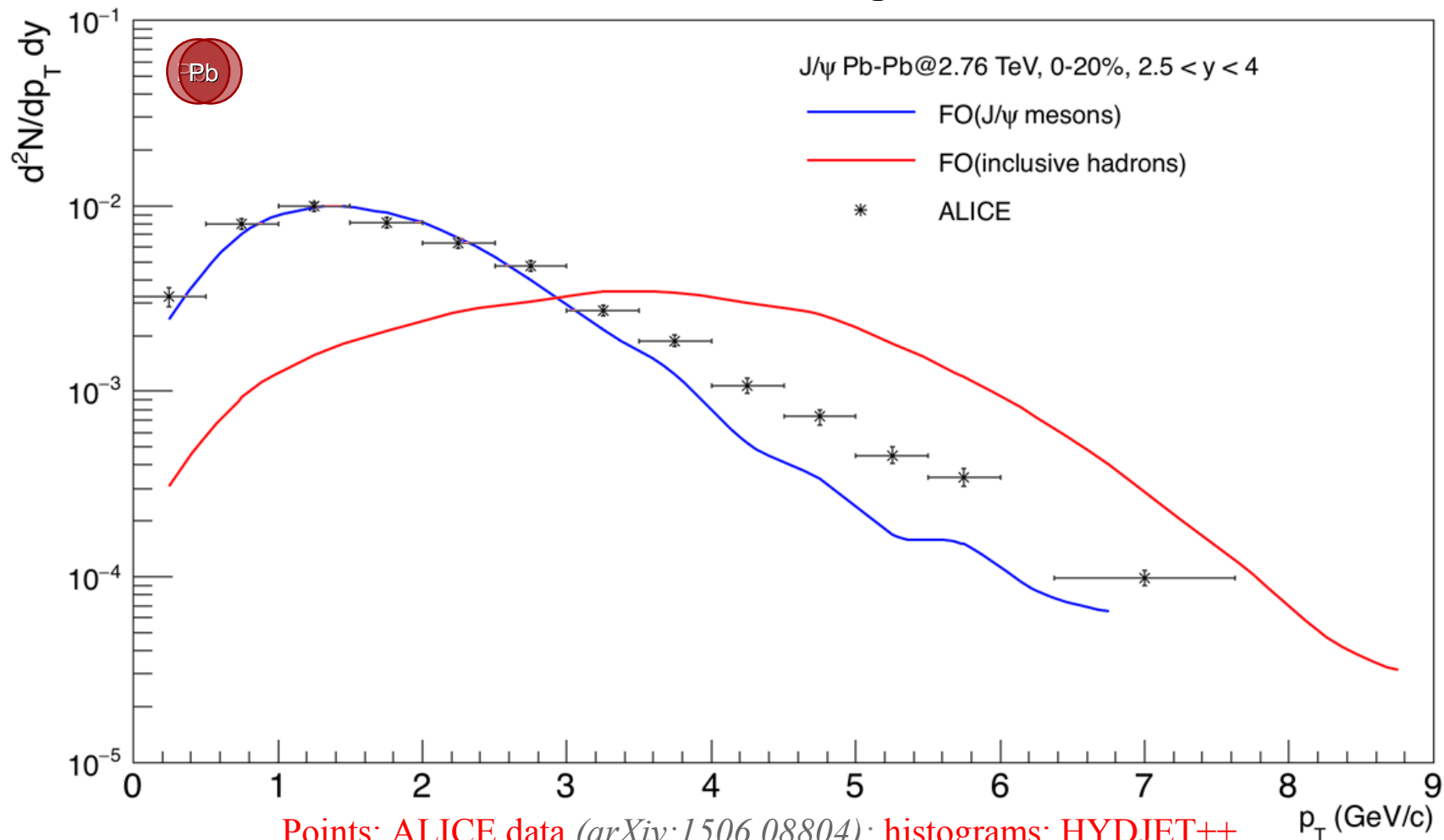
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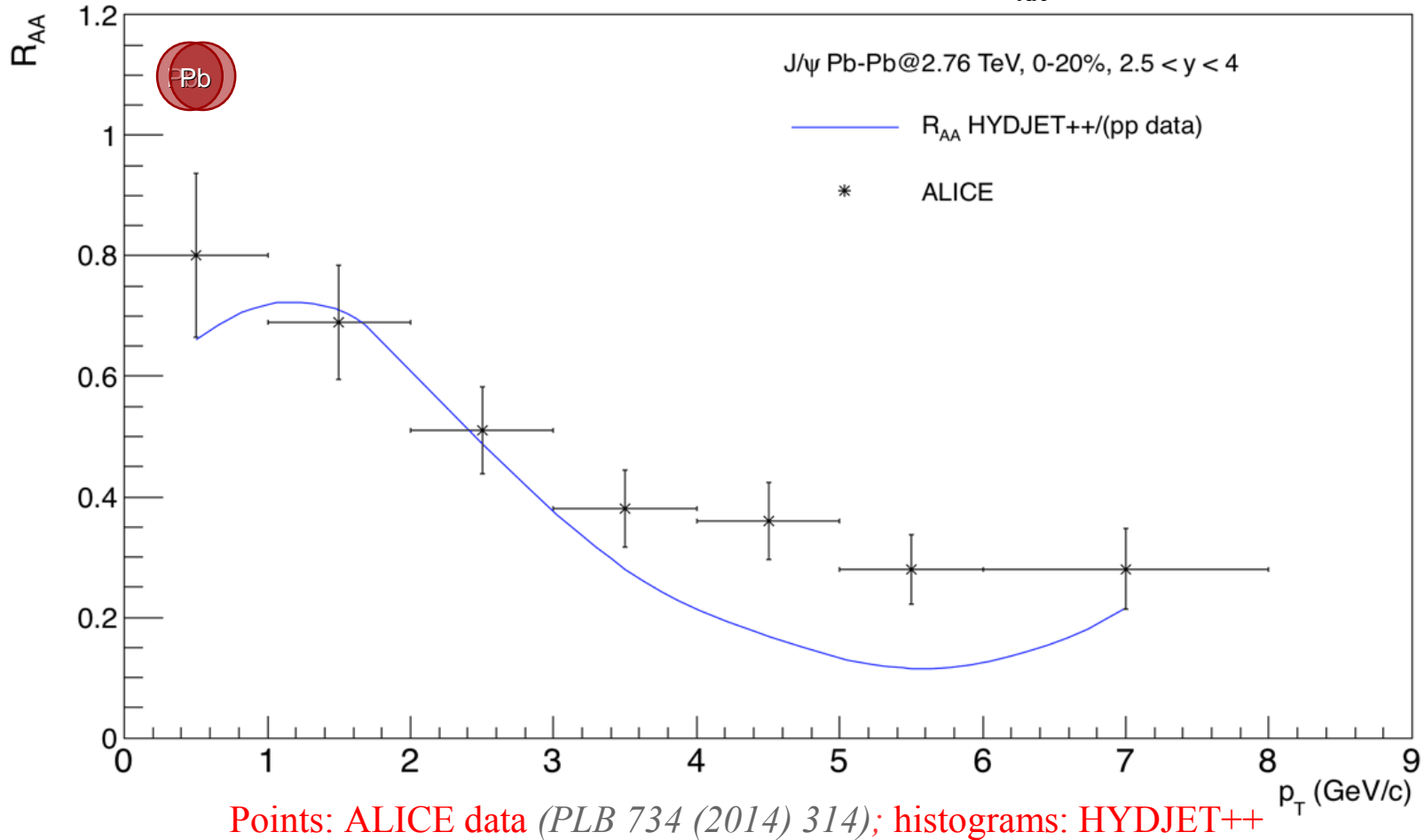
# J/ψ mesons at LHC (p<sub>T</sub>-spectrum)



HYDJET++ reproduces  $J/\psi$ -meson  $p_T$ -spectrum (up to  $\sim 3$  GeV/c) with the *freeze-out parameters different* from ones for inclusive hadrons  $\Rightarrow$  kinetic freeze-out of  $J/\psi$  thermal component occurs before freeze-out of light hadrons; non-thermal component is important at intermediate & high  $p_T$

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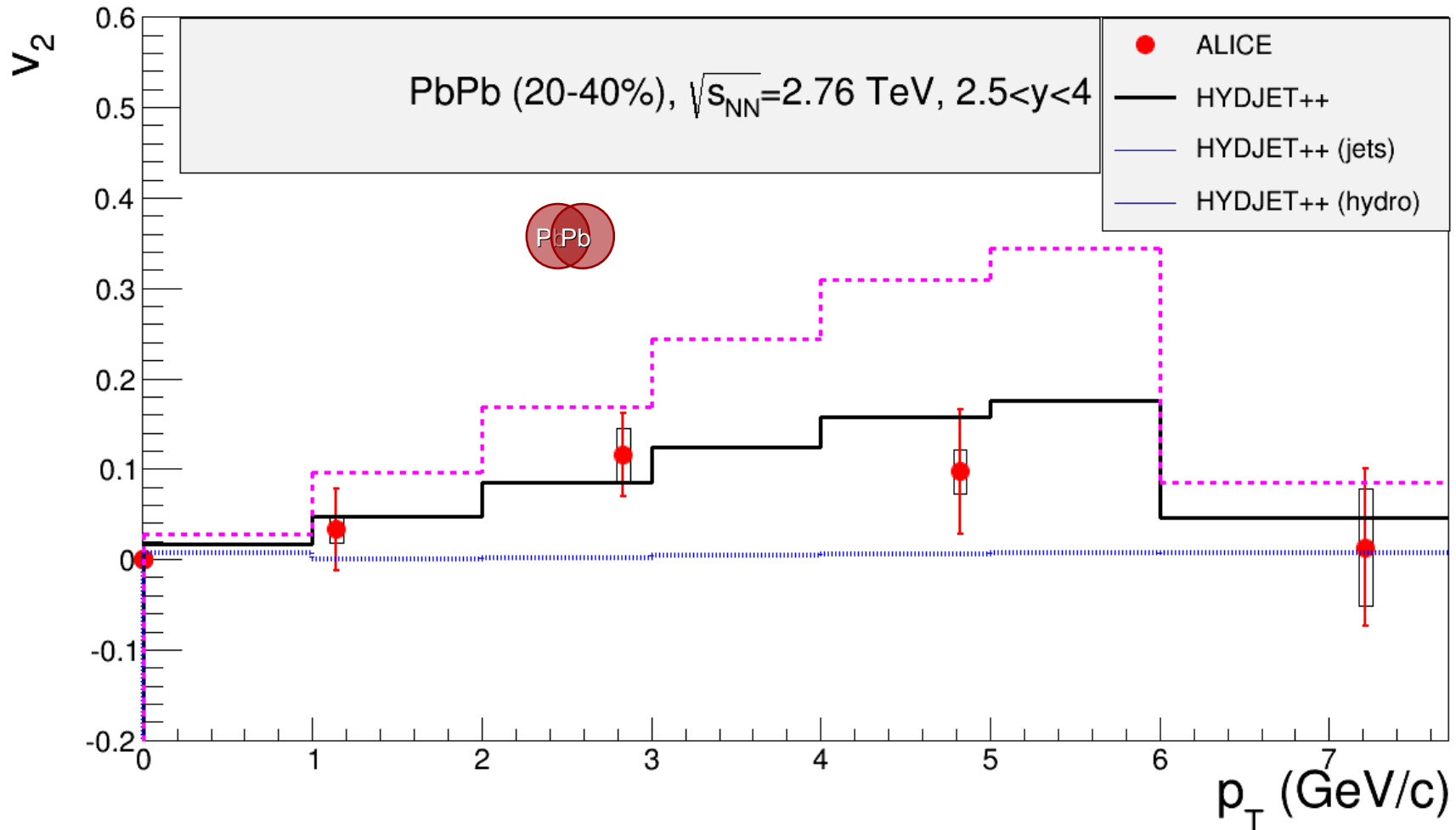
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Superposition of *thermal* and *non-thermal* components in HYDJET++ allows us qualitatively to reproduce **momentum dependence of J/ψ suppression factor** in PbPb collisions at the LHC (but PYTHIA@HYDJET++ tuning is needed for adequate J/ψ modeling at high  $p_T$ )<sup>24</sup>



# J/ψ mesons at LHC (elliptic flow $v_2 = \langle \cos(2\varphi - \psi_R) \rangle$ )



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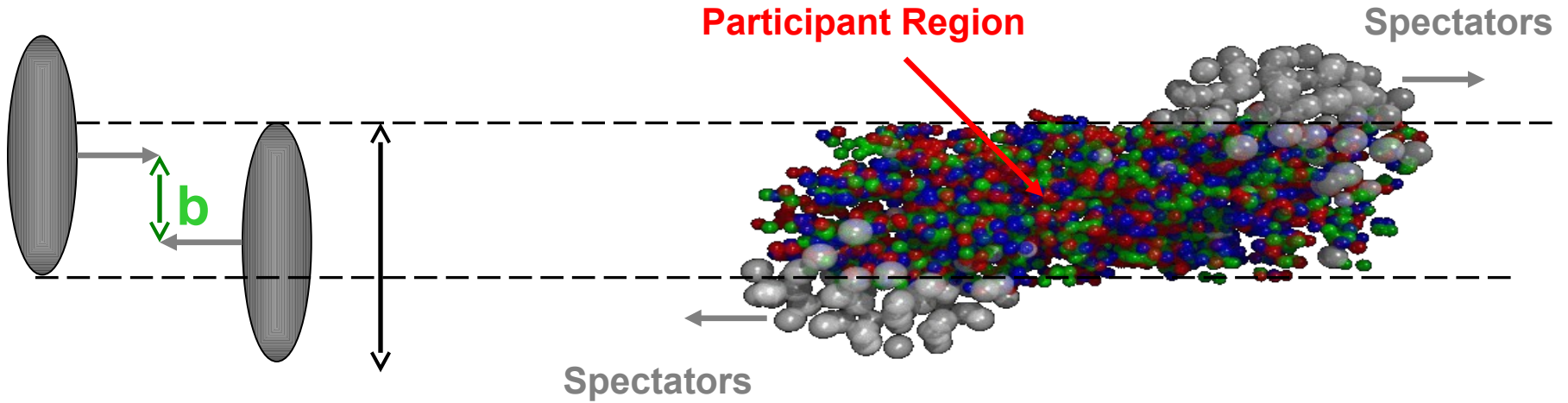
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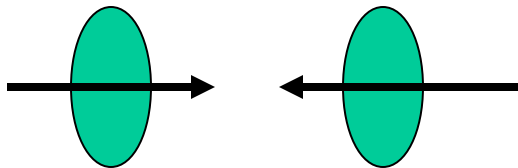
High degree of c-quark thermalization in quark-gluon plasma is achieved in PbPb collisions at the LHC (?)

# BACKUP SLIDES

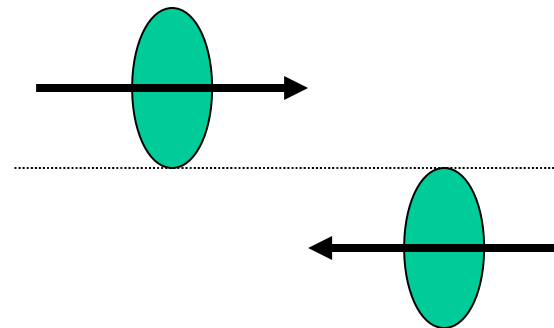
# Centrality of nucleus-nucleus interactions



central collision

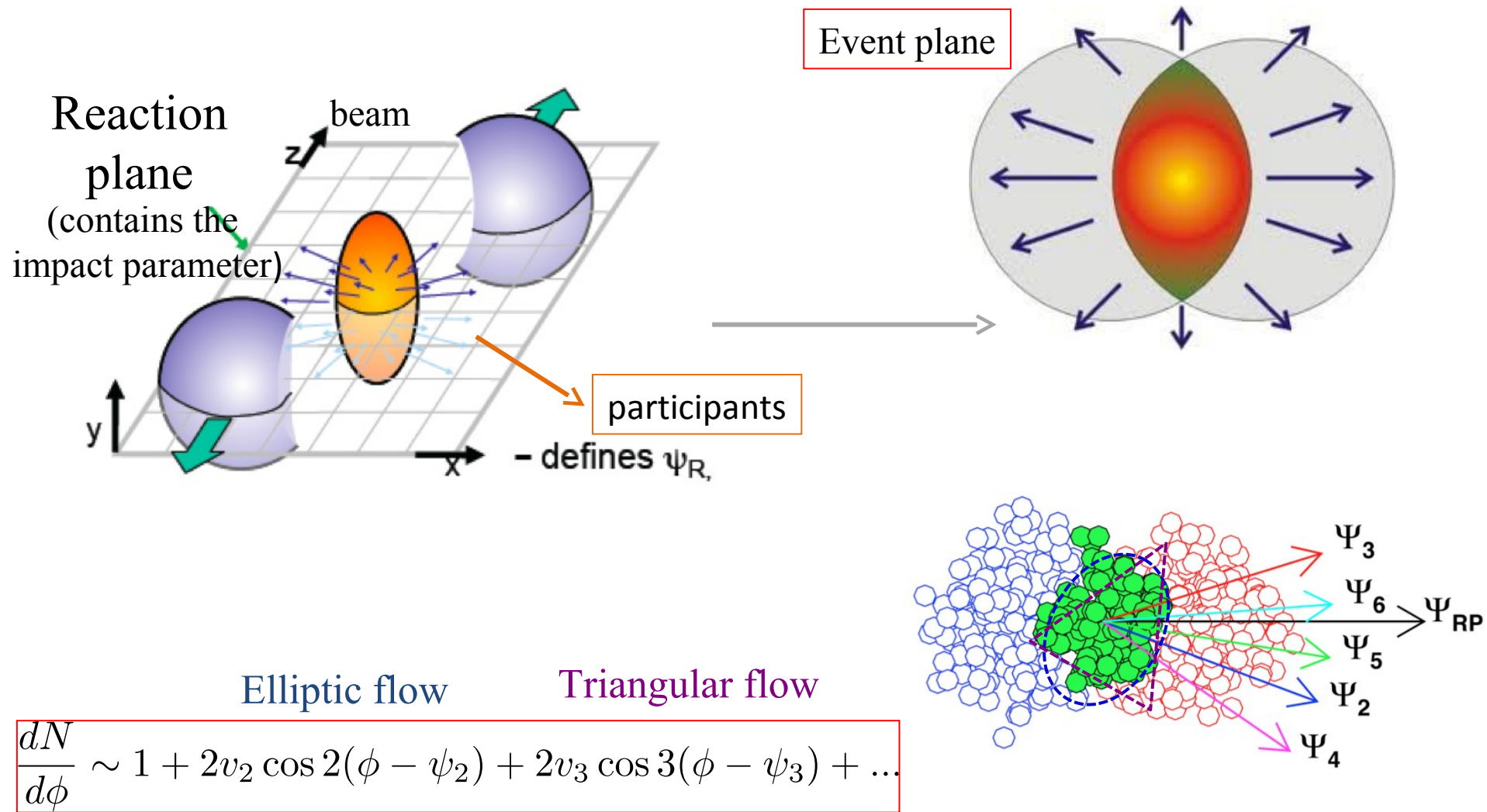


peripheral collision





# Azimuthal correlations and flow



# Monte-Carlo simulation of hard component (including nuclear shadowing) in HYDJET++

- Calculating the number of hard NN sub-collisions  $N_{jet}(b, P_{tmin}, \sqrt{s})$  with  $P_t > P_{tmin}$  around its mean value according to the binomial distribution.
- Selecting the type (for each of  $N_{jet}$ ) of hard NN sub-collisions ( $pp$ ,  $np$  or  $nn$ ) depending on number of protons ( $Z$ ) and neutrons ( $A-Z$ ) in nucleus  $A$  according to the formula:  $Z = A / (1.98 + 0.015A^{2/3})$ .
- Generating the hard component by calling PYQUEN  $n_{jet}$  times.
- Correcting the PDF in nucleus by the accepting/rejecting procedure for each of  $N_{jet}$  hard NN sub-collisions: comparison of random number generated uniformly in the interval  $[0, 1]$  with shadowing factor  $S(r1, r2, x1, x2, Q2) \leq 1$  taken from the adapted impact parameter dependent parameterization based on Glauber-Gribov theory (*K. Tywoniuk et al., Phys. Lett. B 657 (2007) 170*).

# HYDJET++ (soft): main physics assumptions

A hydrodynamic expansion of the fireball is supposed **ends by a sudden system breakup** at given  $T$  and chemical potentials. Momentum distribution of produced hadrons keeps the thermal character of the equilibrium distribution.

**Cooper-Frye formula:** 
$$p^0 \frac{d^3 N_i}{d^3 p} = \int_{\sigma(x)} d^3 \sigma_\mu(x) p^\mu f_i^{eq}(p^\nu u_\mu(x); T, \mu_i)$$

- HYDJET++ avoids straightforward 6-dimensional integration by using the special simulation procedure (like HYDJET): momentum generation in the rest frame of fluid element, then Lorentz transformation in the global frame  $\rightarrow$  uniform weights  $\rightarrow$  effective von-Neumann rejection-acceptance procedure.

## Freeze-out surface parameterizations

1. The Bjorken model with hypersurface

$$\tau = (t^2 - z^2)^{1/2} = \text{const}$$

2. Linear transverse flow rapidity profile

$$\rho_u = \frac{r}{R} \rho_u^{\max}$$

3. The total effective volume for particle production at

$$- V_{\text{eff}} = \int_{\sigma(x)} d^3 \sigma_\mu(x) u^\mu(x) = \tau \int_0^R \gamma_r r dr \int_0^{2\pi} d\phi \int_{\eta_{\min}}^{\eta_{\max}} d\eta = 2\pi\tau\Delta\eta \left( \frac{R}{\rho_u^{\max}} \right)^2 (\rho_u^{\max} \sinh \rho_u^{\max} - \cosh \rho_u^{\max} + 1)$$

# HYDJET++ (soft): hadron multiplicities

1. The hadronic matter created in heavy-ion collisions is considered as a hydrodynamically expanding fireball with EOS of an ideal hadron gas.

2. “Concept of effective volume”  $T=\text{const}$  and  $\mu=\text{const}$ : the total yield of particle species is  $N_i = \rho_i(T, \mu_i)V_{eff}$  .

3. Chemical freeze-out :  $T, \mu_i = \mu_B B_i + \mu_S S_i + \mu_c C_i + \mu_Q Q_i$ ;  $T, \mu_B$  –can be fixed by particle ratios, or by phenomenological formulas

$$T(\mu_B) = a - b\mu_B - c\mu_B^4; \mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

4. Chemical freeze-out: all macroscopic characteristics of particle system are determined via a set of equilibrium distribution functions in the fluid element rest frame:

$$f_i^{eq}(p^{0*}; T, \mu_i) = \frac{1}{(2\pi)^3} \frac{g_i}{\exp([p^{0*} - \mu_i]/T) \pm 1}$$

$$\rho_i^{eq}(T, \mu_i) = \int_0^\infty d^3 \vec{p}^* f_i^{eq}(p^{0*}; T(x^*), \mu(x^*)_i) = 4\pi \int_0^\infty dp^* p^{*2} f_i^{eq}(p^{0*}; T, \mu_i)$$

# HYDJET++ (soft): thermal and chemical freeze-outs

1. The particle densities at the **chemical freeze-out** stage are too high to consider particles as free streaming and to associate this stage with the **thermal freeze-out**
2. Within the **concept of chemically frozen evolution**, assumption of the conservation of the particle number ratios from the chemical to thermal freeze-out :

$$\frac{\rho_i^{eq}(T^{ch}, \mu_i^{ch})}{\rho_\pi^{eq}(T^{ch}, \mu_\pi^{ch})} = \frac{\rho_i^{eq}(T^{th}, \mu_i^{th})}{\rho_\pi^{eq}(T^{th}, \mu_\pi^{th})}$$

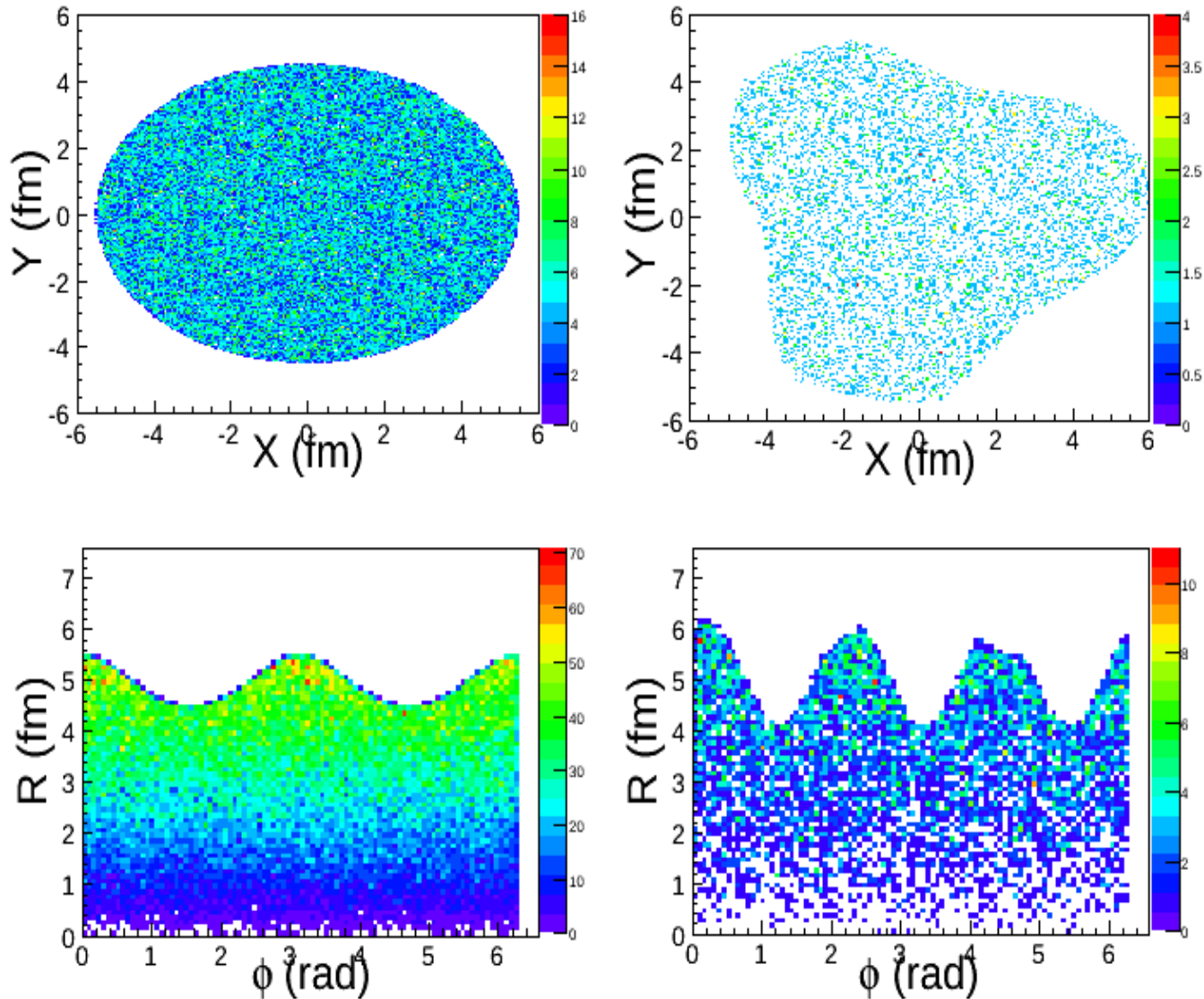
3. The absolute values  $\rho_i^{eq}(T^{th}, \mu_i^{th})$  are determined by the choice of the **free parameter of the model: effective pion chemical potential**  $\mu_\pi^{eff,th}$  at  $T^{th}$ . Assuming for the other particles (heavier than pions) the Boltzmann approximation :

$$\mu_i^{th} = T^{th} \ln \left( \frac{\rho_i^{eq}(T^{ch}, \mu_i^{ch})}{\rho_i^{eq}(T^{th}, \mu_i = 0)} \frac{\rho_\pi^{eq}(T^{th}, \mu_\pi^{eff,th})}{\rho_\pi^{eq}(T^{ch}, \mu_i^{ch})} \right)$$

**Particle momentum spectra** are generated on the **thermal freeze-out hypersurface**, the hadronic composition at this stage is defined by the parameters of the system at chemical freeze-out

# Anisotropic flow generation in HYDJET++ (soft component)

L.V. Bravina et al., *EPJC* 74 (2014) 2807



# Anisotropic flow generation in HYDJET++ (soft component)

**Elliptic flow  $v_2$**   $\frac{dN}{d\phi} \sim 1 + 2v_2 \cos 2(\phi - \psi_2) + 2v_3 \cos 3(\phi - \psi_3) + \dots$

- spatial modulation of freeze-out surface
- fluid velocity modulation

$$v_2 \propto \frac{2(\delta - \epsilon)}{(1 - \delta^2)(1 - \epsilon^2)}$$

Spatial anisotropy

$$\epsilon(b) = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}$$

$R(b)$  – surface radius

Momentum anisotropy

$$\tan \varphi_u = \sqrt{\frac{1 - \delta(b)}{1 + \delta(b)}} \tan \varphi$$

$\varphi_u$  : azimuthal angle of fluid velocity

$\varphi$  : spatial azimuthal angle

## Triangular flow $v_3$

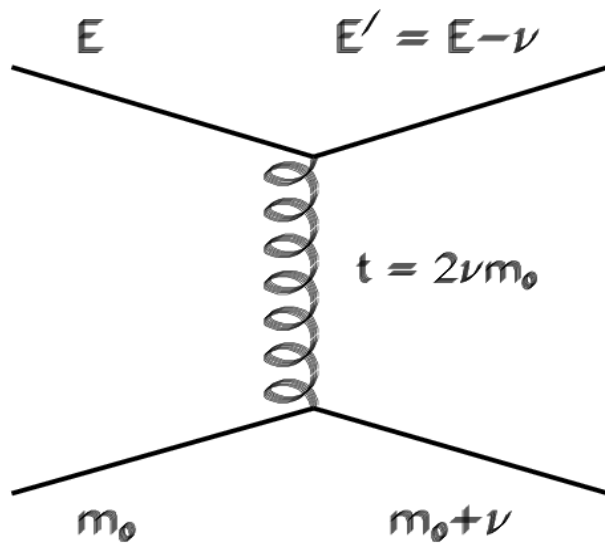
*Spatial modulation of freeze-out surface as  $\cos(3\varphi)$  with independent phase  $\Psi_3$  and parameter  $\epsilon_3$*

$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

Three parameters  $\epsilon(b_0)$ ,  $\epsilon_3(b_0)$  и  $\delta(b_0)$  is tuned to fit the data

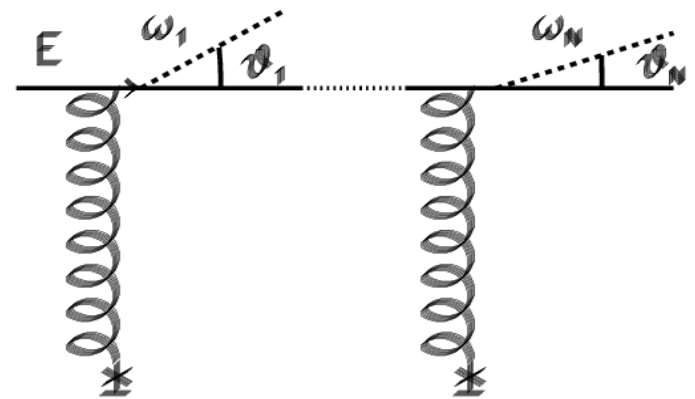
# Medium-induced partonic rescattering and energy loss («jet quenching»)

Collisional loss  
*(high momentum transfer approximation)*



+

Radiative loss  
*(BDMPS model, coherent radiation)*





# PYQUEN: physics frames

## General kinetic integral equation:

$$\Delta E(L, E) = \int_0^L dx \frac{dP}{dx}(x) \lambda(x) \frac{dE}{dx}(x, E), \quad \frac{dP}{dx}(x) = \frac{1}{\lambda(x)} \exp(-x/\lambda(x))$$

### 1. Collisional loss and elastic scattering cross section:

$$\frac{dE}{dx} = \frac{1}{4T\lambda\sigma} \int_{\mu_D^2}^{t_{max}} dt \frac{d\sigma}{dt} t, \quad \frac{d\sigma}{dt} \simeq C \frac{2\pi\alpha_s^2(t)}{t^2}, \quad \alpha_s = \frac{12\pi}{(33-2N_f)\ln(t/\Lambda_{QCD}^2)}, \quad C = 9/4(gg), 1(gq), 4/9(qq)$$

### 2. Radiative loss (BDMPS):

$$\frac{dE}{dx}(m_q=0) = \frac{2\alpha_s C_F}{\pi\tau_L} \int_{E_{LPM} \sim \lambda_g \mu_D^2}^E d\omega \left[ 1 - y + \frac{y^2}{2} \right] \ln |\cos(\omega_1 \tau_1)|, \quad \omega_1 = \sqrt{i \left( 1 - y + \frac{C_F}{3} y^2 \right) \bar{k} \ln \frac{16}{\bar{k}}}, \quad \bar{k} = \frac{\mu_D^2 \lambda_g}{\omega(1-y)}, \quad \tau_1 = \frac{\tau_L}{2\lambda_g}, \quad y = \frac{\omega}{E}, \quad C_F = \frac{4}{3}$$

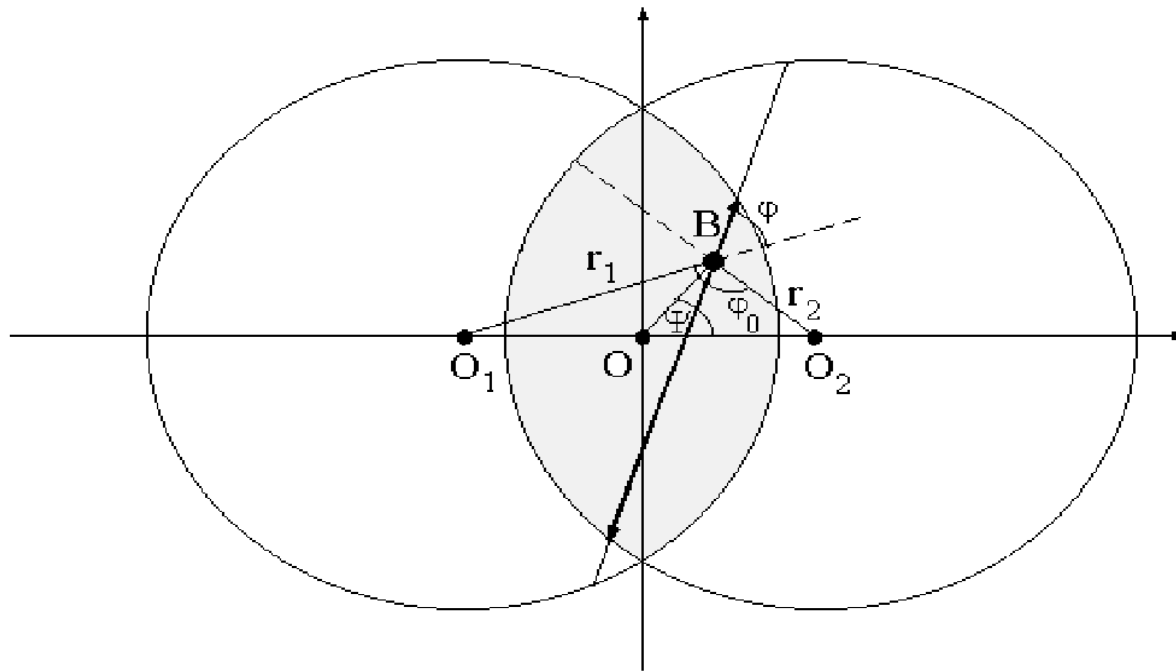
“dead cone” approximation for massive quarks:

$$\frac{dE}{dx}(m_q \neq 0) = \frac{1}{(1+(l\omega)^{3/2})^2} \frac{dE}{dx}(m_q=0), \quad l = \left( \frac{\lambda}{\mu_D^2} \right)^{1/3} \left( \frac{m_q}{E} \right)^{4/3}$$

# Nuclear geometry and QGP evolution

impact parameter  $b \equiv |O_1 O_2|$  - transverse distance between nucleus centers

$$\varepsilon(r_1, r_2) \propto T_A(r_1) * T_A(r_2) \quad (T_A(b) - \text{nuclear thickness function})$$



Space-time evolution of QGP, created in region of initial overlapping of colliding nuclei, is described by Lorenz-invariant Bjorken's hydrodynamics *J.D. Bjorken, PRD 27 (1983) 140*

# Monte-Carlo simulation of parton rescattering and energy loss in PYQUEN

- Distribution over jet production vertex  $V(r \cos \psi, r \sin \psi)$  at im.p.  $b$

$$\frac{dN}{d\psi dr}(b) = \frac{T_A(r_1) T_A(r_2)}{\int_0^{2\pi} d\psi \int_0^{r_{\max}} r dr T_A(r_1) T_A(r_2)}$$

- Transverse distance between parton scatterings  $l_i = (\tau_{i+1} - \tau_i) E/p_T$

$$\frac{dP}{dl_i} = \lambda^{-1}(\tau_{i+1}) \exp\left(-\int_0^{l_i} \lambda^{-1}(\tau_i + s) ds\right), \quad \lambda^{-1} = \sigma \rho$$

- Radiative and collisional energy loss per scattering

$$\Delta E_{tot,i} = \Delta E_{rad,i} + \Delta E_{col,i}$$

- Transverse momentum kick per scattering

$$\Delta k_{t,i}^2 = \left(E - \frac{t_i}{2m_{0i}}\right)^2 - \left(p - \frac{E}{p} \frac{t_i}{2m_{0i}} - \frac{t_i}{2p}\right)^2 - m_q^2$$

# Angular structure of energy loss in PYQUEN

**Radiative loss**, three options (simple parametrizations) for angular distribution of in-medium emitted gluons:

Collinear radiation  $\theta=0$

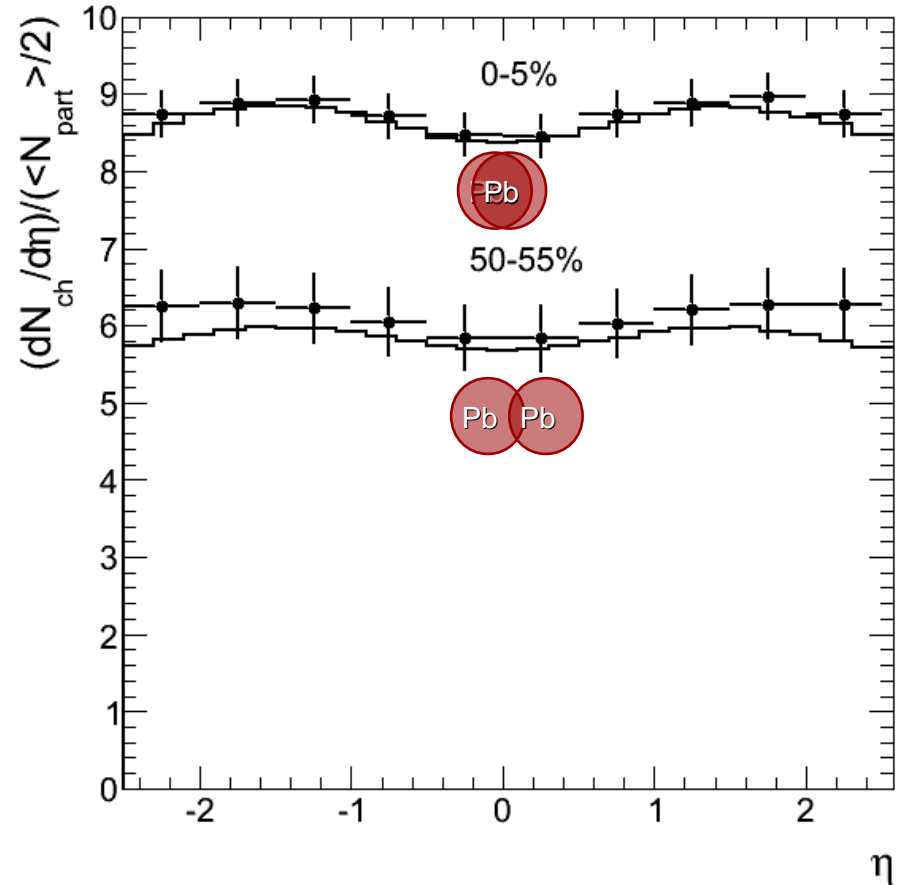
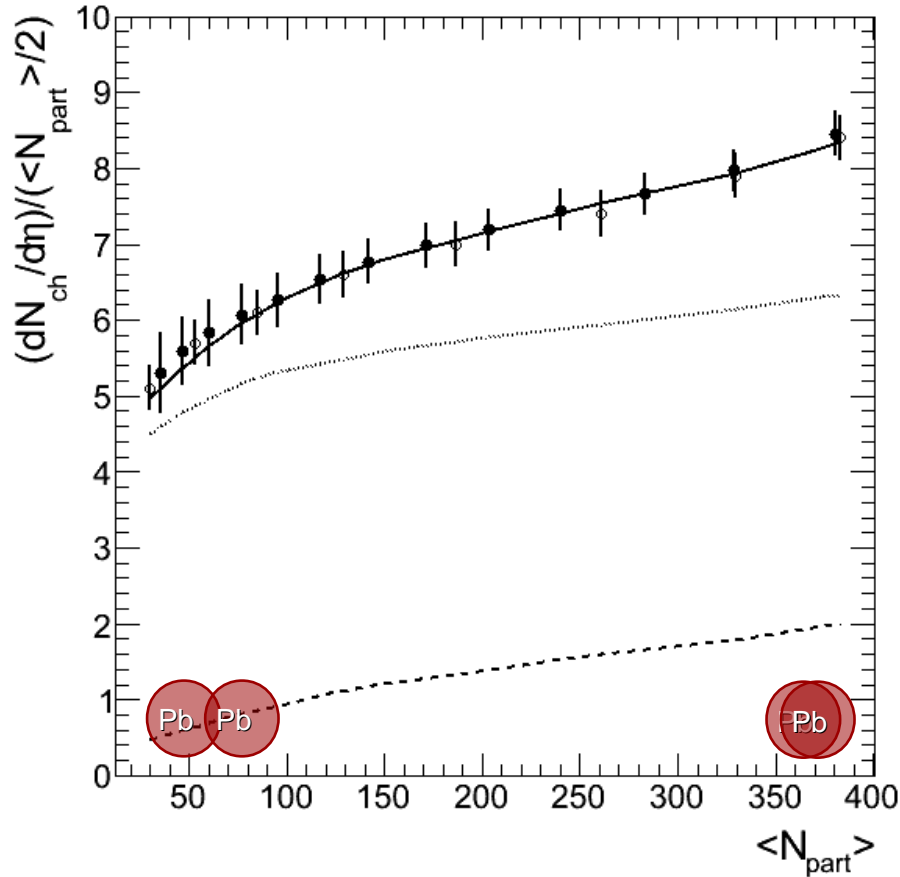
Small-angular radiation  $\frac{dN^g}{d\theta} \propto \sin\theta \exp\left(\frac{-(\theta-\theta_0)^2}{2\theta_0^2}\right)$ ,  $\theta_0 \sim 5^\circ$

Wide-angular radiation  $\frac{dN^g}{d\theta} \propto \frac{1}{\theta}$

**Collisional loss** always “out-of-cone” (energy is absorbed by medium)

# Charged multiplicity vs. centrality and pseudorapidity

I.P. Lokhtin et al., EPJC 72(2012) 2045

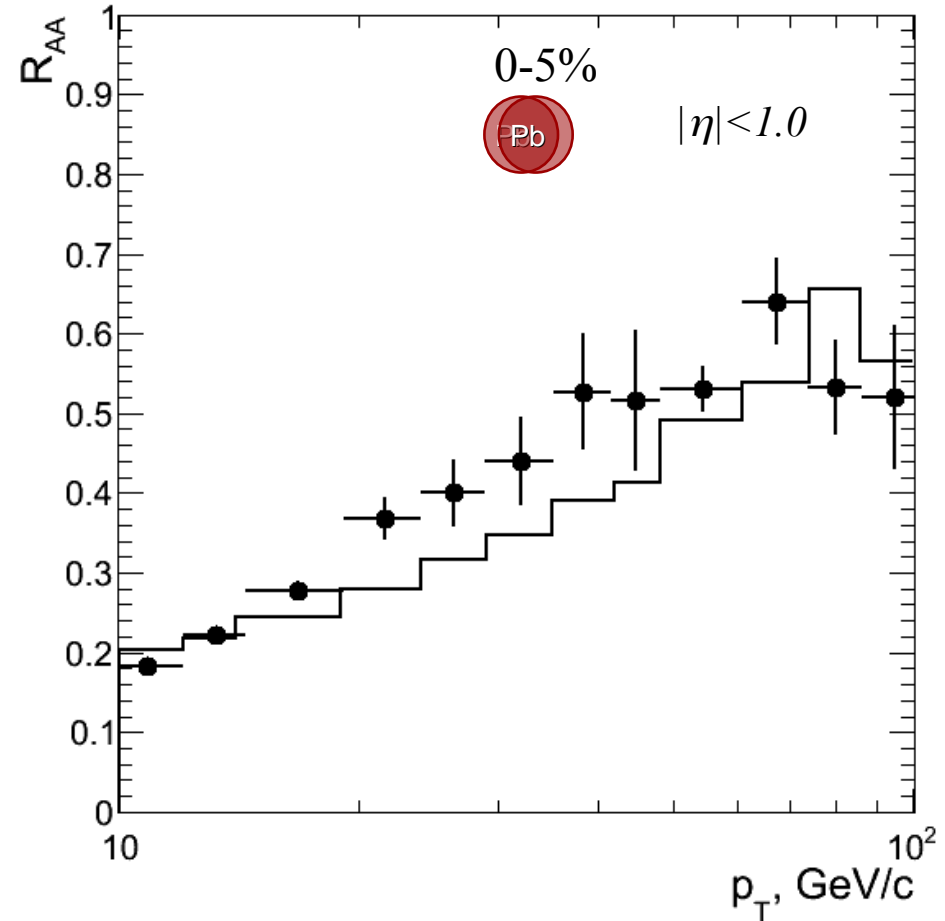
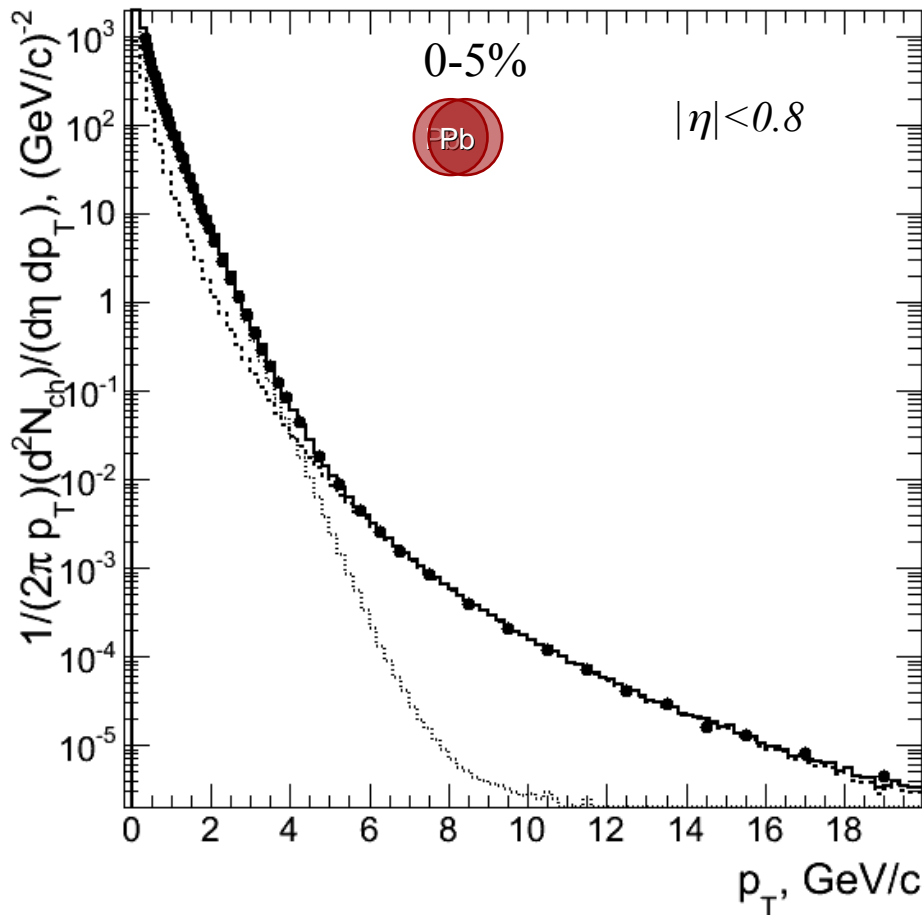


Open points: ALICE data (*PRL* 106 (2011) 032301), closed points: CMS data (*JHEP* 1108 (2011) 141);  
histograms: HYDJET++

Tuned HYDJET++ reproduces multiplicity vs. event centrality (down to very peripheral events) with contribution of hard component to multiplicity in mid-rapidity for central PbPb  $\sim 30\%$ , as well as approximately flat pseudorapidity distribution.

# $P_T$ -spectrum and $R_{AA}$ for inclusive charged hadrons

I.P. Lokhtin et al., EPJC 72(2012) 2045



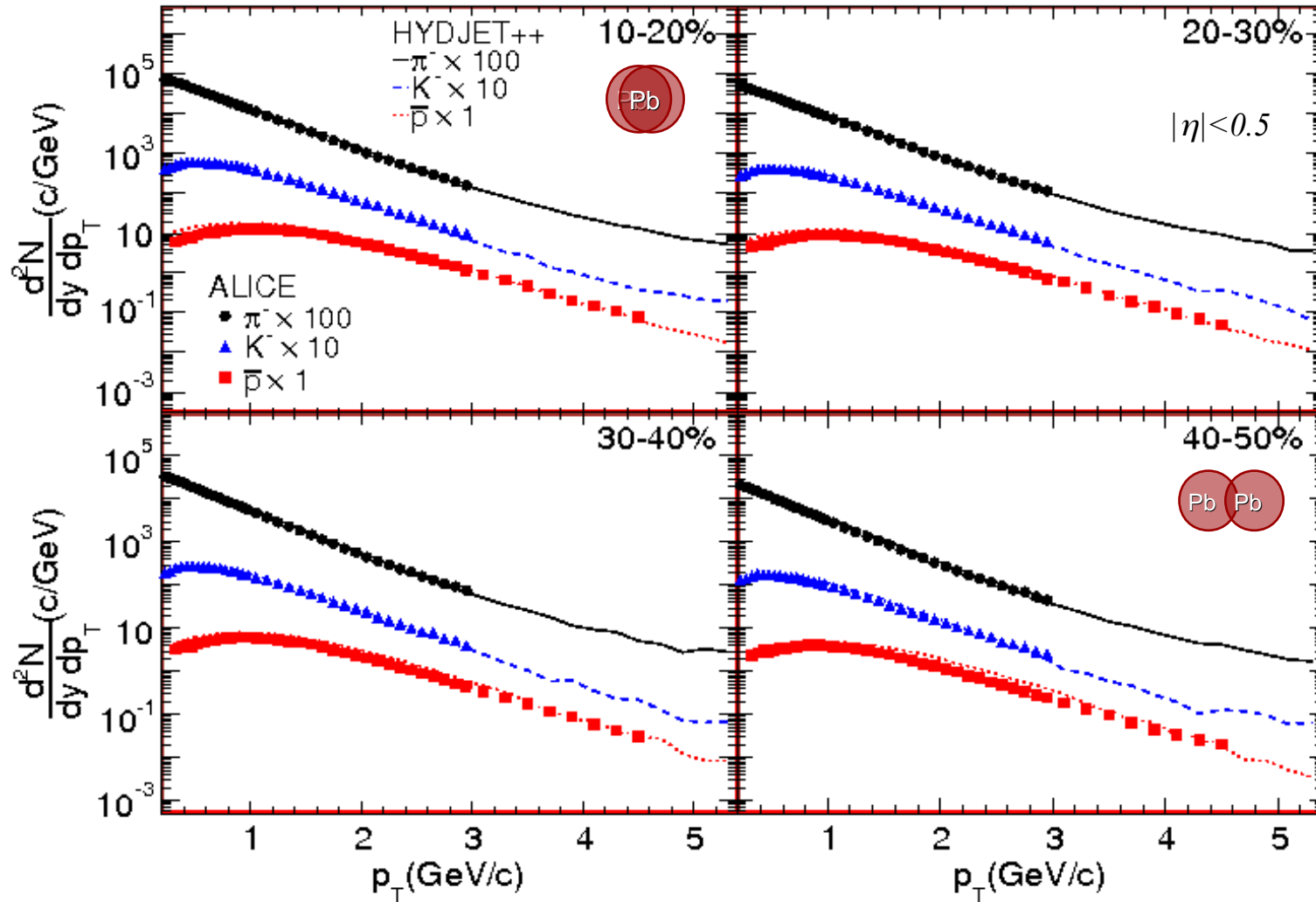
$$R_{AA} = \frac{\sigma_{pp}^{inel}}{\langle N_{coll} \rangle} \frac{d^2 N_{AA} / dp_T d\eta}{d^2 \sigma_{pp} / dp_T d\eta} \sim \begin{cases} \text{“QCD Medium”} \\ \text{“QCD Vacuum”} \end{cases} \left\{ \begin{array}{l} R_{AA} > 1: \text{enhancement} \\ R_{AA} = 1: \text{no medium effect} \\ R_{AA} < 1: \text{suppression} \end{array} \right.$$

Points: ALICE (left) (PLB 696 (2011) 30) & CMS (right) (EPJ C 72 (2012) 1945) data;

histograms: HYDJET++

# $P_T$ -spectra of identified hadrons

L.V. Bravina et al., *EPJC* 74 (2014) 2807

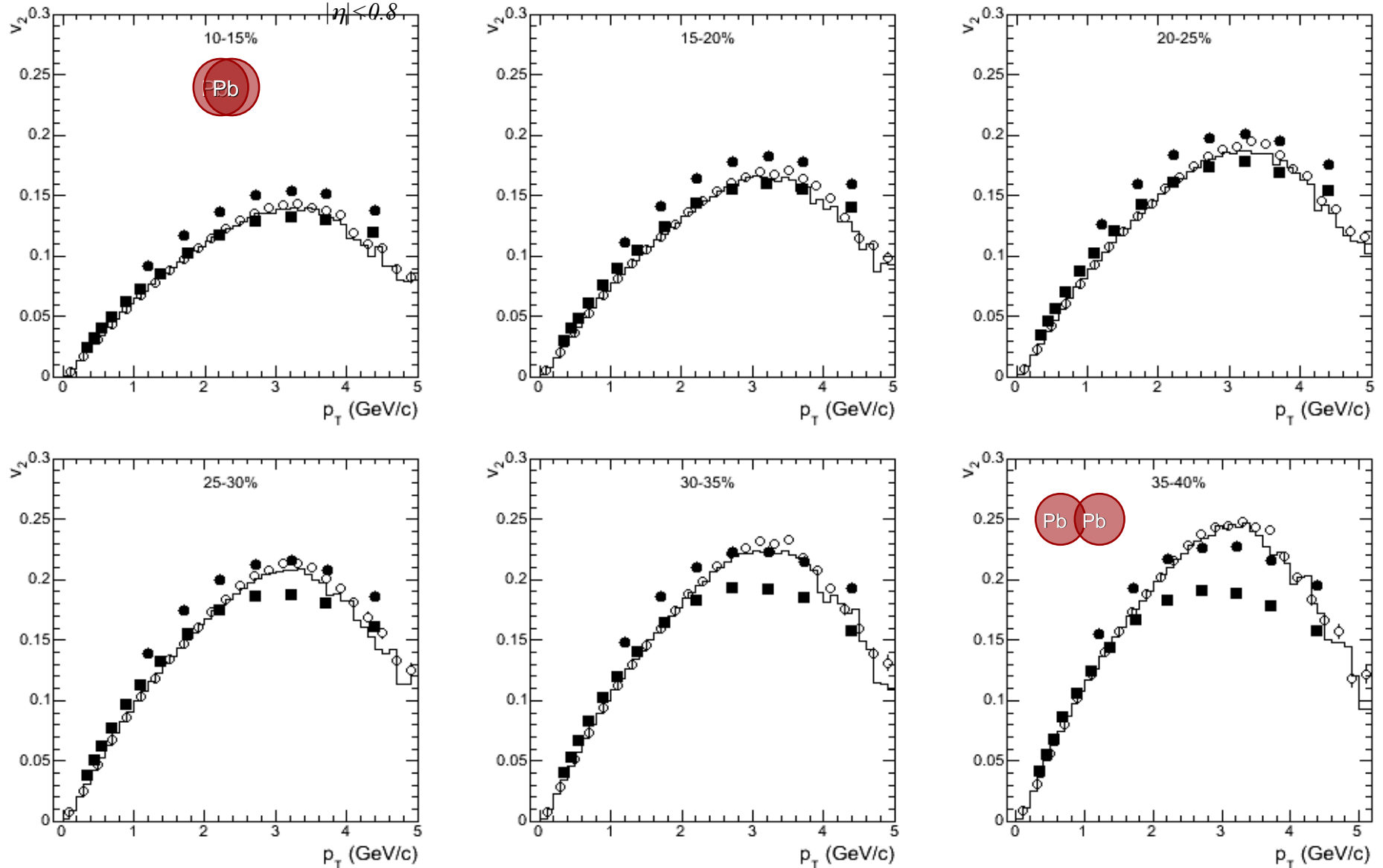


Points: ALICE data (*APP B* 43 (2012) 555); histograms: HYDJET++

HYDJET++ reproduces  $p_T$ -spectrum of pions, kaons and (anti-)protons as well

# Elliptic flow of inclusive charged hadrons

L.V. Bravina et al., *EPJC* 74 (2014) 2807



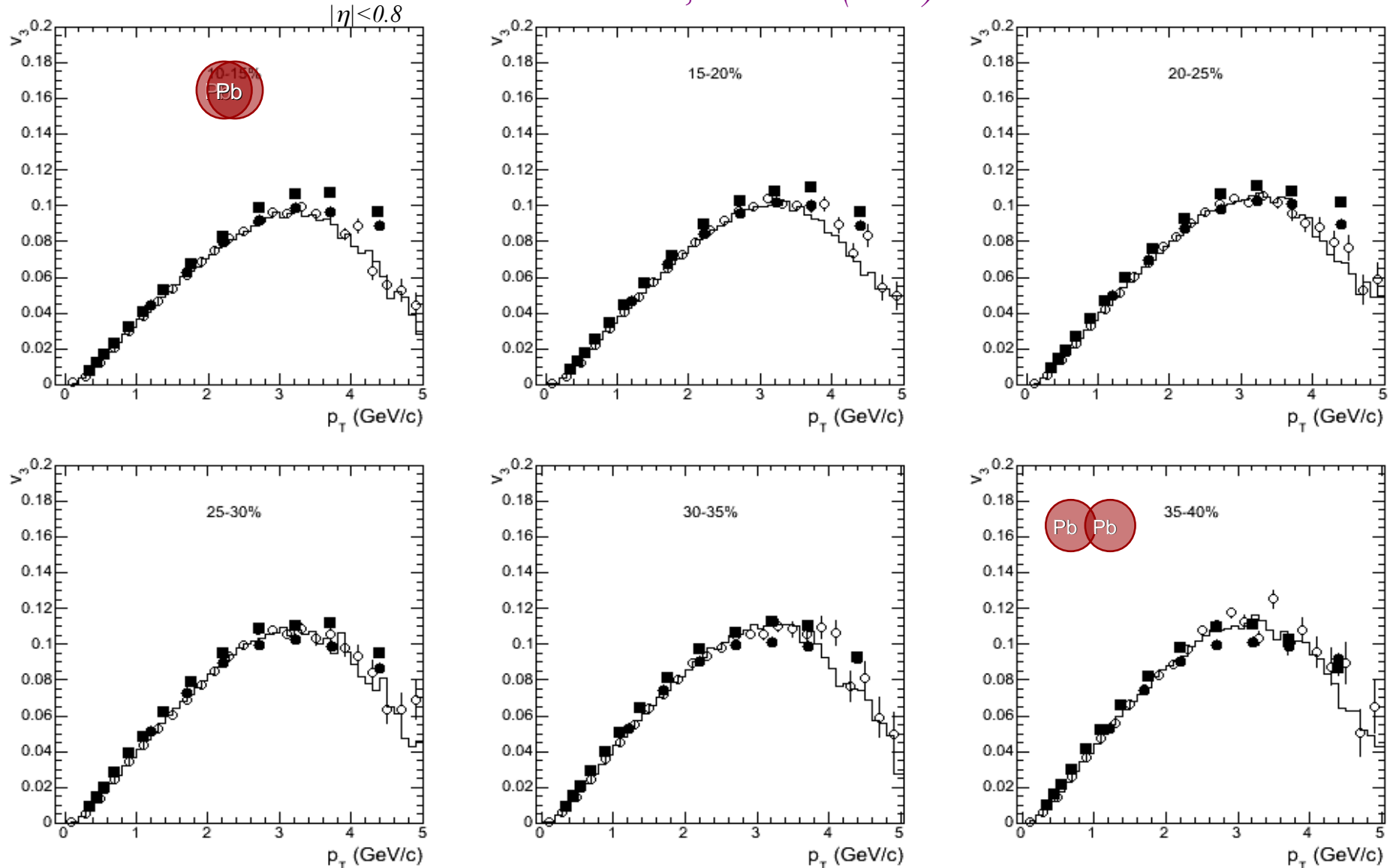
Closed circles and squares: CMS data  $v_2\{2\}$  &  $v_2\{\text{LYZ}\}$  (*PRC* 87 (2013) 014902);

histograms and open circles: HYDJET++ (“true”  $v_2(\psi_2)$  &  $v_2\{\text{EP}\}$ )



# Triangular flow of inclusive charged hadrons

L.V. Bravina et al., *EPJC* 74 (2014) 2807

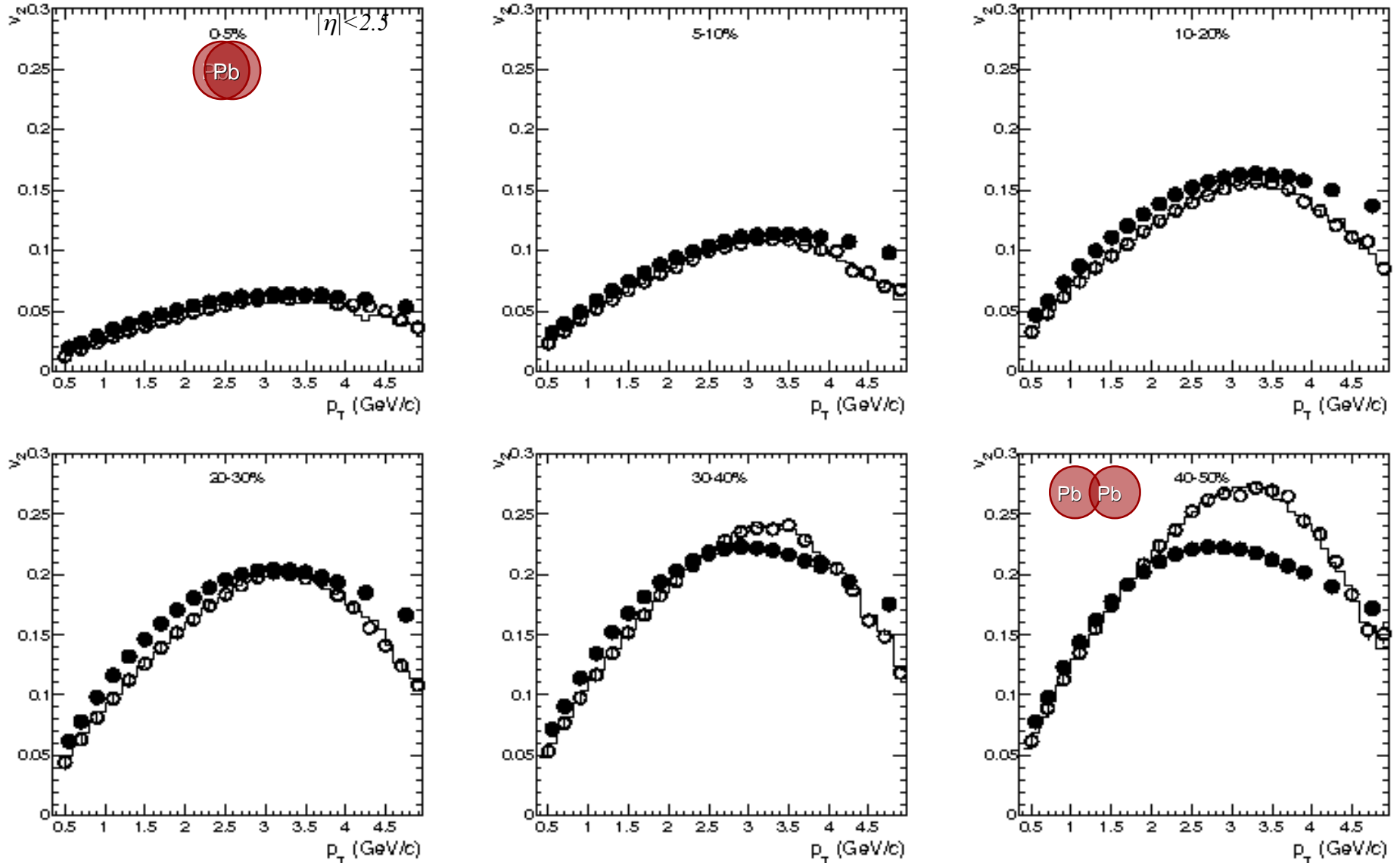


Closed circles and squares: CMS data  $v_3\{2\}$  &  $v_3\{\text{EP}\}$  (*PRC* 89 (2014) 044906);

histograms and open circles: HYDJET++ (“true”  $v_3(\psi_3)$  &  $v_3\{\text{EP}\}$ )

# Elliptic flow of inclusive charged hadrons

L.V. Bravina et al., *EPJC* 74 (2014) 2807

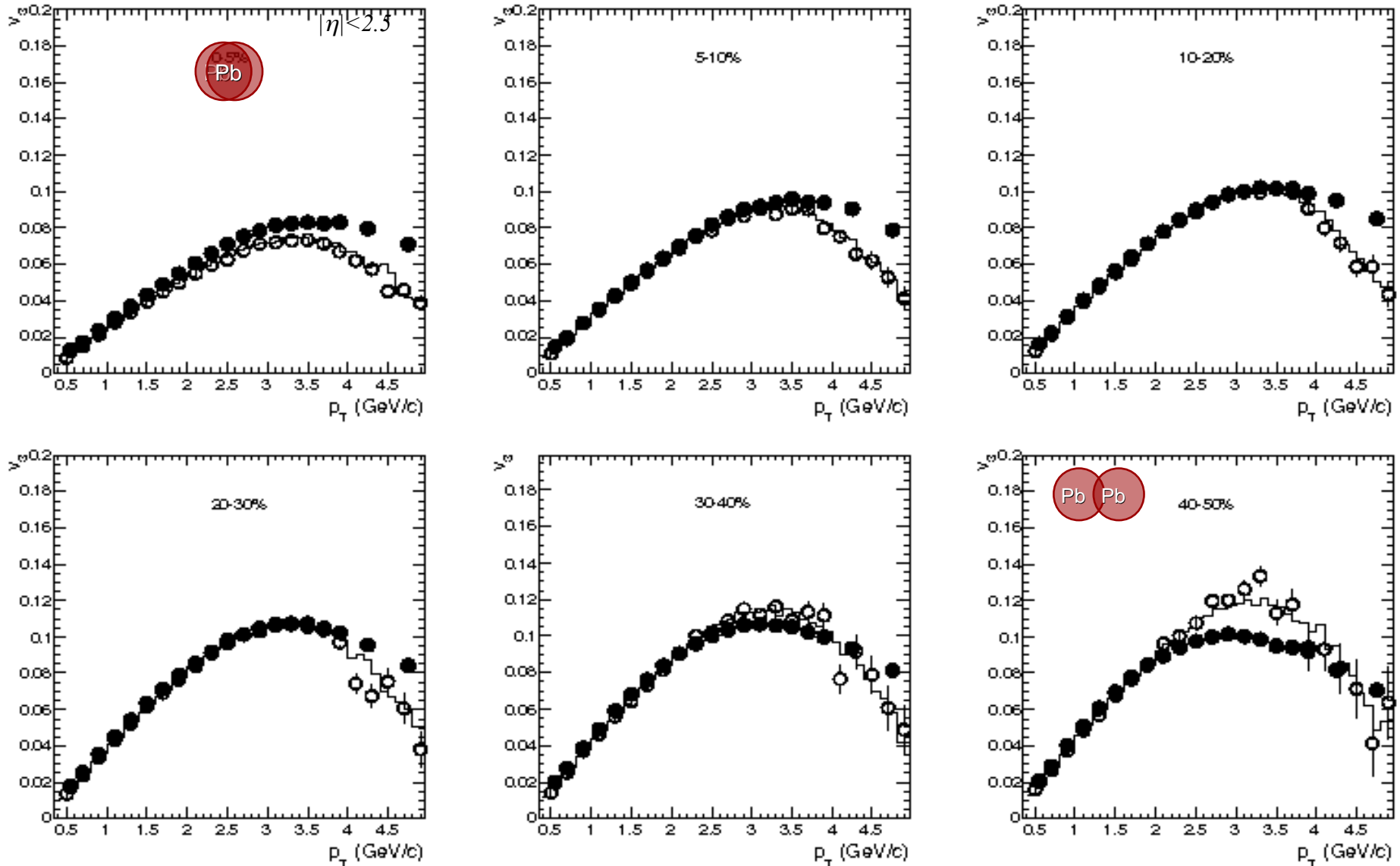


Closed circles: ATLAS data  $v_2\{\text{EP}\}$  (*PRC* 86 (2012) 014907);

histograms and open circles: HYDJET++ (“true”  $v_2(\psi_2)$  &  $v_2\{\text{EP}\}$ )

# Triangular flow of inclusive charged hadrons

L.V. Bravina et al., *EPJC* 74 (2014) 2807

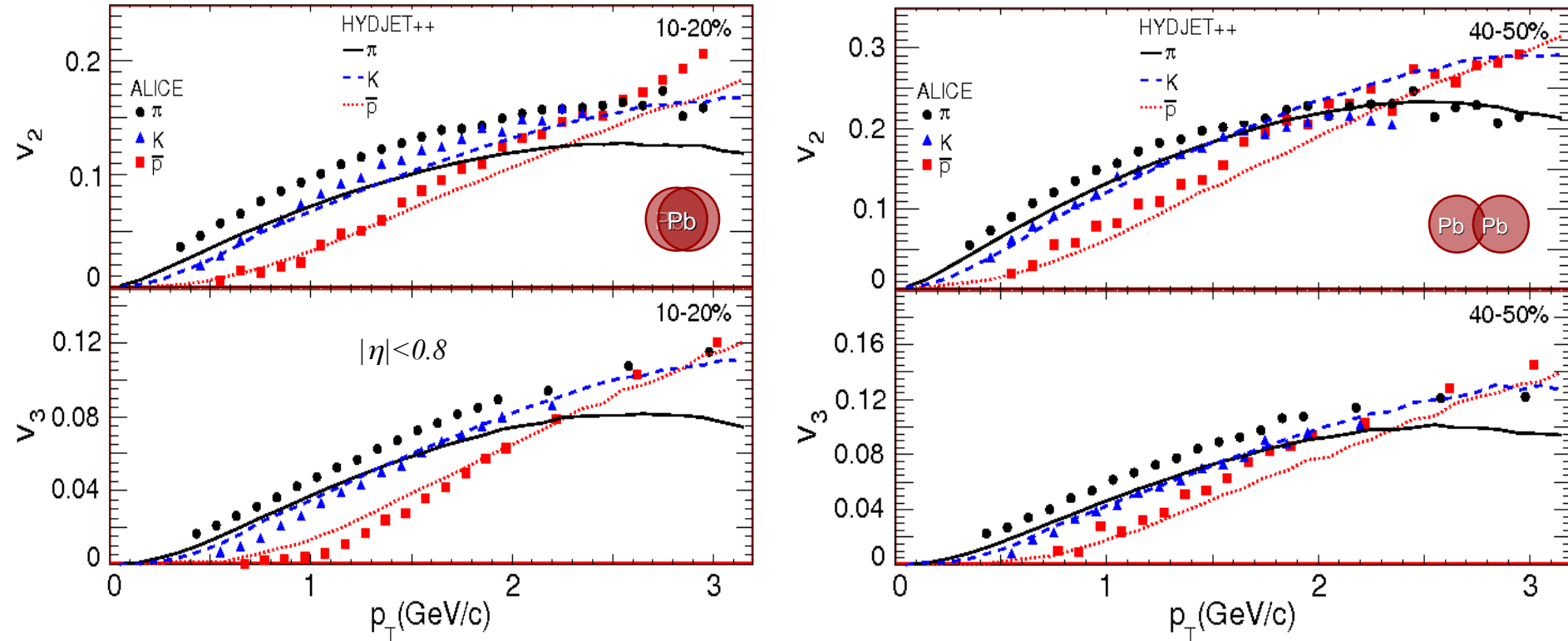


Closed circles: ATLAS data  $v_3\{EP\}$  (*PRC* 86 (2012) 014907);

histograms and open circles: HYDJET++ (“true”  $v_3(\psi_3)$  &  $v_3\{EP\}$ )

# Elliptic and triangular flows of identified hadrons

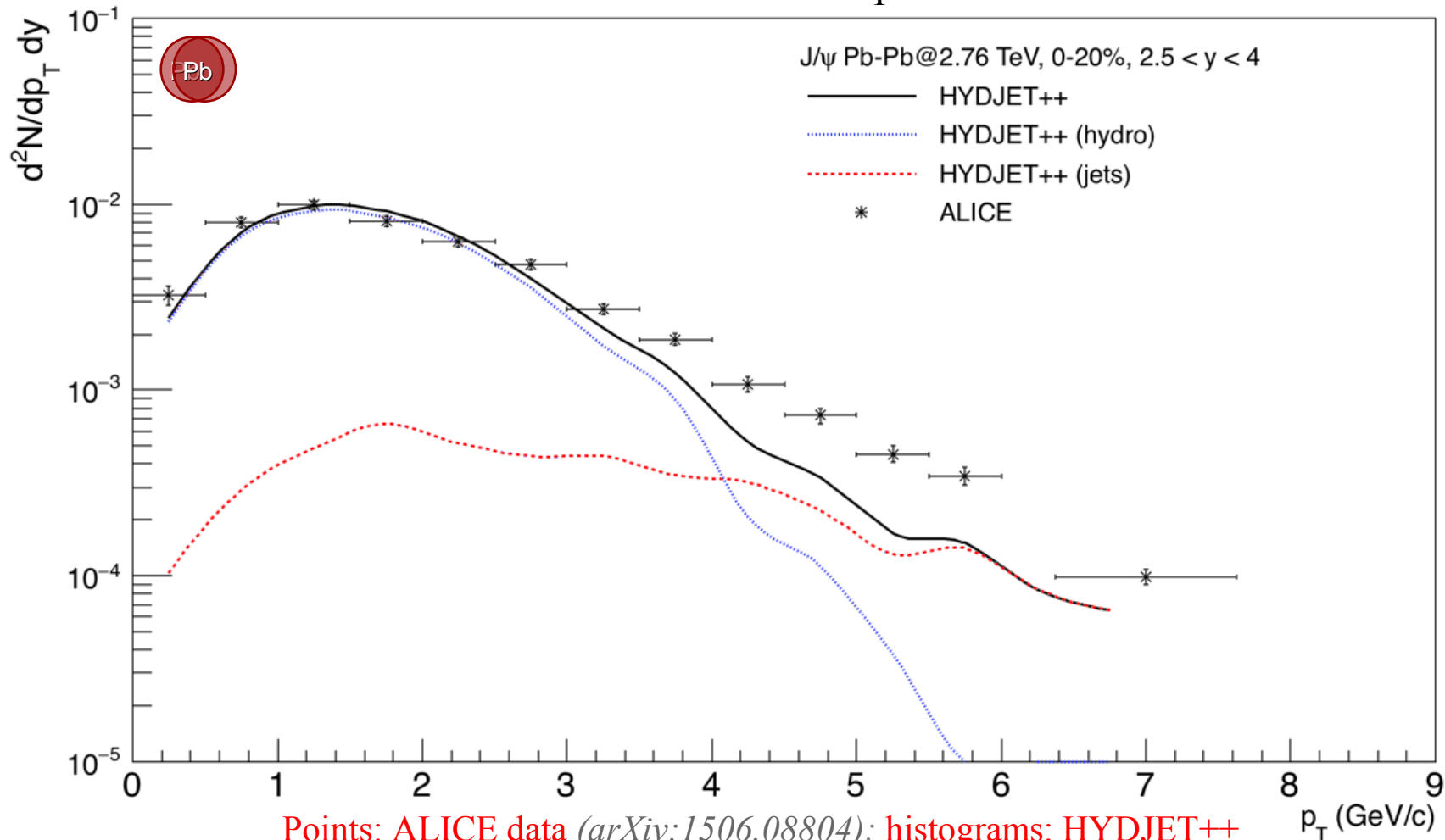
L.V. Bravina et al., *EPJC* 74 (2014) 2807



Points: ALICE data (*JPG* 38 (2011) 124047); histograms: HYDJET++

HYDJET++ reproduces  $v_2$  and  $v_3$  for kaons and (anti-)protons, but rather underestimates the data for pions (stronger non-flow correlations in the data than in the model?)<sup>52</sup>

# J/ $\psi$ mesons at LHC ( $p_T$ -spectrum)



Points: ALICE data (*arXiv:1506.08804*); histograms: HYDJET++  
 ( $T_{th} = T_{ch} = 165$  MeV,  $\eta^{\max} = 2.3$ ,  $\rho_u^{\max} = 0.6$ ,  $\gamma_c = 11.5$ ,  $P_{T\min} = 3.0$  GeV/c)

HYDJET++ reproduces J/ $\psi$ -meson  $p_T$ -spectrum (up to  $\sim 3$  GeV/c) with the *freeze-out parameters different* from ones for inclusive hadrons  $\Rightarrow$  *kinetic freeze-out of J/ $\psi$  thermal component occurs before freeze-out of light hadrons; non-thermal component is important at intermediate & high  $p_T$*