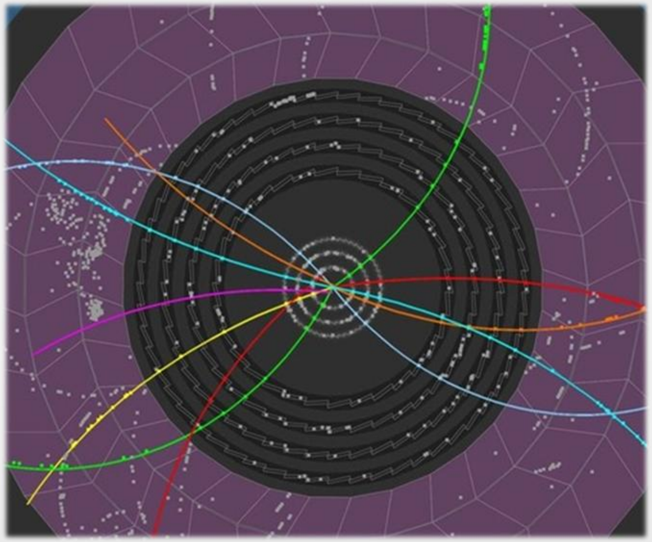
DYNAMICAL UPSILON-SUPPRESSION IN THE STOCHASTIC-SCHRÖDINGER APPROACH



Roland Katz

SQM 2015 Dubna – 7th of July 2015

PhD supervisor: P.B. Gossiaux

Subatech and TOGETHER Pays de la Loire

In few words ?

Initial quasi stationary
Sequential Suppression
assumption
(Matsui & Satz 86)

Final quasi stationary
Statistical Hadronisation
assumption
(Andronic, Braun-Munzinger & Stachel)

Dynamical Models
implicit hope to measure T above Tc

???

➡ **an effective dynamical point of view :**

✓ **QGP genuine time dependent scenario**

✓ **quantum description of the $Q\bar{Q}$**

✓ **binding potentials, screening, thermalisation**

Summary

- Background and motivations
- Schrödinger-Langevin dynamical model
- Application to the $b\bar{b}$ system

Common views on quarkonia suppression

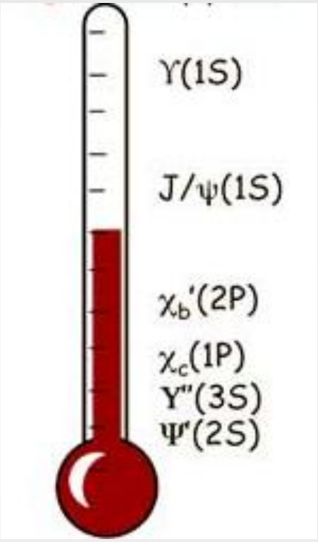
Sequential suppression by Matsui and Satz …

Each state has a dissociation T_{diss}

+ Early states formation in a stationnary QGP at T

= if $T > T_{diss}$ the state is dissociated for ever

=> *quarkonia as QGP thermometer*



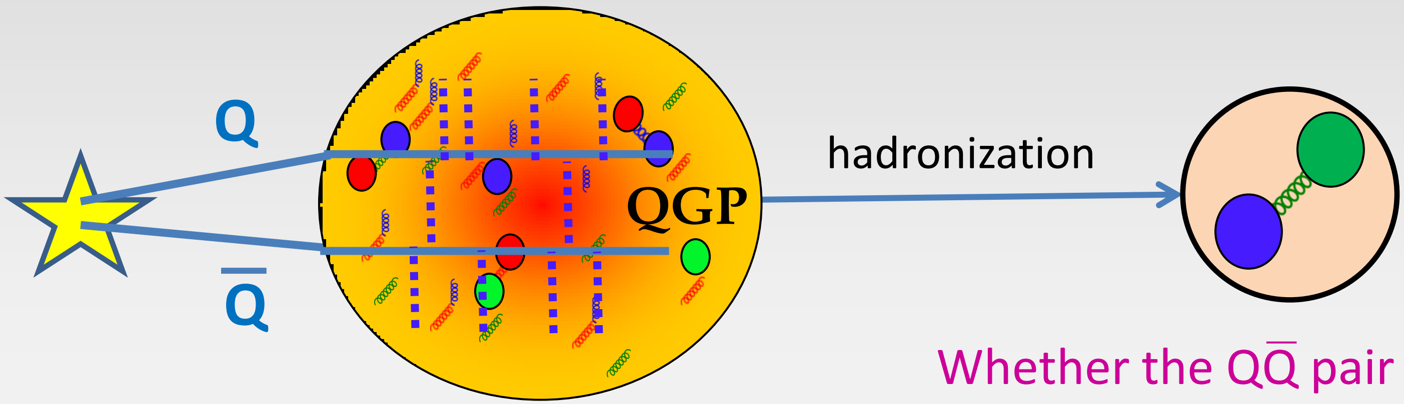
… and recombination

collision energy ↗

⇒ number of $Q\bar{Q}$ in the medium ↗

⇒ probability that a Q re-associates with another $\bar{Q}$ ↗

Reality -> back to concepts



Very complicated QFT problem at finite T(t) !!!

Whether the $Q\bar{Q}$ pair emerges as a quarkonia or as open mesons is <u>only resolved at the end of the evolution</u>

Beware of quantum coherence during the evolution !

Need for full quantum treatment

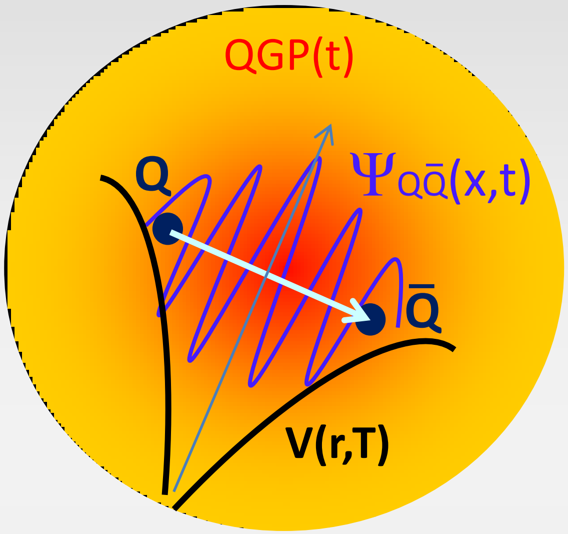
Dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Assumptions

Sequential suppression VS Dynamical model

Sequential suppression	Dynamical model
✓ Sequential suppression at an early stage temperature	-> State formation only at the end of the evolution
✓ Stationnary QGP	-> Reality is closer to a cooling QGP
✓ Adiabatic evolution if formed; fast decorrelation if suppressed	-> Quantum description of the correlated $Q\bar{Q}$ pair

Model ingredients ?



« Dipole » $Q\bar{Q}$ wavefunction

+

Mean field: color screened binding potential V(r,T)

Interactions due to color charges

+

QGP temperature scenarios T(t)

Cooling QGP

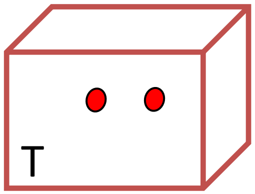
+

Thermalisation and diffusion

Direct interactions with the thermal bath

Mean color field : screened V(Tred, r) binding the $Q\bar{Q}$

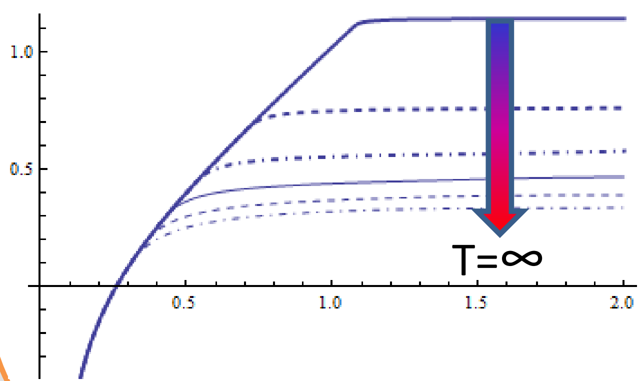
Static lQCD calculations (maximum heat exchange with the medium):



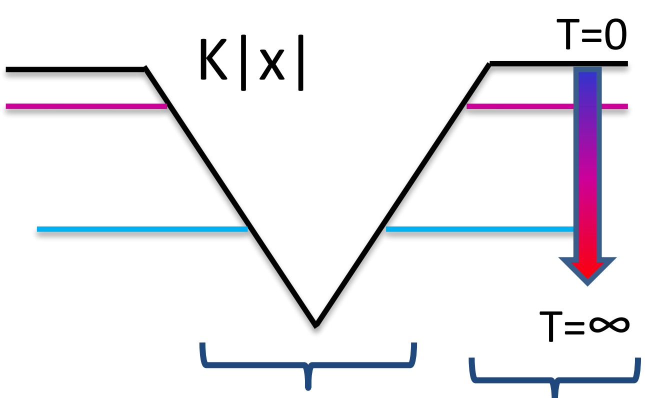
F : free energy
S : entropy

U=F+TS : internal energy
(no heat exchange)

- "Weak potential" $F<V_{weak}<U$ * => some heat exchange
- "Strong potential" V=U ** => adiabatic evolution



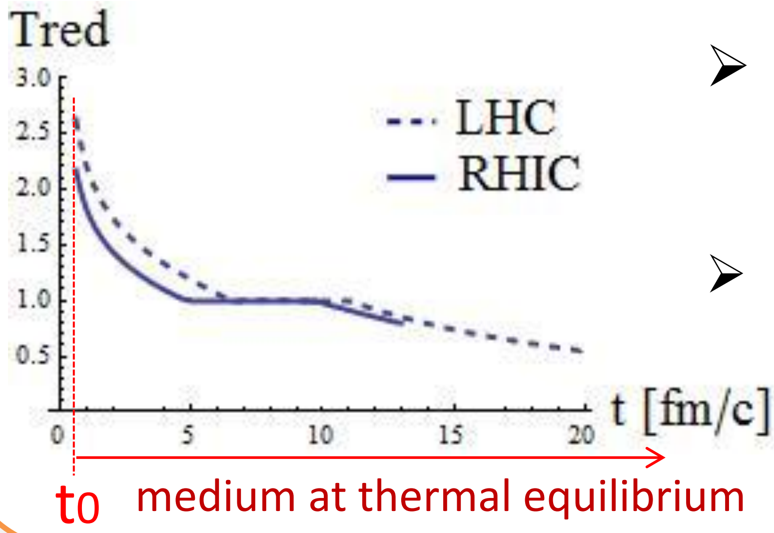
1D
simplification



K chosen such that: $E_2-E_0 = E(Y(2S))-E(Y(1S))$

* Mócsy & Petreczky Phys.Rev.D77:014501,2008
** Kaczmarek & Zantow arXiv:hep-lat/0512031v1

QGP homogeneous temperature(t) scenarios



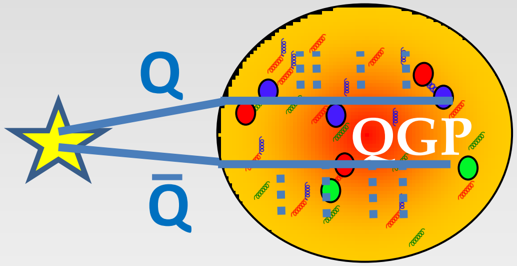
- Cooling over time by Kolb and Heinz* (hydrodynamic evolution and entropy conservation)
- At LHC ($\sqrt{s_{NN}} = 2.76\ \mathrm{TeV}$) and RHIC ($\sqrt{s_{NN}} = 200\ \mathrm{GeV}$) energies

Initial QQ̄ wavefunction

- Produced at the very beginning : $\tau_f^{Q\bar{Q}} \sim \hbar/(2m_Q\, c^2) < 0.1\ \mathrm{fm/c}$
- We assume either a formed state (Υ(1S) or Υ(2S)) OR a more realistic Gaussian wavefunction with parameter $a_{b\bar{b}} = 0.045\ \mathrm{fm}$ (from Heisenberg principle)

* arXiv:nucl-th/0305084v2

Thermalisation ?



The common open quantum approach

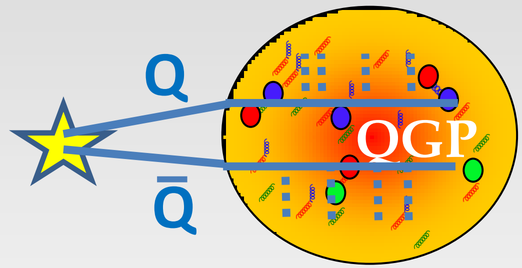
- **Idea:** density matrix and {quarkonia + bath} => bath integrated out
  ⇒ non unitary evolution + decoherence effects

Akamatsu* -> complex potential
Borghini** -> a master equation

- **But** defining the bath is complicated and the calculation entangled…

* Y. Akamatsu Phys.Rev. D87 (2013) 045016 ; ** N. Borghini et al., Eur. Phys. J. C 72 (2012) 2000

Thermalisation ?



The common open qu ... approach

NOT EFFECTIVE

- **Idea:** density matrix and f ... ath integrated out
  ⇒ non uni ... e effects

... otential
... aster equation

- **But** defining the ba ... complicated and the calculation entangled…

* Y. Akamatsu Phys.Rev. D87 (2013) 045016 ; ** N. Borghini et al., Eur. Phys. J. C 72 (2012) 2000

Subatech

Thermalisation ?

Langevin-like approaches ✓

Quarkonia are Brownian particles ($M_{Q\bar{Q}} >> T$)

+ **Drag A(T) => need for a Langevin-like eq.**

(A(T) from single heavy quark observables or lQCD calculations)

- **Idea:** Effective equations to unravel/mock the open quantum approach

Young and Shuryak * -> semi-classical Langevin

Akamatsu and Rothkopf ** -> stochastic and complex potential

Semi-classical	Schrödinger-Langevin equation	Others
See our SQM 2013 proceeding ***		Failed at low/medium temperatures

Effective thermalisation from fluctuation/dissipation

* C. Young and Shuryak E 2009 Phys. Rev. C 79: 034907 ; ** Y. Akamatsu and A. Rothkopf. Phys. Rev. D 85, 105011 (2012) ;
*** R. Katz and P.B. Gossaiux J.Phys.Conf.Ser. 509 (2014) 012095

Schrödinger-Langevin (SL) equation

Derived from the Heisenberg-Langevin equation*, in Bohmian mechanics** …

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big) \right) \Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Hamiltonian
includes the
Mean Field
(color binding potential)

* Kostin The J. of Chem. Phys. 57(9):3589–3590, (1972)
** Garashchuk et al. J. of Chem. Phys. 138, 054107 (2013)

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Fluctuations

taken as a « classical » stochastic force

White quantum noise *

$$\langle F_R(t)F_R(t+\tau)\rangle = 2mA\,E_0 \left[\coth\left(\frac{E_0}{kT_{\mathrm{bath}}}\right) - 1 \right] \delta(\tau)$$

Color quantum noise **

$$\langle N[F_R(t)F_R(t+\tau)]\rangle = \frac{2mA}{\pi} \int_0^\infty \frac{\hbar\omega}{\exp(\hbar\omega/kT_{\mathrm{bath}}) - 1} \cos(\omega\tau)\, d\omega.$$

* I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124}, 642 (1961).
** G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. 6, 504 (1965).

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Dissipation

$$S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$$

✓non-linearly dependent on $\Psi_{Q\bar{Q}}$

✓ real and ohmic

✓ ***A*** = drag coefficient (inverse relaxation time)

✓ Brings the system to the lowest state

Properties of the SL equation

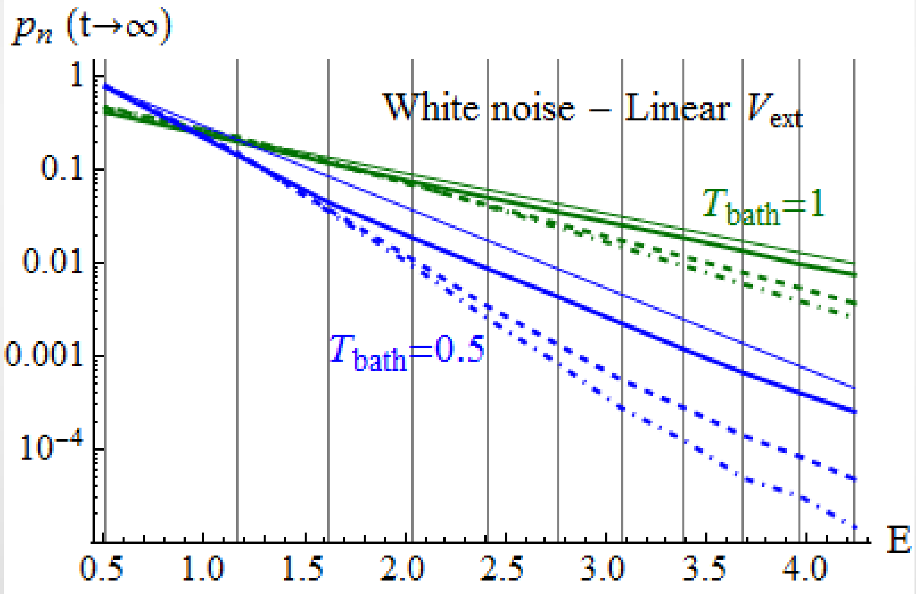
- 2 parameters: A (Drag) and T (temperature)
- Unitarity (no decay of the norm as with imaginary potentials)
- Heisenberg principle satisfied at any T
- Non linear => Violation of the superposition principle (=> decoherence)
- Gradual evolution from pure to mixed states (large statistics)
- Mixed state observables:

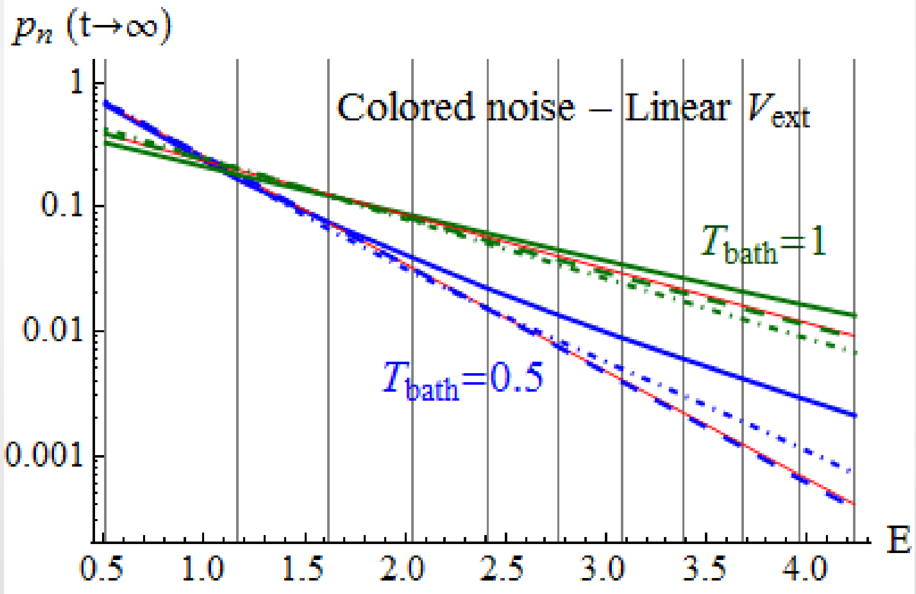
$$\left\langle \langle\psi(t)|\hat{O}|\psi(t)\rangle \right\rangle_{\text{stat}} = \lim_{n_{\text{stat}}\to\infty} \frac{1}{n_{\text{stat}}} \sum_{r=1}^{n_{\text{stat}}} \langle\psi^{(r)}(t)|\hat{O}|\psi^{(r)}(t)\rangle$$

- Easy to implement numerically (especially in Monte-Carlo generator)

Equilibration with SL equation

Leads the subsystem to thermal equilibrium (Boltzmann distributions) for at least the low lying states





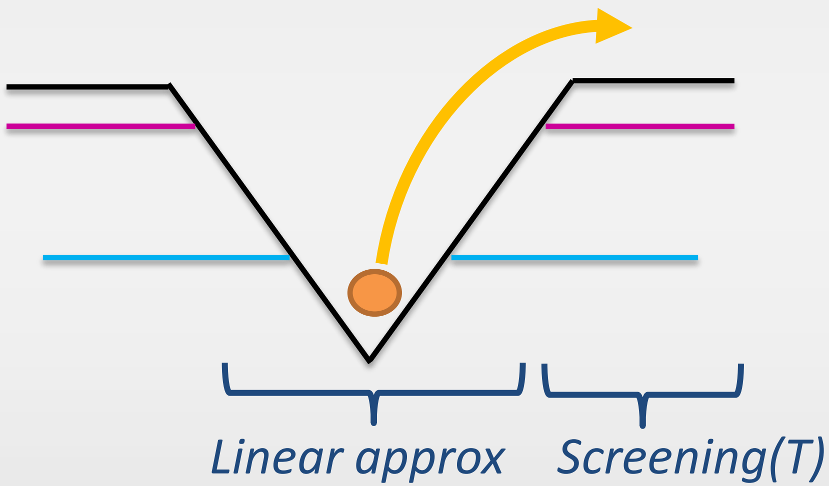
See R. Katz and P. B. Gossiaux, arXiv:1504.08087 [quant-ph]

Dynamics of $Q\bar{Q}$ with SL equation

- Drag coeff. for b quarks*: $A_b(T) = 0.92T + 0.64T^2 \ (\mathrm{c/fm})$

Typically T ∈ [0.1 ; 0.43] GeV => A ∈ [0.09 ; 0.51] $(\mathrm{fm/c})^{-1}$

- Simplified Potential:



Stochastic forces =>
- feed up of higher states
- leakage

- Observables: « **Weight :** » $W_i(t) = |\langle \Psi_i(T=0) | \Psi_{Q\bar{Q}(t)} \rangle|^2$

« **Survivance** : » $S_i(t) = W_i(t)/W_i(t=0)$

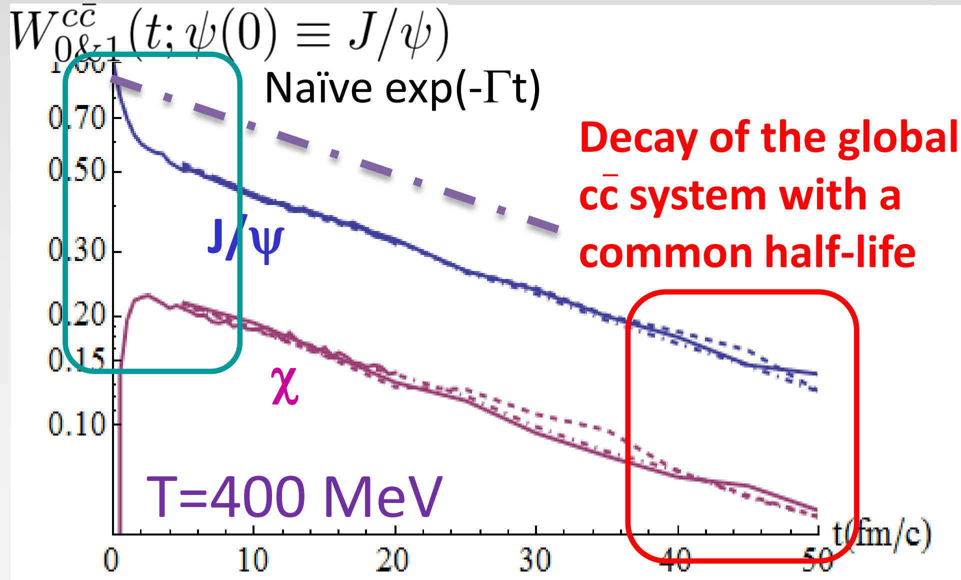
* P.B. Gossiaux and J. Aichelin 2008 Phys. Rev. C 78 014904

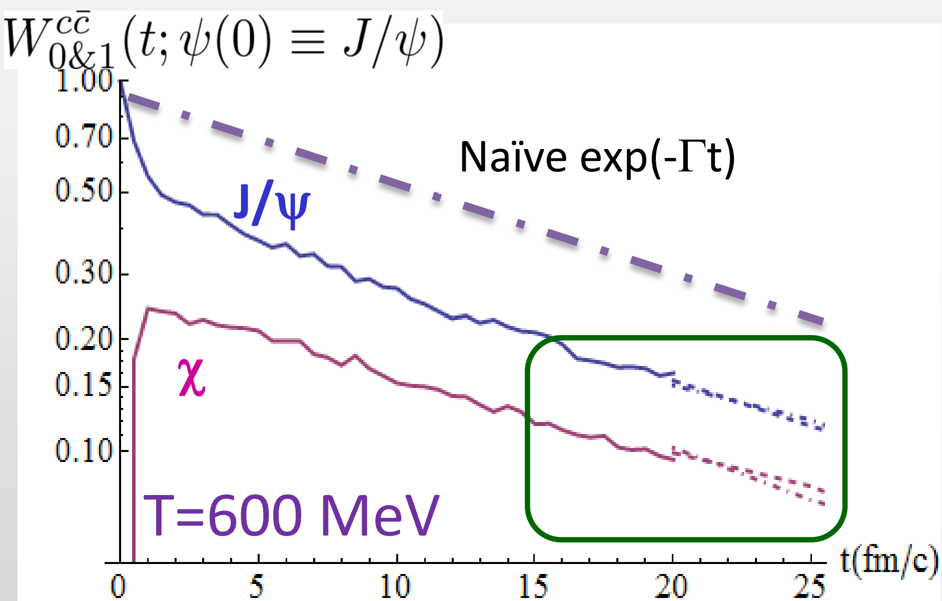
Evolutions at constant T

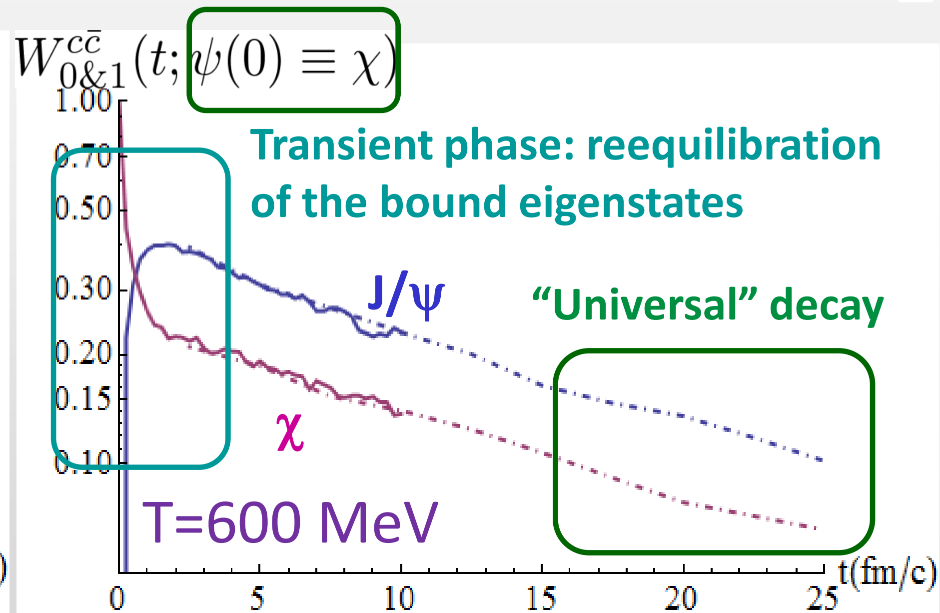
Evolution with V(T=0) + Fstocha

V(T=0) => NO Debye screening

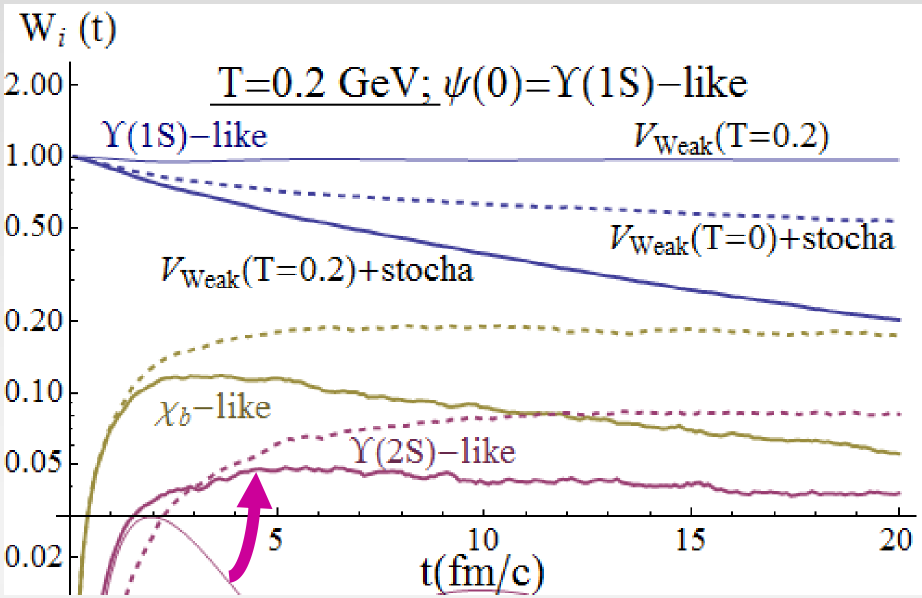
Results for charmonia (equivalent behaviour for bottomonia)

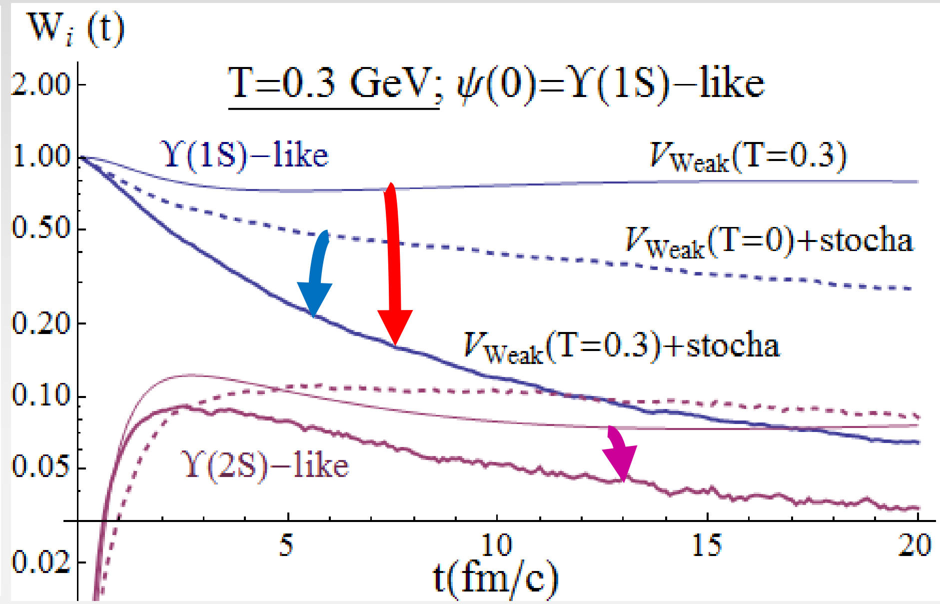




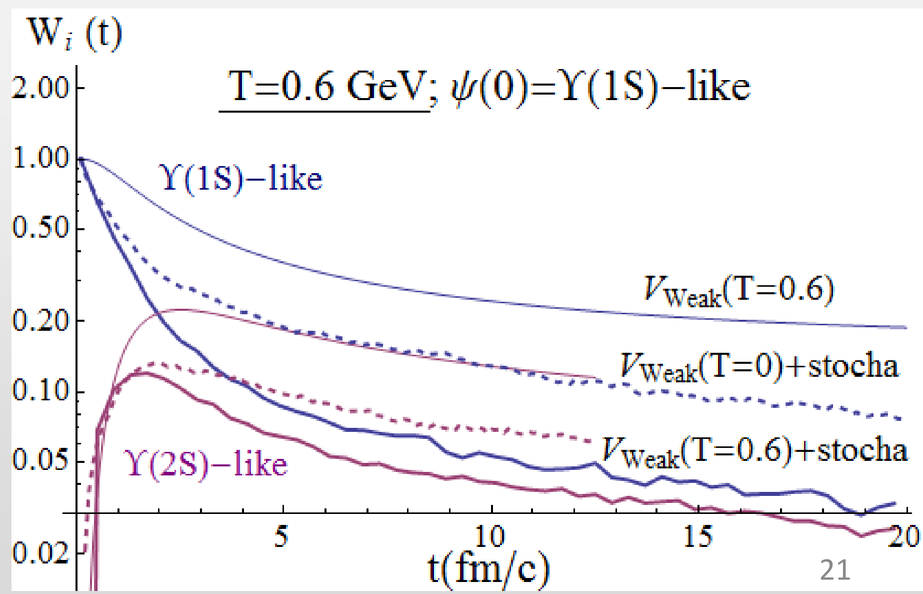


Evolutions with V(T=cst) + Fstocha

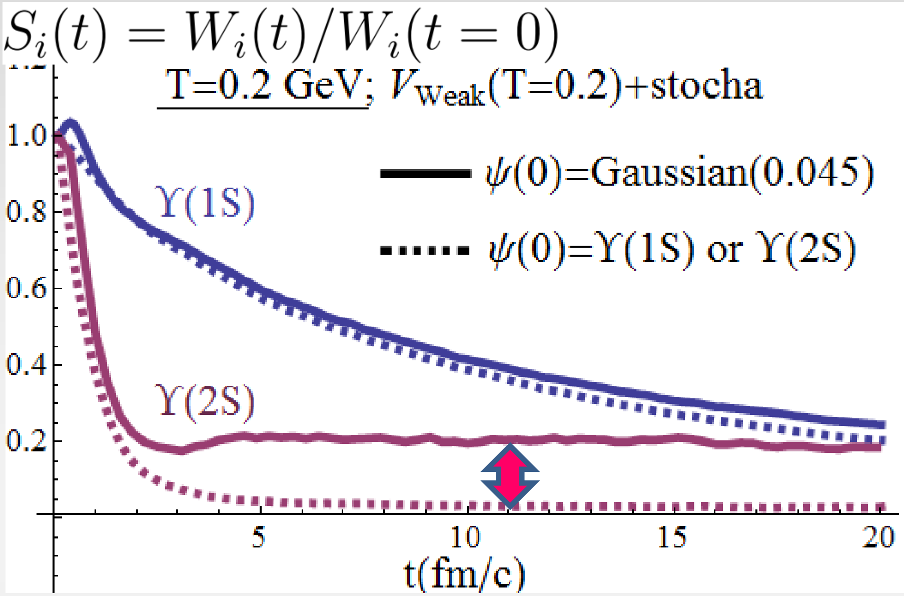


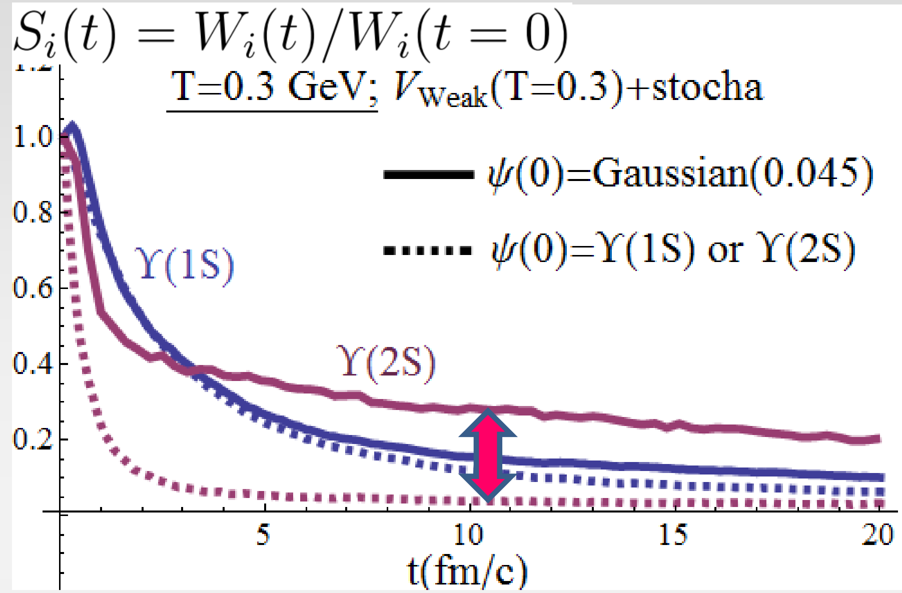


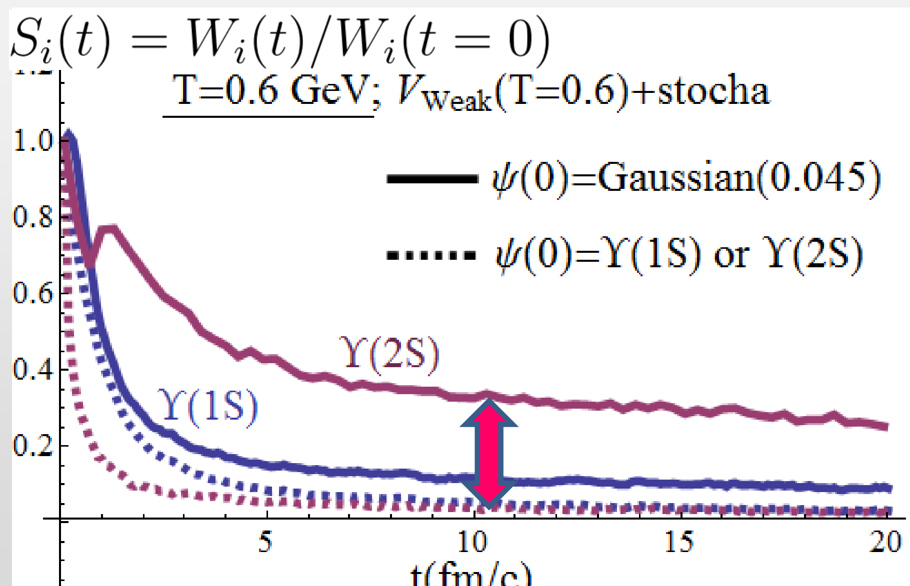
- ✓ Υ(1S): The thermal forces leads to larger suppressions
- ✓ Υ(2S): ……… for T≥0.3 only
- ✓ The screening also leads to larger suppression
- ✓ The decays increases with T



Evolutions with V(T=cst) + Fstocha





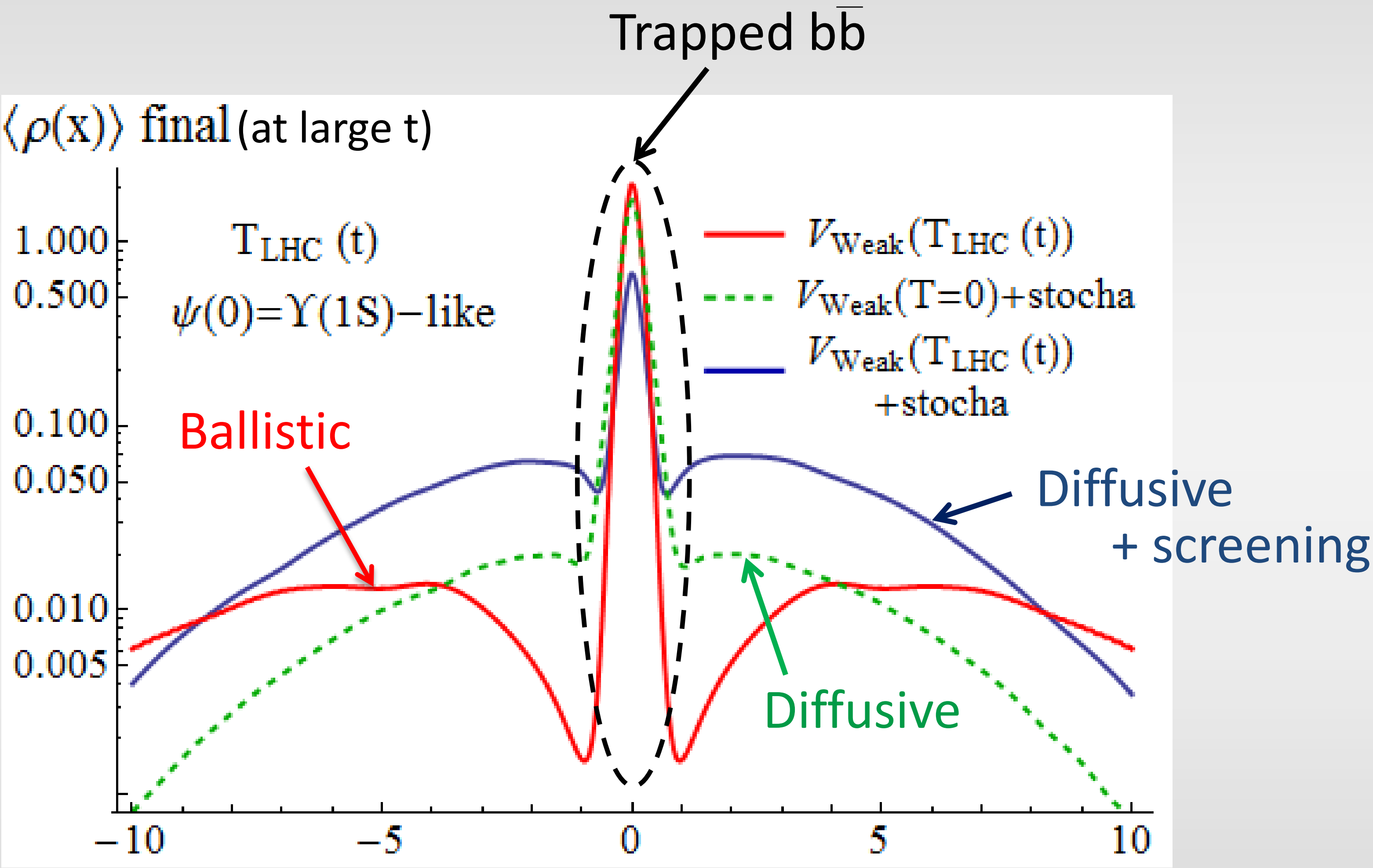


✓ S quite depends on the initial quantum state !

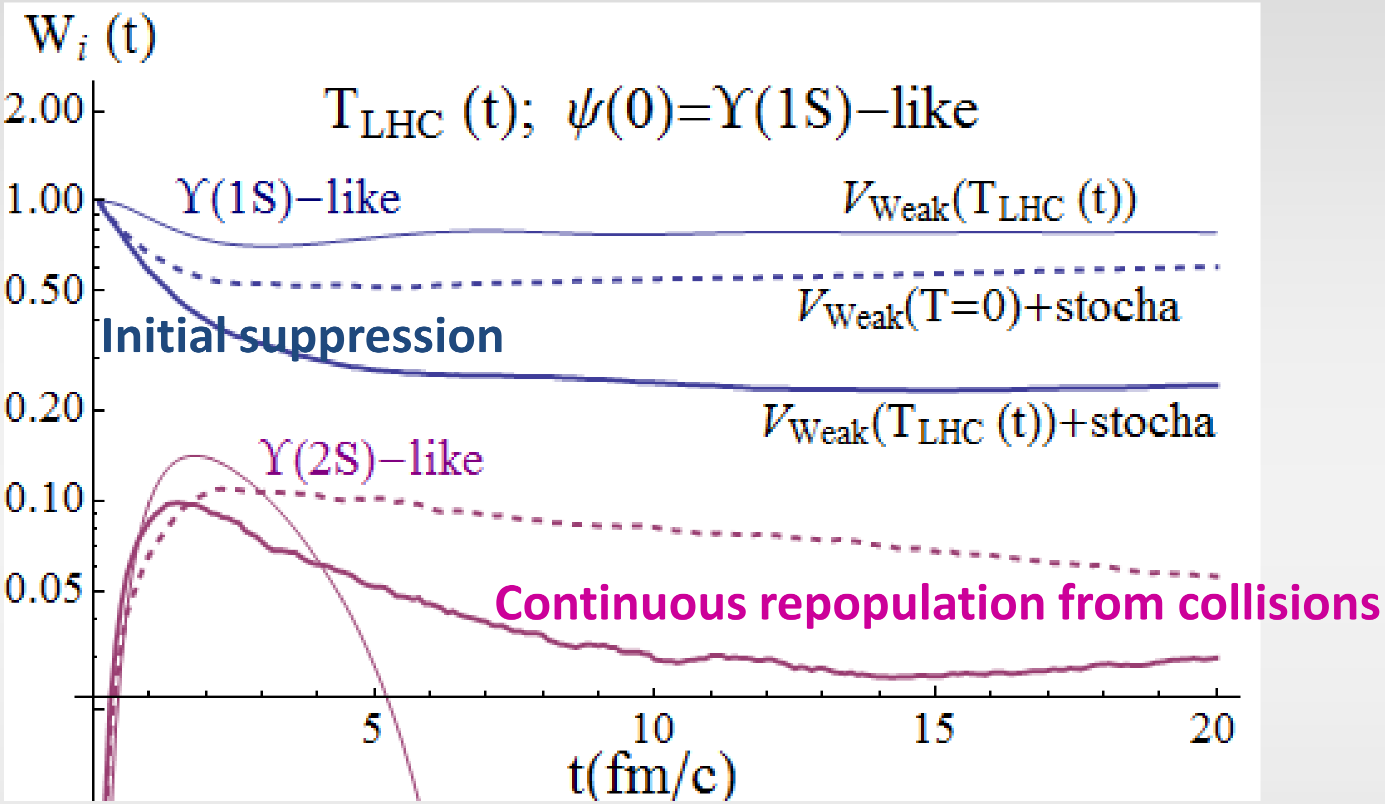
=> Kills the assumption of quantum decoherence at t=0

Evolutions with $T_{LHC}(t)$

Density with V(TLHC(t)) and initial Υ(1S)

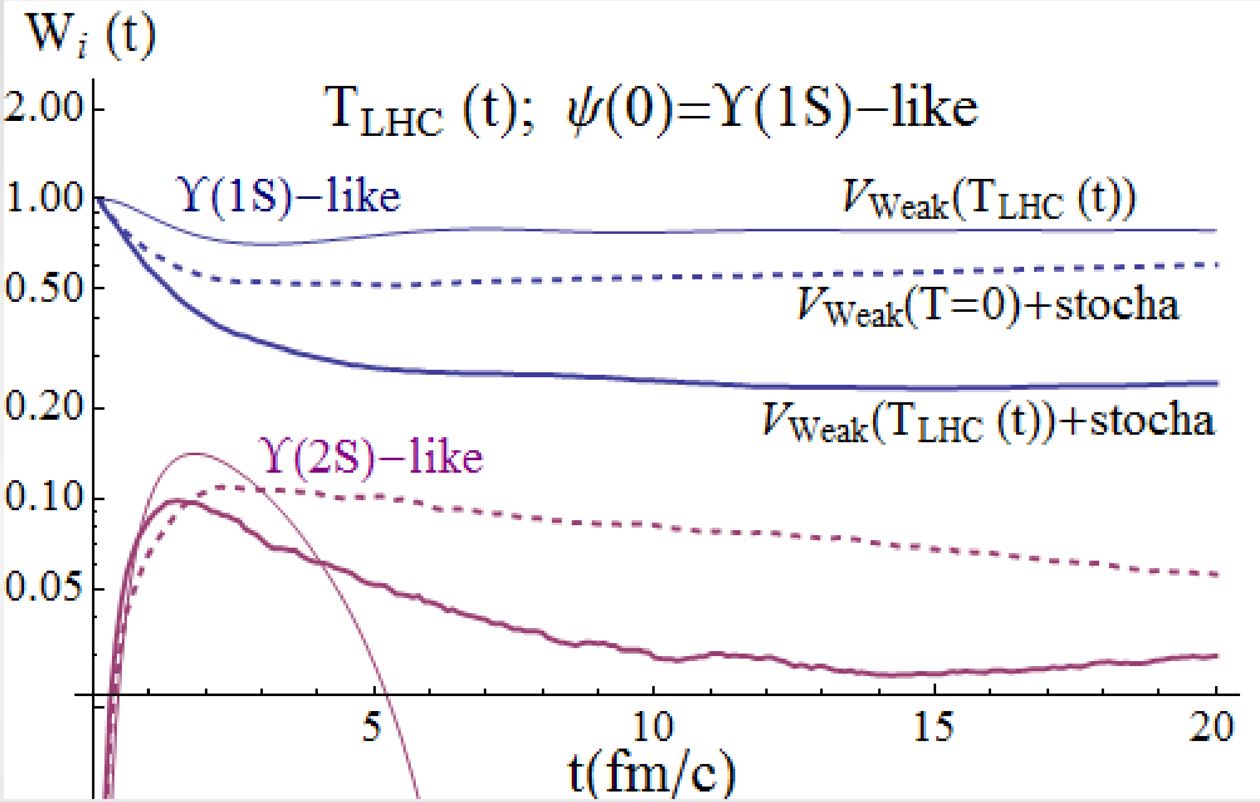


Evolutions with V(TLHC(t)) and initial Υ(1S)



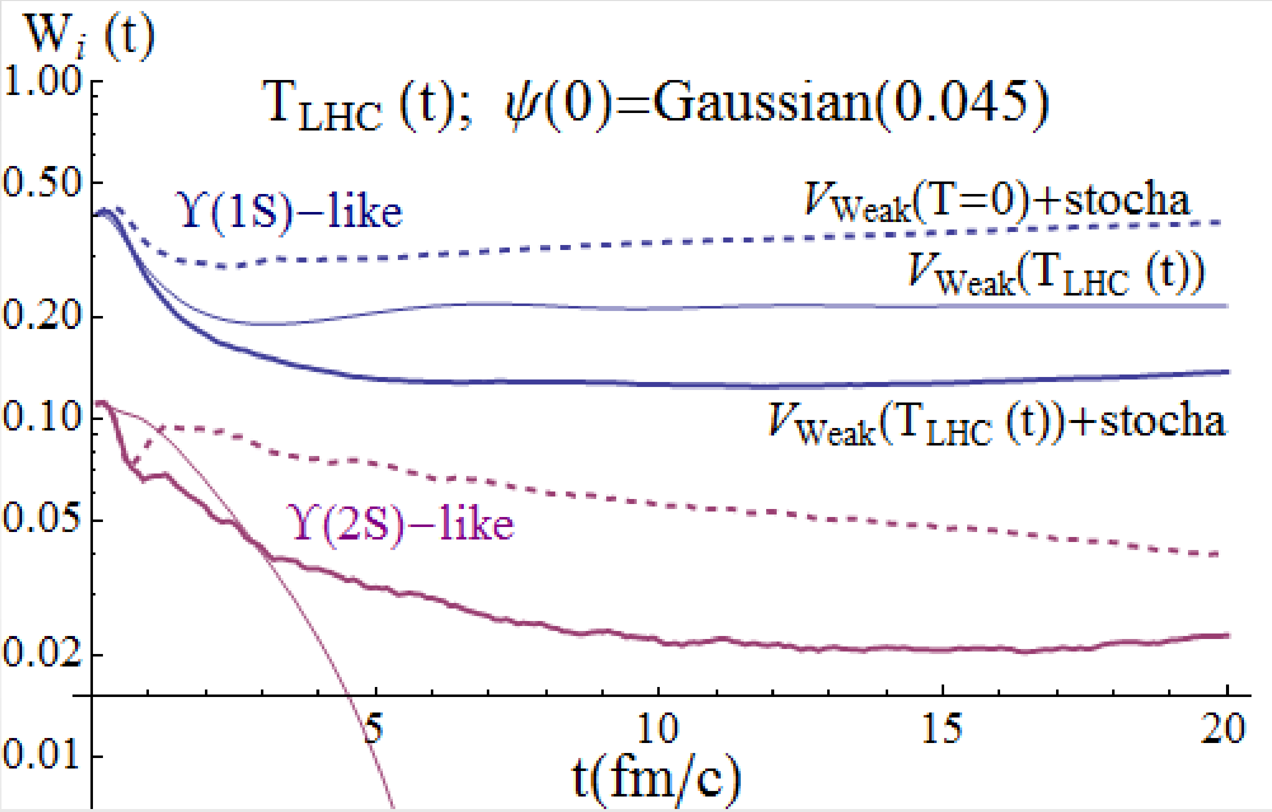
Fast suppression in MF only

Evolutions with V(TLHC(t)) and initial Y(1S)



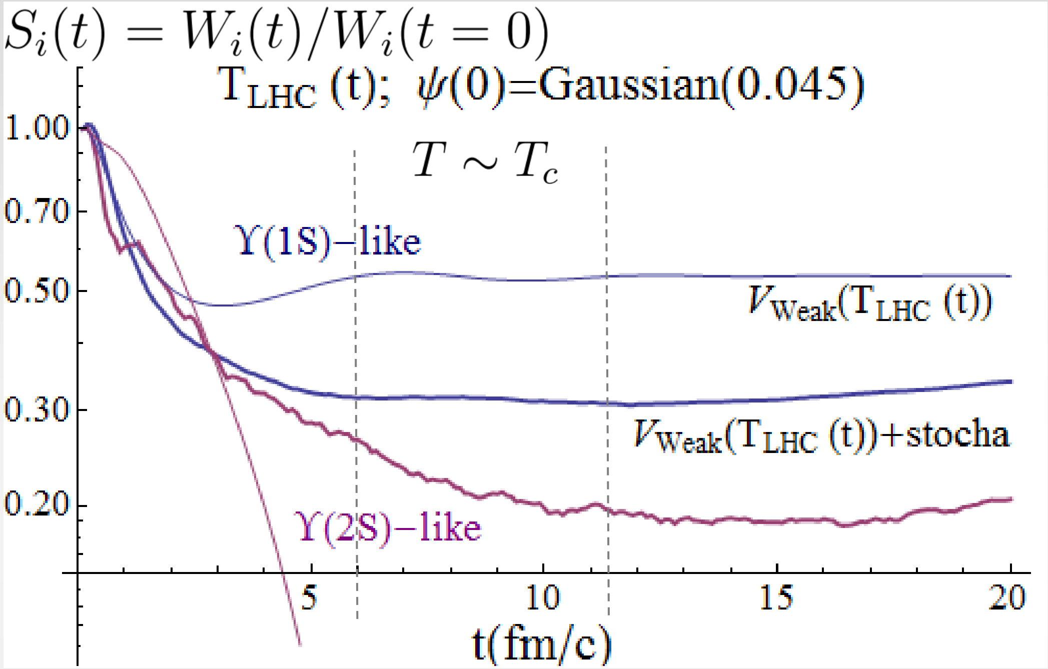
- ✓ Y(2S) strongly suppressed while Y(1S) partially survives
- ✓ Thermal forces lead to a larger suppression for Y(1S) and a smaller for Y(2S)

Evolutions with V(TLHC(t)) and initial Gaussian



✓ Initial weights have no big influence on large time weights

Results at hadronisation and data



	Y(1S)	Y(2S)
R_{AA} CMS Data*	0.31±0.05	0.075±0.04
From realistic Gaussian	~0.3	~0.2
From Y(1S)	~0.25	~0.03

Feed downs from exited states and CNM to be implemented…

* Most central suppression: CMS-PAS-HIN-15-001

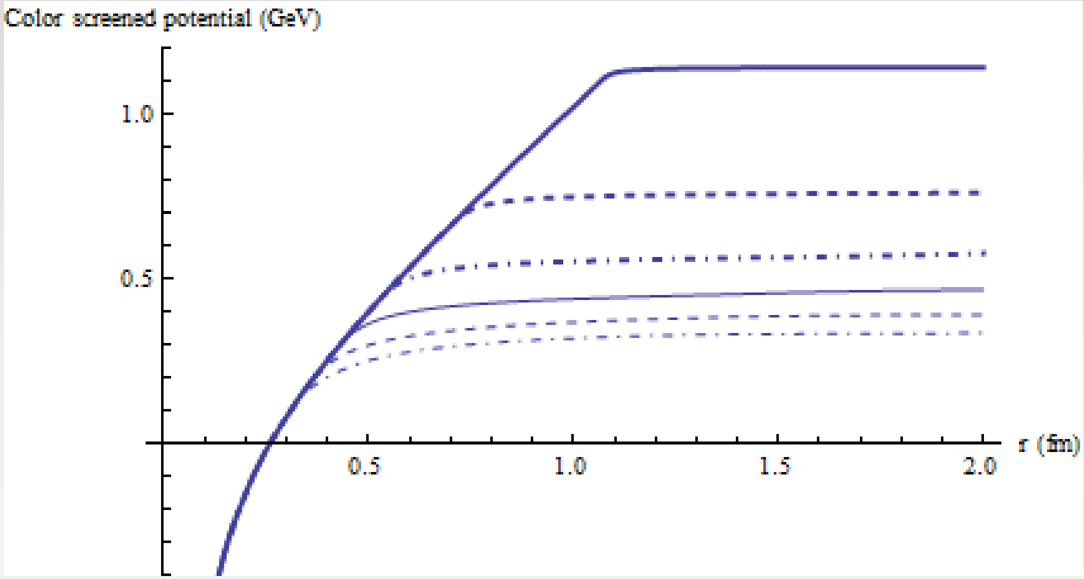
Conclusion

- Framework satisfying all the fundamental properties of quantum evolution in contact with a heat bath
- Easy to implement numerically
- First tests passed with success
- Rich suppression patterns
- Assumptions of early decoherence and adiabatic evolution ruled out.

➢ Future:
  - ❑ To be included in a more realistic collision
  - ❑ Identify the limiting cases and make contact with the other models (a possible link between statistical hadronization and dynamical models)
  - ❑ 3D ?

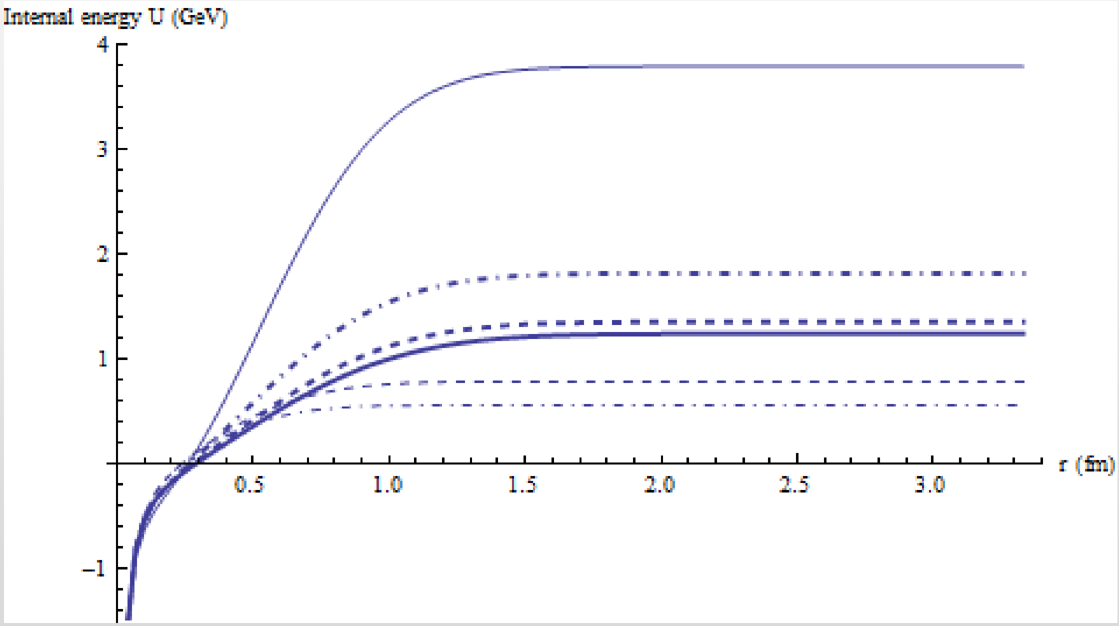
BACK UP SLIDES

Some plots of the potentials



With weak potential F<V<U with Tred from 0.4 to 1.4

With strong potential V=U with Tred from 0.4 to 1.4

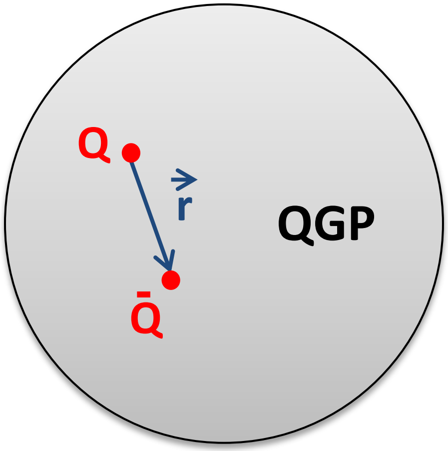


Quantum approach

- **Schrödinger equation for the $Q\bar{Q}$ pair evolution**

Where
$$\widehat{H} = 2m_q - \frac{(\hbar c)^2}{m_q}\nabla^2 + V(r, T_{\text{red}})$$

$$\Psi_{Q\bar{Q}}(\mathbf{r}, t) = R_{Q\bar{Q}}(r, t) \times \cancel{Y_{Q\bar{Q}}(\theta, \phi)}$$



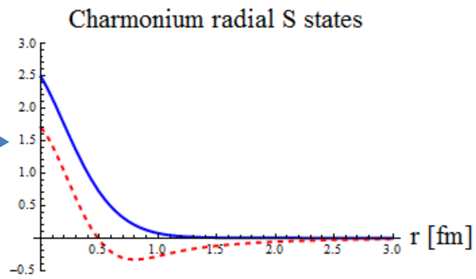
Initial wavefunction:
$$R_{Q\bar{Q}}\,(r, t=0) = \left(\frac{1}{\pi a^2}\right)^{3/4} e^{-\frac{r^2}{2a^2}}$$

where $a_{c\bar{c}} = 0.165$ fm and $a_{b\bar{b}} = 0.045$ fm

- **Projection onto the S states: the S weights**

$$W_S(t) = \left(4\pi\,\text{Abs}\left[\int_0^\infty R_{Q\bar{Q}}\,(r, t, T_{red}) \times \underline{R_S(r, T_{red}^{had})}\ r^2\,dr\right]\right)^2$$

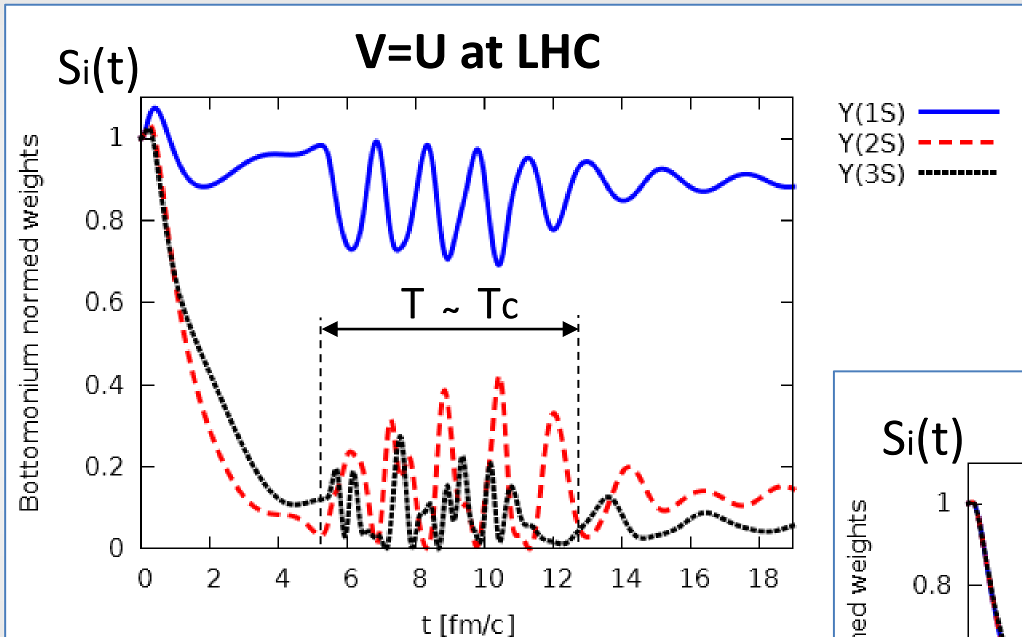
Radial eigenstates of the hamiltonian



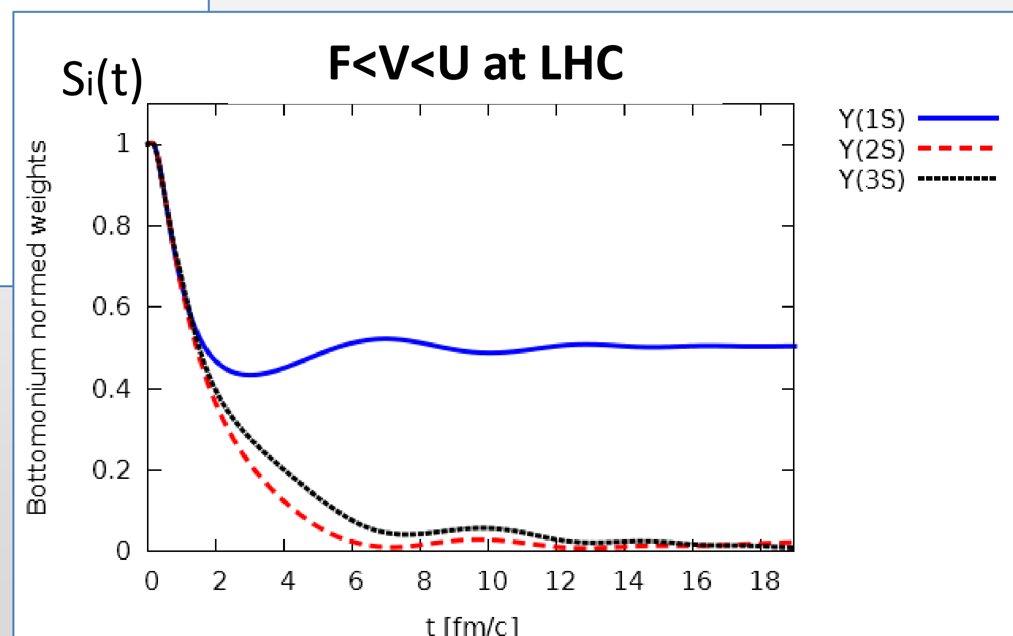
Only mean field + T(t)

« **Weight :** » $W_i(t) = |\langle \Psi_i(T=0) | \Psi_{Q\bar{Q}(t)} \rangle|^2$

« **Survivance :** » $S_i(t) = W_i(t)/W_i(t=0)$

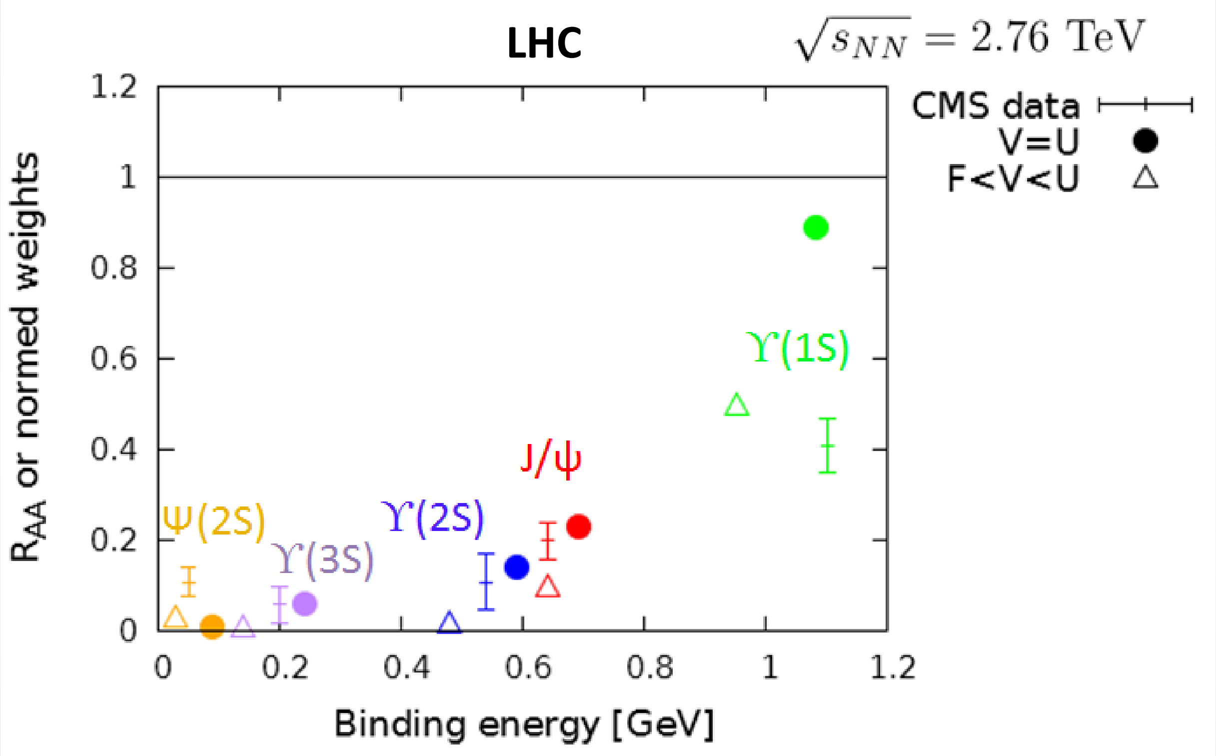


Already an actual evolution => the scenario can not be reduced to its very beginning



Only mean field + T(t)

Results (at the end of the evolution) – data comparison to some extent:



CMS data: High pT and most central data => color screening relatively more important

- **Quite encouraging for such a simple scenario !**
- **Feed downs from exited states and CNM effects to be implemented**
- **Similarly to the data, less J/ψ suppression at RHIC than at LHC.**

CMS Collaboration JHEP 05 (2012) 063 ; CMS-PAS-HIN-12-007 ; Phys. Rev. Lett. 109 (2012) 222301

Semi-classical approach

The“Quantum” **Wigner distribution of the cc pair**:

$$F\left(\vec{x},\vec{p},t\right)=\int e^{\frac{i\vec{p}\vec{y}}{\hbar}}\,\Psi^{*}\left(\vec{x}+\frac{\vec{y}}{2}\right)\Psi\left(\vec{x}-\frac{\vec{y}}{2}\right)d\vec{y}$$

… is **evolved** with the “classical”, 1st order in ħ, Wigner-Moyal equation + FP:

$$\left[\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\frac{\partial}{\partial\vec{x}}\right)-\frac{\partial}{\partial\vec{p}}\frac{\partial}{\partial\vec{x}}V\left(\vec{x}\right)\right]F\left(\vec{x},\vec{p},t\right)=\vec{\nabla}_{p}\left(Af+\vec{\nabla}_{p}(Bf)\right)$$

Finally the **projection** onto the J/ψ state is given by:

$$W_{S}(t)=\int F\left(\vec{r},\vec{p},t\right)F_{S}\left(\vec{r},\vec{p}\right)\frac{d^{3}\vec{p}d^{3}\vec{r}}{\left(\hbar c\right)^{3}}$$

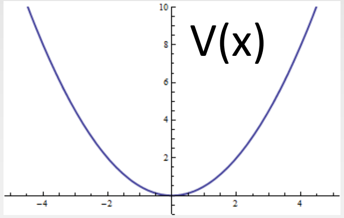
But in practice: N test particles (initially distributed with the same gaussian distribution in (r, p) as in the quantum case), that evolve with Newton’s laws, and give the J/ψ weight at t with:

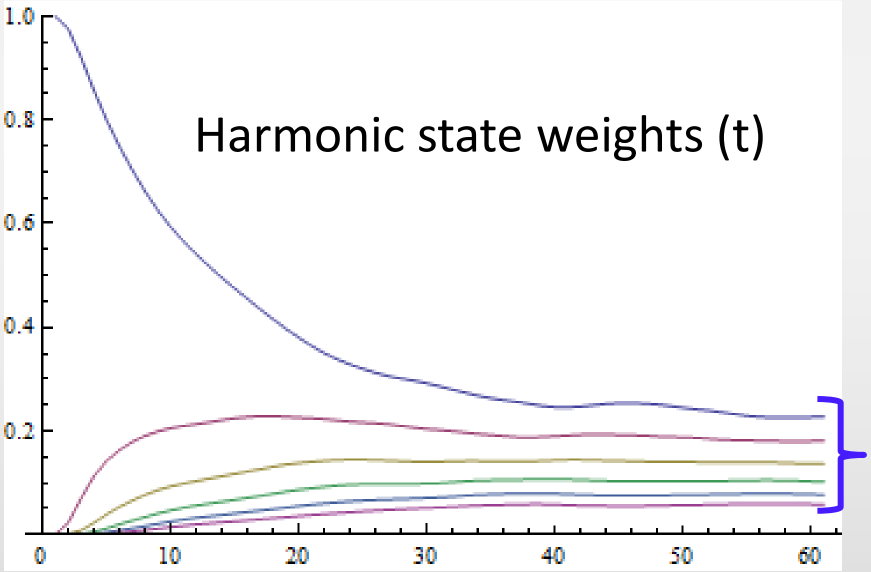
$$W_{S}(t)=\frac{1}{N}\sum_{i=1}^{N}F_{S}\left(r_{i}(t),p_{i}(t)\right)$$

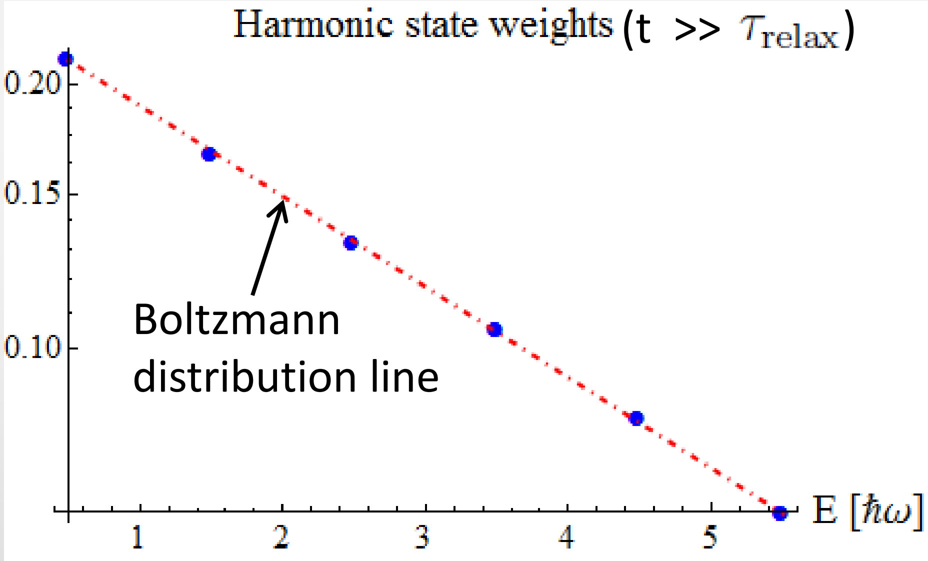
SL: numerical test of thermalisation

Harmonic potential

Asymptotic Boltzmann distributions ? YES
for any (A,B,σ) and from any initial state







Evolution with V(T=cst) and initial Gaussian