Quarkonium suppression in an anisotropic QGP

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Strangeness in Quark Matter
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Outline

- **Introduction**

- **Dissociation of Quarkonium from color screening**
  - Self energy and propagators
  - Heavy quark effective potential
  - Quarkonium Properties
- **Dissociation of Quarkonium from Landau damping**
  - Imaginary part of the Potential
  - Decay Width
  - Quarkonium Properties

- **Conclusion**

❖ **Quarkonium serve as a diagnostic tool to study the matter produced in heavy ion collisions.**

❖ **In 1986 Matsui and Satz :**

Dissociation of J/Ψ ➡ Color screening in deconfined medium ➡ QGP

❖ **Potential Model serve as a basic tool to study the properties of Quarkonium States.**

❖ **In-medium (T≠0) the potential V(r,T) between quark-antiquark is screened due to Debye Screening.**

Quarkonium dissociation: Two approaches

- **Solve Schrodinger equation with a temperature-dependent potential V (r, T)**

- **Calculate the quarkonium spectrum directly in finite temperature lattice QCD**

❖ **Effective Field Theories(EFT) (T=0): NRQCD, PNRQCD** $m_Q \gg m_Q v \gg m_Q v^2$

❖ **EFT (T ≠ 0) :** $T \gg gT \gg g^2T$

➡ **Singlet to octet breakup**

N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky PRD78 (08)

➡ **Landau damping**

Laine, O.Philipsen, P.Romatschke and M.Tassler, JHEP 03 (07)

❖Imaginary part provides a contribution to the width (Γ) of quarkonium bound states which in turn determines their dissociation temperatures.

❖ Quarkonium effectively dissociates at a lower temperature although binding energy is non-zero but overtaken by the Landau-damping induced thermal width.

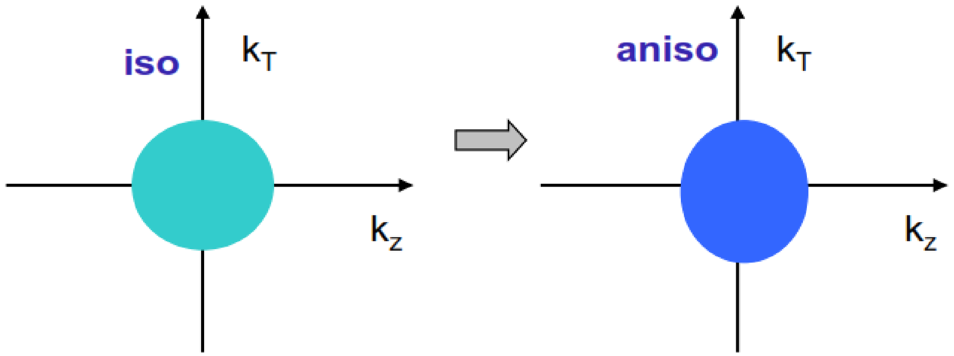
Y. Burnier, M. Laine and M. Vepsalainen JHEP 0801(08)

Anisotropic QGP

❖ At the early stage of ultrarelativistic heavy ion collisions at RHIC or LHC, the generated parton system has an anisotropic distribution.

$$f_{aniso}(\vec{k}) = f_{iso}\left(\sqrt{k^2 + \xi(\vec{k}.\hat{n})^2}\right), \qquad \xi = \frac{1}{2}\frac{\langle k_\perp^2 \rangle}{\langle k_z^2 \rangle} - 1$$

❖ The parton momentum distribution is strongly elongated along the beam direction.



❖ By time quarkonia are formed in plasma rest frame, system may not be then highly anisotropic rather closer to isotropic distribution

Dissociation of Quarkonium from color screening

In linear response approximation, retarded gluon self energy in Anisotropic medium

$$\Pi^{ij}(P) = m_D^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v.n})n^l}{(1+\xi(\mathbf{v.n})^2)^2}\left(\delta^{jl} + \frac{v^j p^l}{P\cdot V + i\epsilon}\right)$$

P. Romatschke, M. Strickland, PRD68(2003)

Gluon propagator

$$\lim_{\omega\to 0}\Delta_{ij}(\omega,k) = -\frac{\left(k^2+m_\alpha^2+m_\gamma^2\right)\, k_i k_j}{\omega^2\left[(k^2+m_\alpha^2+m_\gamma^2)(k^2+m_\beta^2)-m_\delta^4\right]}$$

In the static limit

$$m_\alpha^2 = \lim_{\omega\to 0}\alpha,\;\; \mathrm{m}_\beta^2 = \lim_{\omega\to 0} -\frac{k^2}{\omega^2}\beta,\;\; m_\gamma^2 = \lim_{\omega\to 0}\gamma,\;\; \mathrm{m}_\delta^2 = \lim_{\omega\to 0}\frac{\tilde{n}k^2}{\omega}\,\mathrm{Im}\,\delta$$

Dielectric permittivity for anisotropic medium

$$\epsilon^{-1}(k) = -\lim_{\omega\to 0}\omega^2\frac{k_i k_j}{k^2}\Delta_{ij}(\omega,k) = \frac{k^2(k^2+m_\alpha^2+m_\gamma^2)}{(k^2+m_+^2)(k^2+m_-^2)}$$

In the small ξ limit

$$m_+^2 = \left(1+\frac{\xi}{6}(3\cos 2\beta_n - 1\right)m_D^2 \qquad m_-^2 = -\frac{\xi}{3}\cos 2\beta_n m_D^2$$

Heavy quark effective potential

➢ By correcting full Cornell potential with a dielectric function ε(k) encoding the effects of the deconfined medium.

➢ Fourier transform of potential at vanishing frequency gives the desired non-relativistic potential at finite temperature

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$$V(\mathbf{r},\xi,T) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{V(k)}{\epsilon(k)}$$

where $V(k) = -\sqrt{(2/\pi)}\frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}$

$$= -\frac{\alpha}{2\pi^2}\int \frac{d^3\mathbf{k}\ \mathbf{e}^{\mathbf{ik\cdot r}}}{k^2 + m_D^2\left(1+\frac{\xi}{6}(3\cos 2\beta_n - 1)\right)}$$

$$-\frac{4\sigma}{(2\pi)^2}\int \frac{d^3\mathbf{k}\ e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2\left[k^2 + m_D^2\left(1+\frac{\xi}{6}(3\cos 2\beta_n - 1)\right)\right]}$$

⇒ Unlike in an isotropic medium, an extra angular dependence appears in the potential

➢ **Two cases for illustration:**

- r is parallel to the direction of anisotropy, n
- r is perpendicular to the direction of anisotropy, n

Parallel Alignment

Medium-modified potential for quark pairs along the direction of anisotropy:

$$V(\vec{r} \,||\, \hat{n}, \xi, T) = \left(\frac{2\sigma}{m_D} - \alpha m_D \right) \frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D$$

$$+ \xi \left[\frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left[2\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{e^{\hat{r}} + 2}{3} - \frac{2}{\hat{r}} - \frac{\hat{r}}{12} \right] - \frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} \left[2\frac{(e^{\hat{r}} - 1)}{\hat{r}^2} - \frac{2}{\hat{r}} - \frac{\hat{r}}{6} - 1 \right] \right]$$

short distance limit ($r m_D \ll 1$):

$$-\frac{\alpha}{r} + \sigma r$$

QQbar is not affected by the medium in anisotropic case too

long distance (screening) limit ($r m_D \gg 1$):

$$-\frac{2\sigma}{m_D \hat{r}} \left(1 + \frac{4}{3}\xi \right) - \alpha m_D$$

The potential in anisotropic medium is screened less than in isotropic medium and is stronger than in isotropic medium

Potential has a long range Coulombic tail, in addition to the standard Yukawa term, like isotropic medium

Perpendicular Alignment

Medium-modification for Perpendicular alignment gives

$$V(\vec{r}\perp\hat{n},\xi,T)=\left(\frac{2\sigma}{m_D}-\alpha m_D\right)\frac{e^{-\hat{r}}}{\hat{r}}-\frac{2\sigma}{m_D\hat{r}}+\frac{2\sigma}{m_D}-\alpha m_D$$

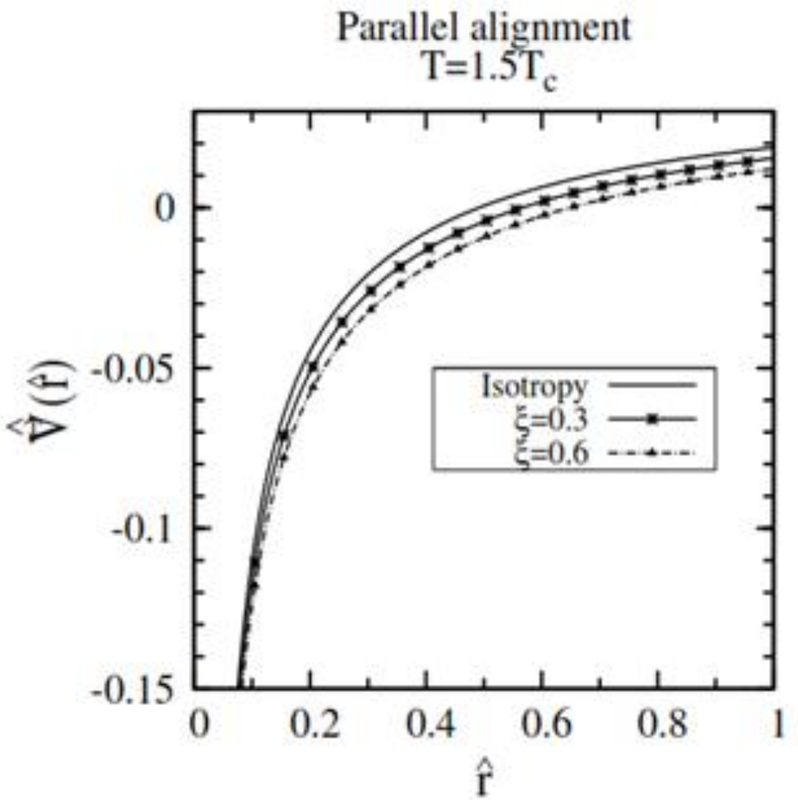
$$-\xi\left[\begin{array}{l}\frac{4\sigma}{m_D\hat{r}}e^{-\hat{r}}\left[\frac{(e^{\hat{r}}-1)}{\hat{r}^2}+\frac{(e^{\hat{r}}-7)}{12}-\frac{1}{\hat{r}}-\frac{\hat{r}}{6}\right]\\ -\frac{\alpha m_D}{\hat{r}}e^{-\hat{r}}\left[\frac{(e^{\hat{r}}-1)}{\hat{r}^2}-\frac{1}{\hat{r}}-\frac{\hat{r}}{3}-\frac{1}{2}\right]\end{array}\right]$$

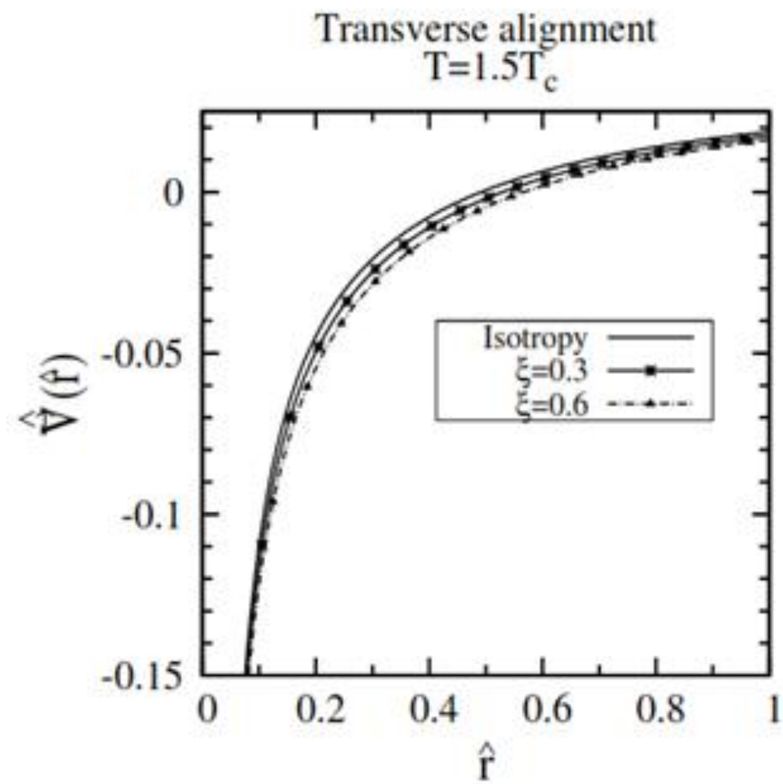
$rm_D \ll 1$: $-\frac{\alpha}{r}+\sigma r$

$rm_D \gg 1$: $-\frac{2\sigma}{m_D\hat{r}}\left(1+\frac{\xi}{6}\right)-\alpha m_D$

➢ The quark pairs along the direction of anisotropy feel more attraction than the transverse alignment because the inter-quark potential along the direction of anisotropy is screened smaller

Effect of anisotropy on the potential





- **Potential in anisotropic medium is always screened less than isotropic medium.**

Complete angular dependence of the potential

$$V(r,\theta_n,T) = V(r,T) + V_{\text{tensor}}(r,\theta_n,T)$$

$$= \left(\frac{2\sigma}{m_D} - \alpha m_D\right)\frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D\hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D \quad \Big\} \; V(r,T)$$

$V_{\text{tensor}}(r,\theta_n,T)$:

$$+\xi\left(\frac{2\sigma}{m_D}\frac{e^{-\hat{r}}}{\hat{r}}\left[\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{5e^{\hat{r}}}{12} - \frac{1}{\hat{r}} + \frac{\hat{r}}{12} - \frac{1}{12}\right]\right.$$

$$- \frac{\alpha m_D}{2}\frac{e^{-\hat{r}}}{\hat{r}}\left[\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{1}{\hat{r}} + \frac{\hat{r}}{6} - \frac{1}{2}\right]$$

$$+\left(\frac{2\sigma}{m_D}\frac{e^{-\hat{r}}}{\hat{r}}\left[3\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{e^{\hat{r}}}{4} - \frac{3}{\hat{r}} - \frac{\hat{r}}{4} - \frac{5}{4}\right]\right.$$

$$\left.\left. - \frac{\alpha m_D}{2}\frac{e^{-\hat{r}}}{\hat{r}}\left[3\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{3}{\hat{r}} - \frac{\hat{r}}{2} - \frac{3}{2}\right]\right)\cos\ 2\theta_n\right)$$

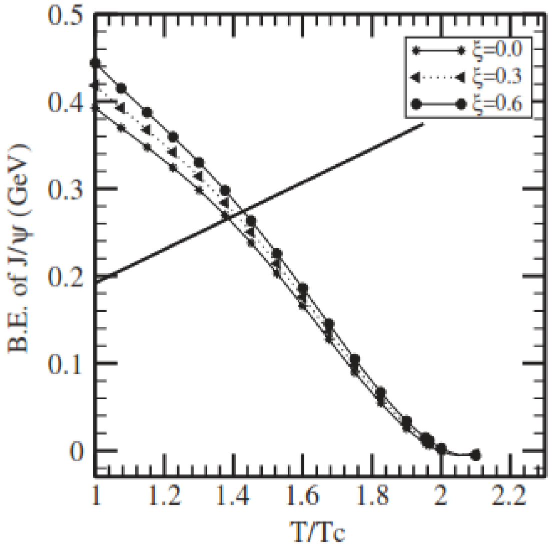
$rm_D \ll 1$:

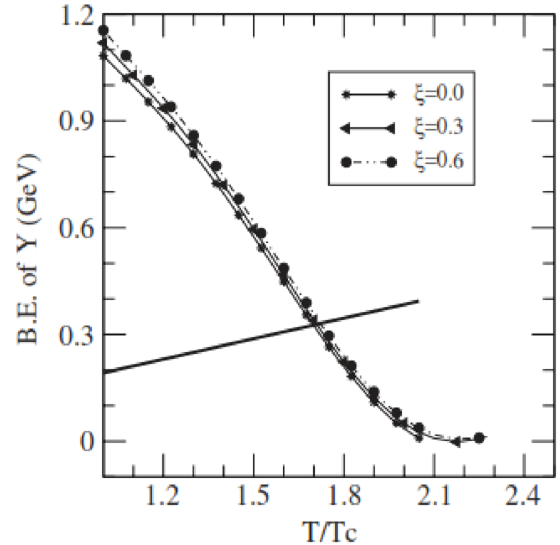
$$-\frac{\alpha}{r} + \sigma r$$

$rm_D \gg 1$:

$$-\frac{2\sigma}{m_D^2 r} - \alpha m_D - \frac{5\xi}{12}\frac{2\sigma}{m_D^2 r}\left(1 + \frac{3}{5}\cos 2\theta_n\right)$$

Variation of Binding energy with temperature in anisotropic medium





Dissociation Criteria for heavy Quarkonia

❖When the binding energy of resonance state drops **below mean thermal energy of parton**, state becomes feebly bound because **thermal fluctuation** can easily dissociate them.

❖ **Resonances have been broadened due to direct thermal activation**, so the dissociatoin of bound state is expected to occur around $E_{bin} \approx T$

$$|E_n| = \frac{E_I}{n^2} = T_D$$

State	$\xi = 0.0$	$\xi = 0.3$	$\xi = 0.6$
J/ψ	1.38	1.41	1.43
Υ	1.70	1.71	1.72

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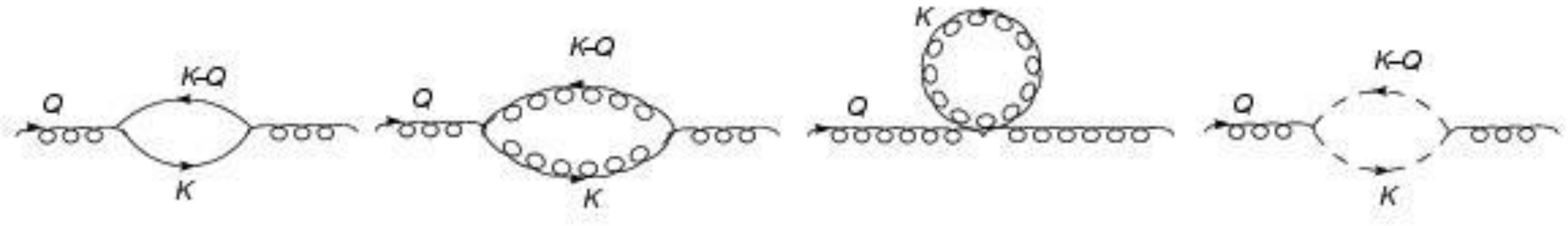
Dissociation of Quarkonium from Landau Damping

Gluon self-energy from diagrammatic approach



Retarded and advanced self energies

$$\Pi^L_{R,A(iso)}(P) = m_D^2 \left(\frac{p_0}{2p} \ln \frac{p_0 + p \pm i\epsilon}{p_0 - p \pm i\epsilon} - 1 \right)$$

$$\Pi^L_{R,A(aniso)}(P) = \frac{m_D^2}{6} \left(1 + \frac{3}{2} \cos 2\theta_p \right) + \Pi^L_{R(iso)}(P) \left(\cos(2\theta_p) - \frac{{p_0}^2}{2p^2} (1 + 3 \cos 2\theta_p) \right)$$

Symmetric self energies

$$\Pi^L_{F(iso)}(P) = -2\pi i m_D^2 \frac{T}{p} \Theta(p^2 - {p_0}^2) \;,$$

$$\Pi^L_{F(aniso)}(P) = \frac{3}{2} \pi i m_D^2 \frac{T}{p} \left(\sin^2\theta_p + \frac{p_0^2}{p^2} (3\cos^2\theta_p - 1) \right) \Theta(p^2 - {p_0}^2)$$

A. Dumitru Y. Guo and M. Strickland, PRD79 (2009)114003

Retarded and advanced self energy contribute to the the real part and symmetric part contribute to the imaginary part of the potential.

Propagators

$$\Re D^{00}_{R,A}(0,p) = -\frac{1}{(p^2+m_D^2)} + \xi \frac{m_D^2}{6(p^2+m_D^2)^2}\left(3\cos 2\theta_p - 1\right)$$

$$\Im D^{00}_{F}(0,p) = \frac{-2\pi T m_D^2}{p(p^2+m_D^2)^2} + \xi \left(\frac{3\pi T m_D^2}{2p(p^2+m_D^2)^2} \sin^2\theta_p - \frac{4\pi T m_D^4}{p(p^2+m_D^2)^3}\left(\sin^2\theta_p - \frac{1}{3}\right)\right)$$

❖ **Relation between dielectric permittivity and propagator**

$$\epsilon^{-1}(p) = -\lim_{\omega\to 0} p^2 D^{00}_{11}(\omega,p)$$

$$\Re D^{00}_{11}(\omega,p) = \frac{1}{2}\left(D^{00}_{R} + D^{00}_{A}\right)$$ and $$\Im D^{00}_{11}(\omega,p) = \frac{1}{2} D^{00}_{F}$$

Retarded and advanced self energy contribute to the the real part and symmetric part contribute to the imaginary part of the potential.

Propagators

$$\Re D^{00}_{R,A}(0,p) = -\frac{1}{(p^2+m_D^2)} + \xi \frac{m_D^2}{6(p^2+m_D^2)^2}(3\cos 2\theta_p - 1)$$

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and

$$\Im D^{00}_{11}(\omega,p) = \frac{1}{2} D^{00}_F$$

Real part of Potential

$$
\begin{aligned}
\Re V_{aniso}(r,\theta_n,T) &= \left(\frac{2\sigma}{m_D} - \alpha m_D\right)\frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D\hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D \\
&+ \xi\left(\frac{2\sigma}{m_D}\frac{e^{-\hat{r}}}{\hat{r}}\left[\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{5e^{\hat{r}}}{12} + \frac{\hat{r}e^{\hat{r}}}{3} - \frac{1}{\hat{r}} + \frac{\hat{r}}{12} - \frac{1}{12}\right]\right. \\
&- \frac{\alpha m_D}{2}\frac{e^{-\hat{r}}}{\hat{r}}\left[\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{1}{\hat{r}} - \frac{\hat{r}e^{\hat{r}}}{3} + \frac{\hat{r}}{6} - \frac{1}{2}\right] \\
&+ \left(\frac{2\sigma}{m_D}\frac{e^{-\hat{r}}}{\hat{r}}\left[3\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{e^{\hat{r}}}{4} - \frac{3}{\hat{r}} - \frac{\hat{r}}{4} - \frac{5}{4}\right]\right. \\
&- \left.\left.\frac{\alpha m_D}{2}\frac{e^{-\hat{r}}}{\hat{r}}\left[3\frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{3}{\hat{r}} - \frac{\hat{r}}{2} - \frac{3}{2}\right]\right)\cos\ 2\theta_r\right)
\end{aligned}
$$

Imaginary Part of Potential

Lata Thakur U.Kakade, B.K.Patra ,PRD 89 (2014) 094020

$$
\begin{aligned}
ImV_{(\mathrm{aniso})}(\mathbf{r},\xi,T) &= -\int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\left(-\sqrt{\frac{2}{\pi}}\frac{\alpha}{p^2}-\frac{4\sigma}{\sqrt{2\pi}p^4}\right)p^2\left[\frac{-\pi T m_D^2}{p(p^2+m_D^2)^2}\right. \\
&\quad +\xi\left[\frac{3\pi T m_D^2}{2p(p^2+m_D^2)^2}\sin^2\theta_p-\frac{4\pi T m_D^4}{p(p^2+m_D^2)^3}(\sin^2\theta_p-\frac{1}{3})\right]\Bigg] \\
&\equiv ImV_{1(aniso)}(\mathbf{r},\xi,T)+ImV_{2(aniso)}(\mathbf{r},\xi,T)\ ,
\end{aligned}
$$

$$
ImV_{1(aniso)}(\mathbf{r},\xi,T) \equiv \alpha T\xi\left[\phi_1(\hat{r},\theta_r)+\phi_2(\hat{r},\theta_r)\right]
$$

$$
\begin{aligned}
\phi_1(\hat{r},\theta_r) &= \frac{\hat{r}^2}{600}\left[123-90\gamma_E-90\log\hat{r}+\cos(2\theta_r)\left(-31+30\gamma_E+30\log\hat{r}\right)\right] \\
\phi_2(\hat{r},\theta_r) &= \frac{\hat{r}^2}{90}(-4+3\cos(2\theta_r))
\end{aligned}
$$

A. Dumitru Y. Guo and M. Strickland, PRD79 (2009)114003

$$
ImV_{2(aniso)}(r,\xi,T) = -\xi\,\frac{2\sigma T}{m_D^2}\left[\psi_1(\hat{r},\theta_r)+\psi_2(\hat{r},\theta_r)\right]
$$

where

$$\psi_1(\hat{r},\theta_r) = \frac{r^2}{10} + \frac{(-739 + 420\gamma_E + 420\log(\hat{r}))\hat{r}^4}{39200} + \left(-\frac{\hat{r}^2}{20} + \frac{(176 - 105\gamma_E - 105\log(\hat{r}))\hat{r}^4}{14700}\right)\cos^2\theta_r$$

$$\psi_2(\hat{r},\theta_r) = -\frac{4}{3}\left[\frac{7\hat{r}^2}{120} - \frac{11\hat{r}^4}{3360} + O(\hat{r}^5)\right] - 4\left[-\frac{\hat{r}^2}{60} + \frac{\hat{r}^4}{840} + O(\hat{r}^5)\right]\cos^2\theta_r\ ,$$

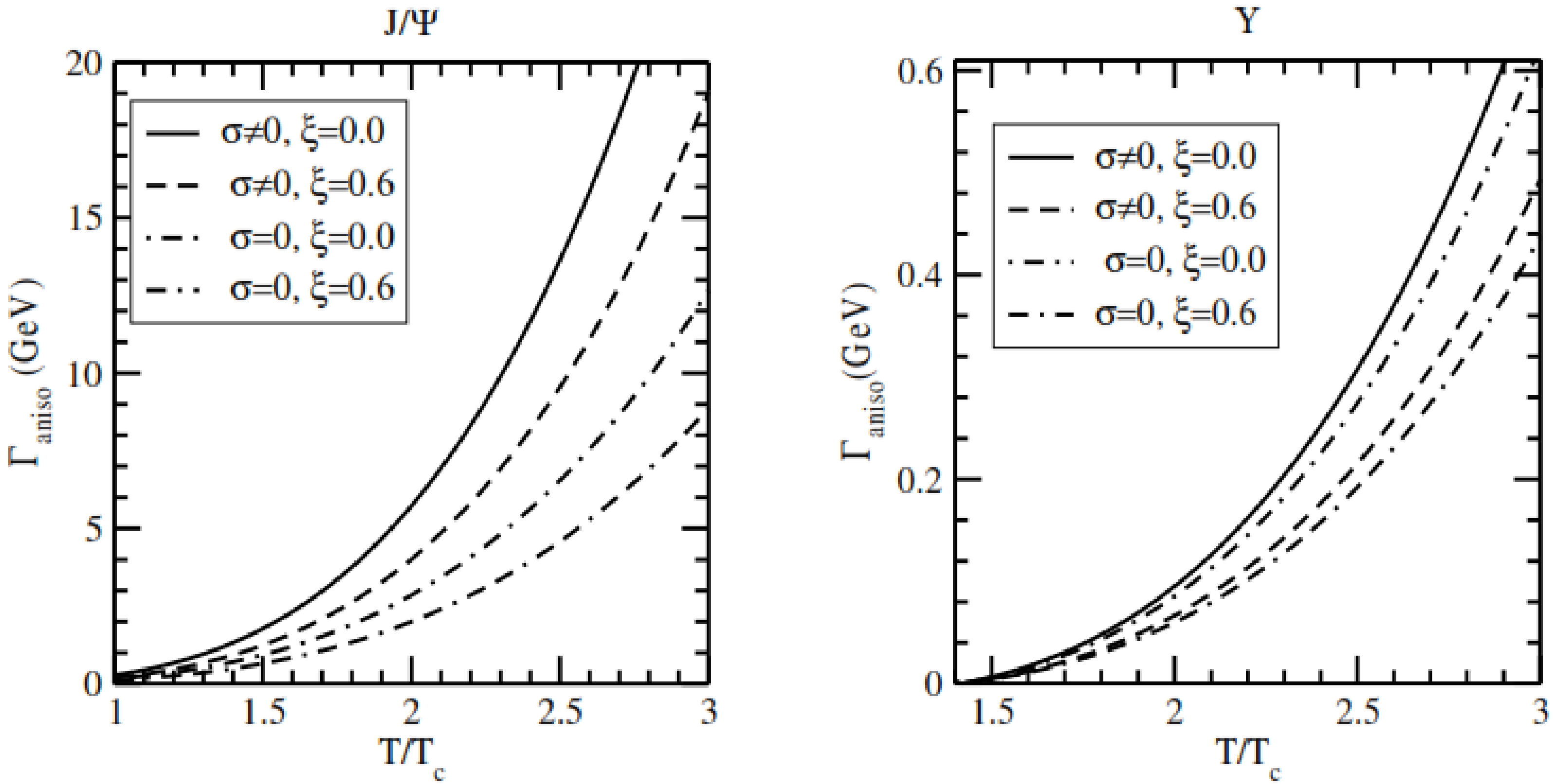
Imaginary potential at the leading logarithmic order, when $rm_D \ll 1$

$$ImV_{(\text{aniso})}(r,\theta_r,T) = -T\left(\frac{\alpha\hat{r}^2}{3} + \frac{\sigma\hat{r}^4}{30m_D^2}\right)\log\left(\frac{1}{\hat{r}}\right) + \xi T\left[\left(\frac{\alpha\hat{r}^2}{5} + \frac{3\sigma\hat{r}^4}{140m_D^2}\right) - \cos^2\theta_r\left(\frac{\alpha\hat{r}^2}{10} + \frac{\sigma\hat{r}^4}{70m_D^2}\right)\right]\log\left(\frac{1}{\hat{r}}\right)$$

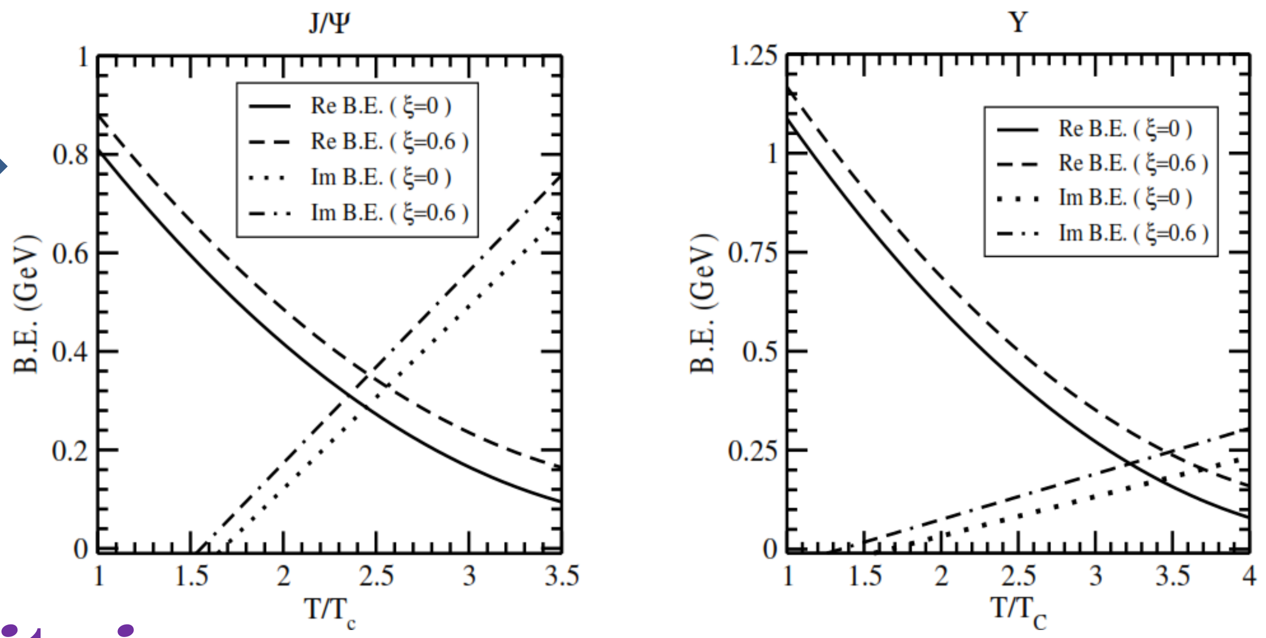
❖ **Decay width** Γ

$$\Gamma_{(\text{aniso})} = \left(\frac{4T}{\alpha m_Q^2} + \frac{12\sigma T}{\alpha^2 m_Q^4}\right)\left(1 - \frac{\xi}{2}\right)m_D^2\log\frac{\alpha m_Q}{2m_D}$$

Variation of Decay width with temperature for different value of anisotropy



Real and Imaginary Binding Energies ⇨



Dissociation Criteria

Dissociation temperature in thermal medium can be calculated in two ways:

1. **From the intersection of the real and imaginary binding energies [M. Strickland NPA 879(2012) 25].**
2. **From the conservative criterion on the width of the resonance as: Γ = 2Re B.E.[A. Mocsy PRL 99 (2007) 211602].**

Method	State	$\xi = 0.0$	$\xi = 0.3$	$\xi = 0.6$
Re B.E.=Im B.E.	J/ψ	1.33	1.34	1.35
	Υ	1.91	1.93	1.94
Γ=2B.E.	J/ψ	1.02	1.06	1.12
	Υ	1.88	1.92	2.02

Conclusion

- **Momemtum anisotropy introduces a characteristic angular dependence in the potential .**
- **Potential in anisotropic medium is always screened less than isotropic medium.**
- **Quarkonia states in anisotropic medium are always more bound than in isotropic medium , so survives much higher temperature.**
- **Inclusion of string term makes the width larger and thus the damping of the exchanged gluon in the heat bath provides larger contribution to the dissociation rate.**
- **The effect of nonperturbative term on the width is relatively more on J/ψ than Υ state because the binding of Υ (1S) state is more Coulombic than J/ψ (1S) state.**
- **Width in anisotropic medium becomes smaller than isotropic medium and gets narrower with the increase of anisotropy,**
- **Quarkonium states are dissociated at higher temperature as compared to the Coulomb term alone.**

Thank You