Dipole toroidal resonance: vortical properties, relation to pygmy mode, anomalous deformation features

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W. Kleinig
JINR, Dubna. Moscow region, Russia

46-я сессия ПКК по ядерной физике, 14.06.2017, ОИЯИ, Дубна

To discuss:

- Exotic E1 resonances:
  - toroidal (TDR),
  - compression (CDR)
  - pygmy (PDR)

J. Kvasil, V.O.N., W. Kleinig, P.-G. Reinhard, P.. Vesely,
PRC 84, 034303 (2011).

- ~~TDR is the most accurate measure of the nuclear vorticity~~

P.-G. Reinhard, V.O.N, A. Repko, and J. Kvasil,
PRC 89, 024321 (2014).

- PDR is a local manifestation of TDR

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,
PRC 87, 024305 (2013).

- Anomalous deformation splitting of TDR as its fingerprint

J. Kvasil, V.O.N, W. Kleinig, and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014).

- Experimental perspectives

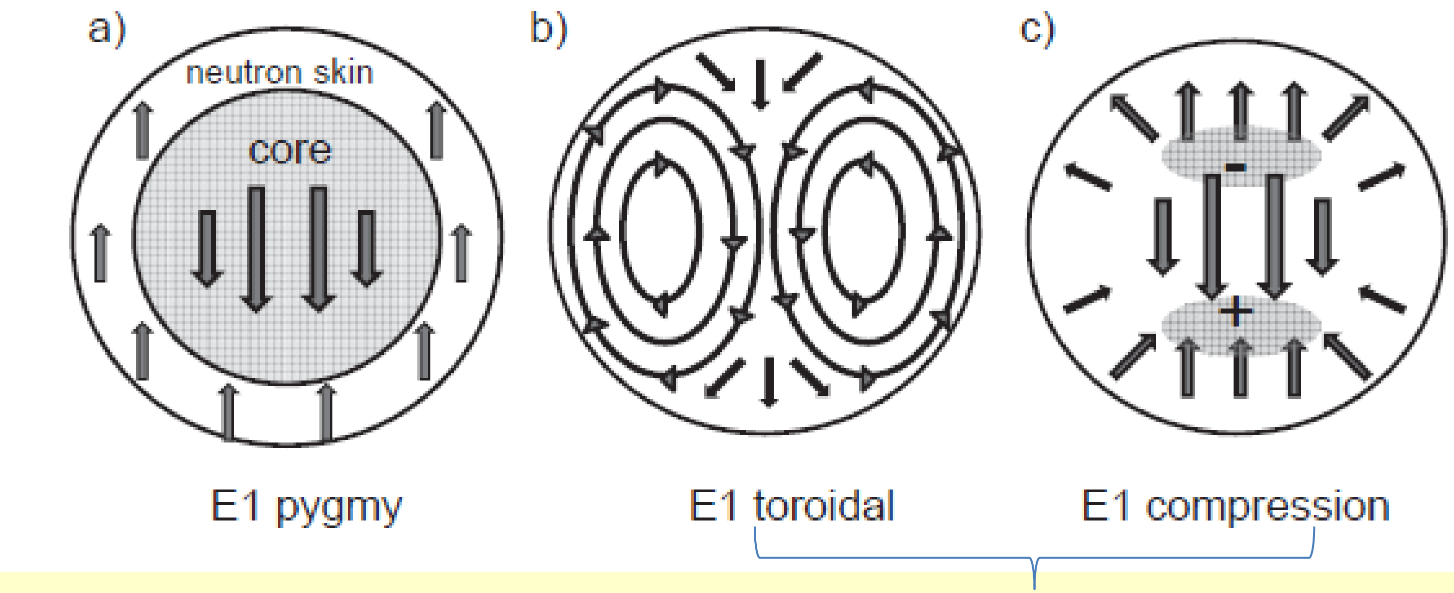
J. Kvasil, A. Repko,V.O.N, and P.-G. Reinhard,
arXive: 1705.05436[nucl-th], subm. to EPJ(A).

See for review: V.O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, P.-G.Reinhard,
Phys. Atom. Nucl. 79, 842 (2016).

Exotic dipole resonances

R. Mohan et al (1971), V.M. Dubovik (1975) M.N. Harakeh (1977)

S.F. Semenko (1981) S. Stringari (1982)



E1 pygmy E1 toroidal E1 compression

Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational vortical irrotational

$$E = 50 \div 60 \; A^{-1/3} MeV \qquad E = 50 \div 70 \; A^{-1/3} MeV \qquad E = 132 \; A^{-1/3} MeV$$

Reviews:
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
V.O. Nesterenko, Phys. Atom. Nucl. 79, 842 (2016).

- Different kinds of dipole oscillations with fixed c.m.
- TR: elastic, at fixed boundaries
- TR: the only known electric vortical mode

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

Toroidal moment:

- appears in multipole decomposition of nuclear current density

Following theorems of Helmholtz and Chandrasekhar/Moffat, the current distribution can be decomposed as

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\chi(\vec{r})]$$

electric moments

magnetic moments

electric + **toroidal** moments

transversal

Multipole electric operator (external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\; j_{\lambda}(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$j_{\lambda}(kr) = \frac{(kr)^{\lambda}}{(2\lambda+1)!!}[1-\frac{(kr)^{2}}{2(2\lambda+3)}+\ldots]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

Toroidal operator appears as the second order term in long-wave expansion of the electric operator

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^{\lambda}Y_{\lambda\mu} \longleftarrow$$

standard electric operator
In long wave approximation

Toroidal E1 operator:

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3} r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}}) \cdot [\underbrace{\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{vortical flow}}]$$

- second-order part of the electric operator

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu}\,[\underbrace{\vec{\nabla} \cdot \hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{irrotational flow}}]$$

$$\Downarrow$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu}$$

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

$$\hat{M}_{tor} = \frac{-i}{2\sqrt{3}c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r}) \cdot \vec{\nabla} \times (\vec{r} \times \vec{\nabla})\underbrace{[r^3 - \frac{5}{3} r < r^2 >_0]}_{\text{compression}}$$

toroidal and compression modes are coupled

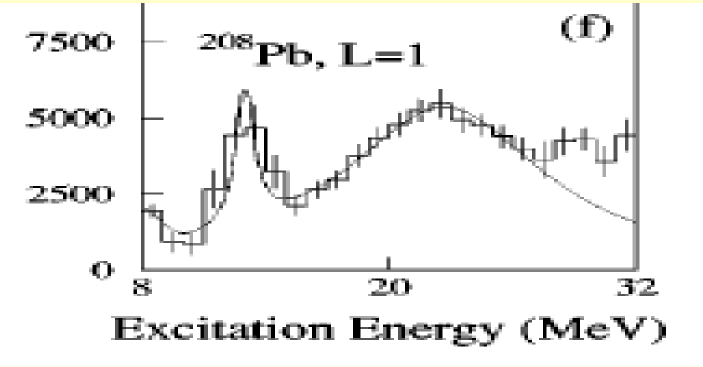
TDR and CDR constitute low- and high-energy ISGDR branches (?)

Experiment: (α,α')

Familiar treatment → LE (toroidal) HE (compression)

^{208}Pb
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PRC 69, 051301(R) (2004)



TDR

CDR

There are also exp ISGDR data in

^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007);

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,
PRC 87, 024305 (2013).

Perhaps Uchida observed at 10-17 MeV not TDR but mixed CDR/TDR low-energy bump. The main peaked TDR must at the lower energy ~ 7-9 MeV.

Relation of E1 toroidal and pygmy resonances

Is PDR a local (peripheral) part of TDR?

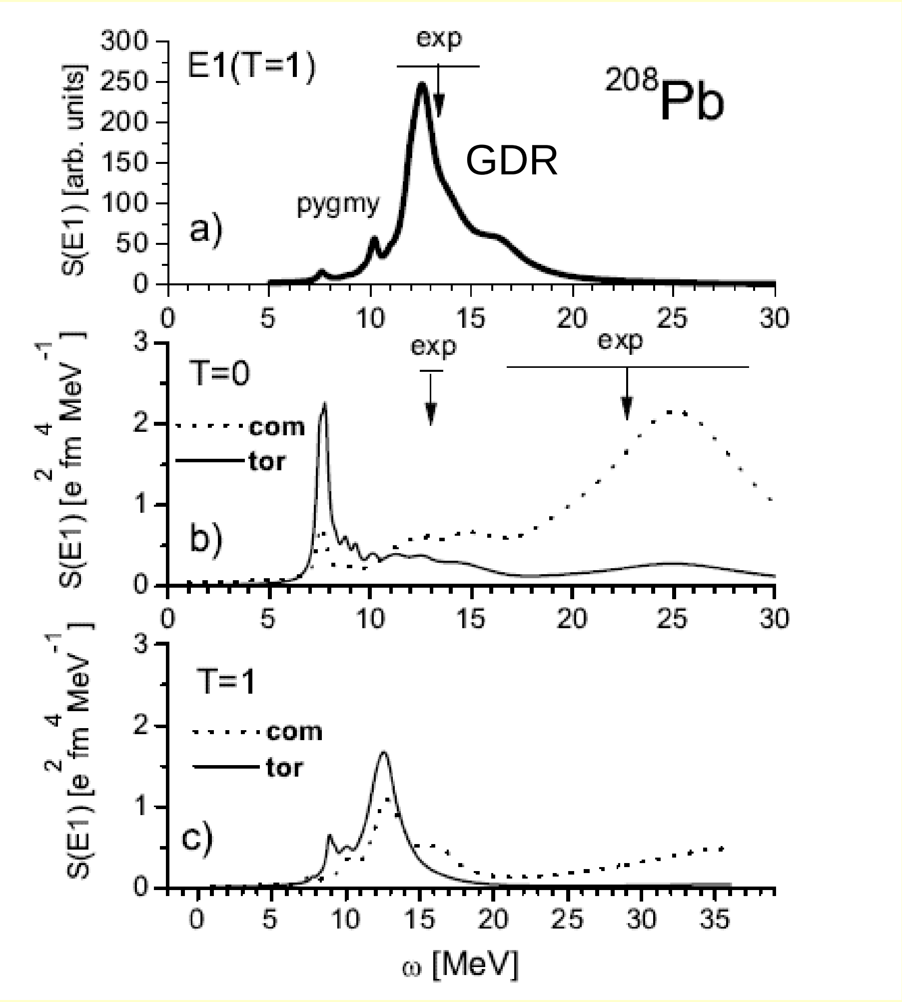
A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
Phys. Rev. C87, 024305 (2013).

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
EPJ Web of Conferences, 93, 01020 (2015); arXiv:1410.5634[nucl-th],

V.O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, P.-G.Reinhard,
“Toroidal Resonance: Relation to Pygmy Mode, Vortical Properties,
and Anomalous Deformation Splitting”
Phys. Atom. Nucl. 79, 842 (2016).

Strength functions SLy6

A. Repko, P.G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013)

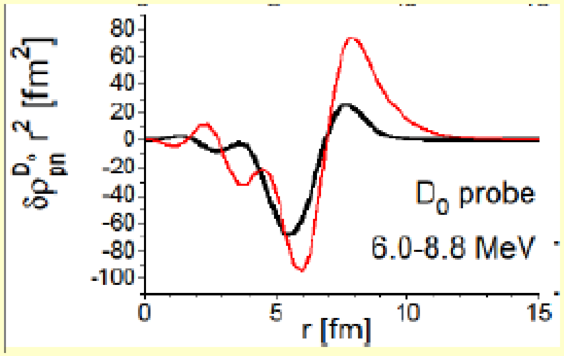


Two peaks at 7.5 and 10.3 MeV in agreement to RMF calculations
(D. Vretenar, N. Paar, P. Ring, PRC, **63**, 047301 (2001))

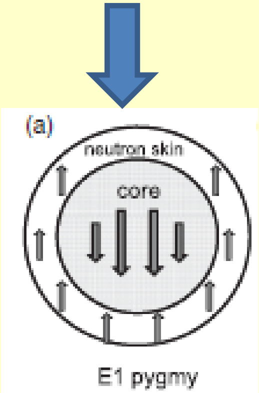
(α,α') experiment
of Uchida et al (2003)

PDR region hosts TDR and CDR!

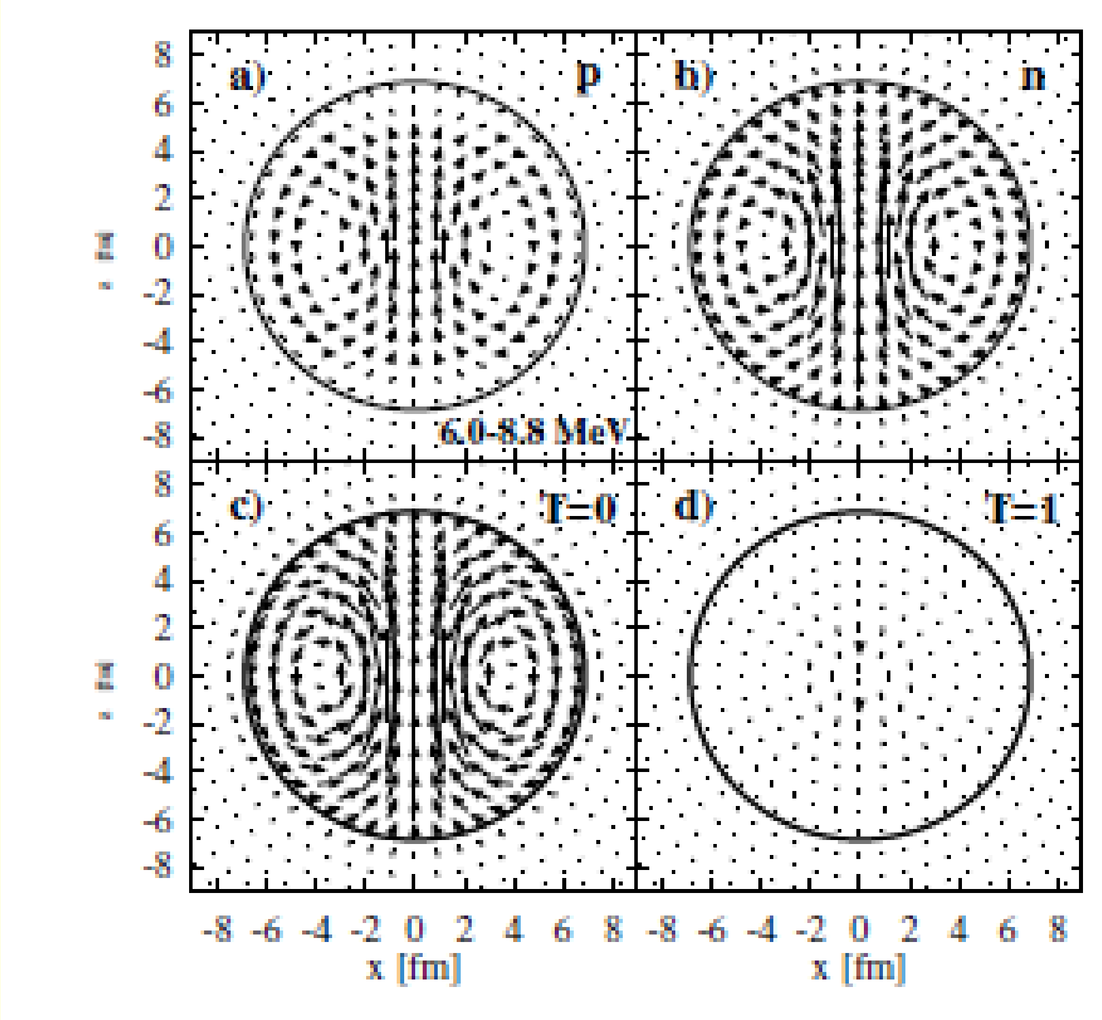
Typical PDR transition density:



- n
- p



QRPA : nuclear current for E1-excitations at 6.0-8.8 MeV (PDR region)

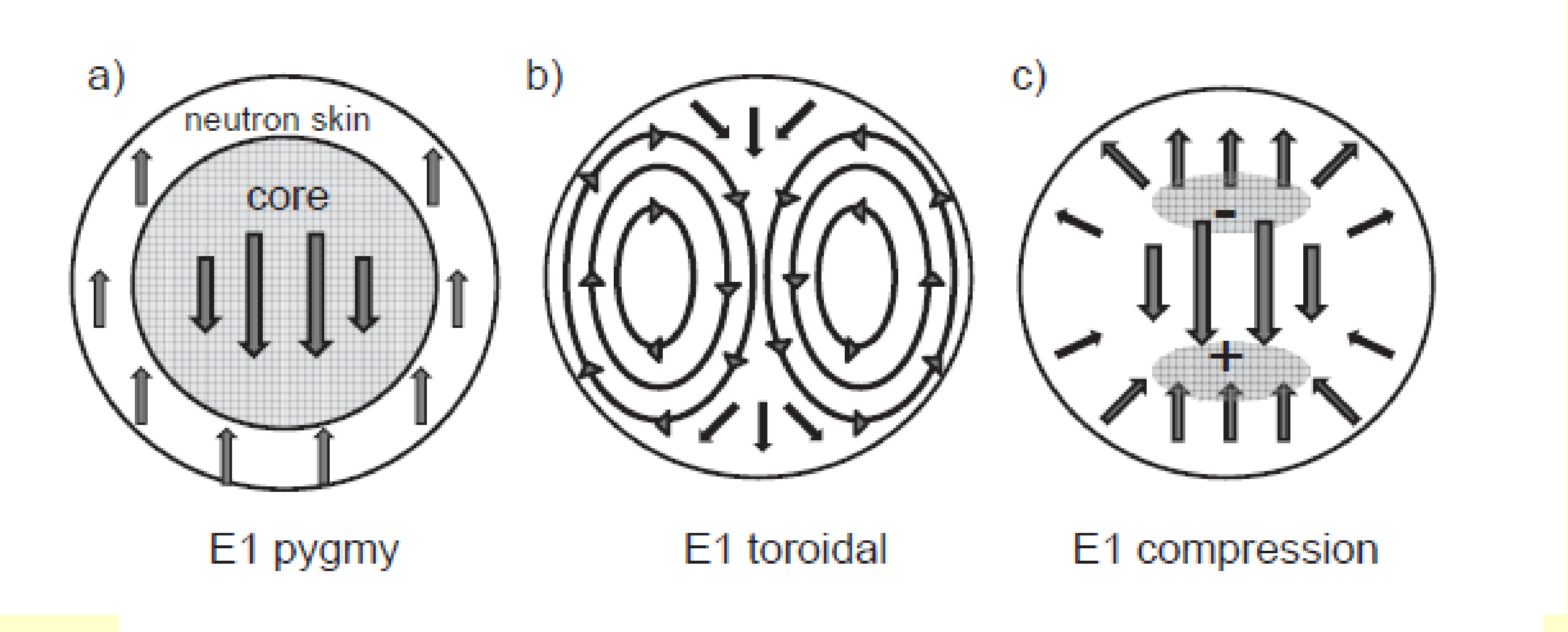


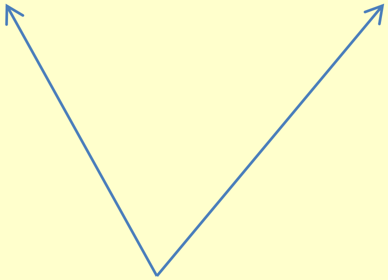
^{208}Pb

- mainly isoscalar toroidal flow in PDR energy region!

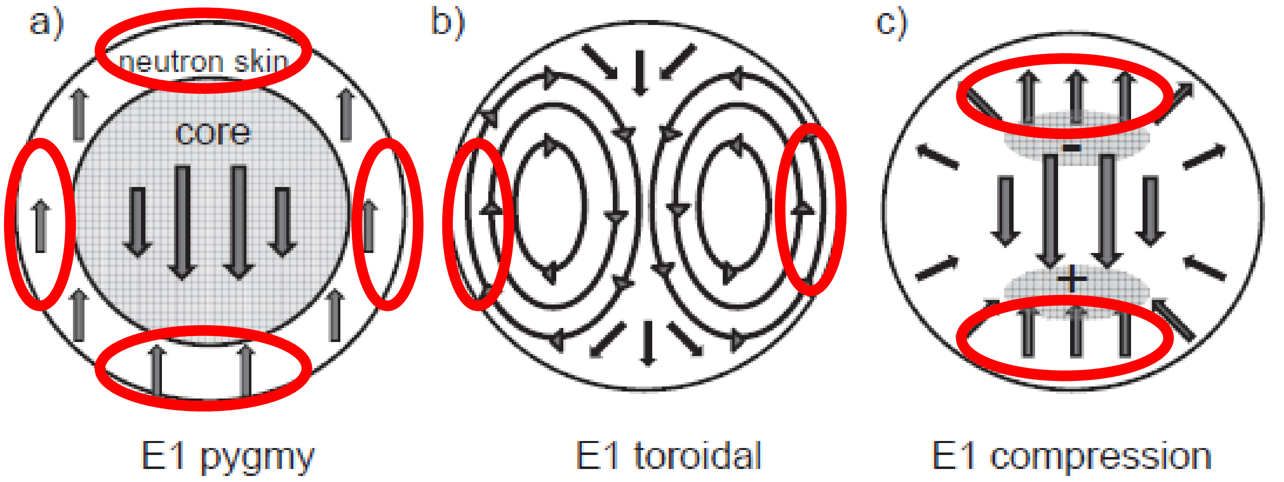
--so PDR is actually the toroidal motion?

Does the toroidal flow contradicts the familiar PRD picture?





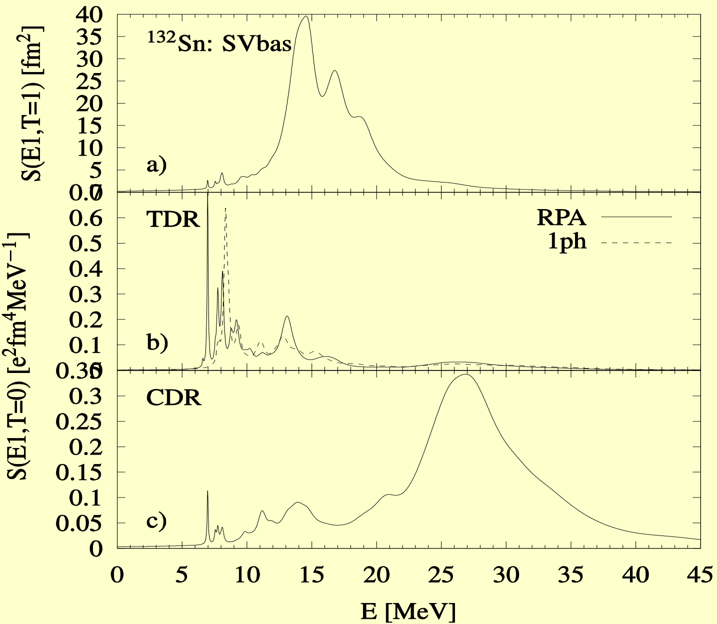
?

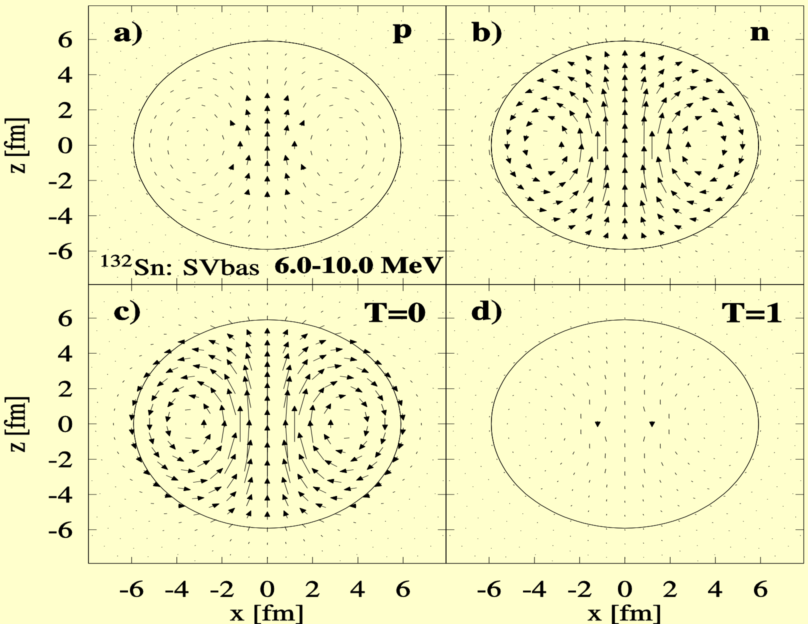


V.O. Nesterenko, A. Repko,
P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal
resonances",
arXiv:1410.5634[nucl-th],

- **PDR can be viewed as a local peripheral part of TDR and CDR**
- **Our calculations demonstrate the TDR motion in PDR energy region for other nuclei: Ni, Zr, Sn, …**

132Sn, SVbas, with PDR





So PDR indeed could be a local peripheral part of the dipole toroidal flow

Anomalous deformation feature of the toroidal resonance

(to be used as TDR fingerprint)

J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in {144-154}Sm: Skyrme-RPA
exploration of deformation effect",
Eur. Phys. J. A, v.49, 119 (2013).

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",
Phys. Scri., v.89, n.5, 054023 (2014).

V.O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, P.-G.Reinhard,
“Toroidal resonance: relation to pygmy mode, vortical properties,
and anomalous deformation splitting”
Phys. Atom. Nucl. 79, 842 (2016).

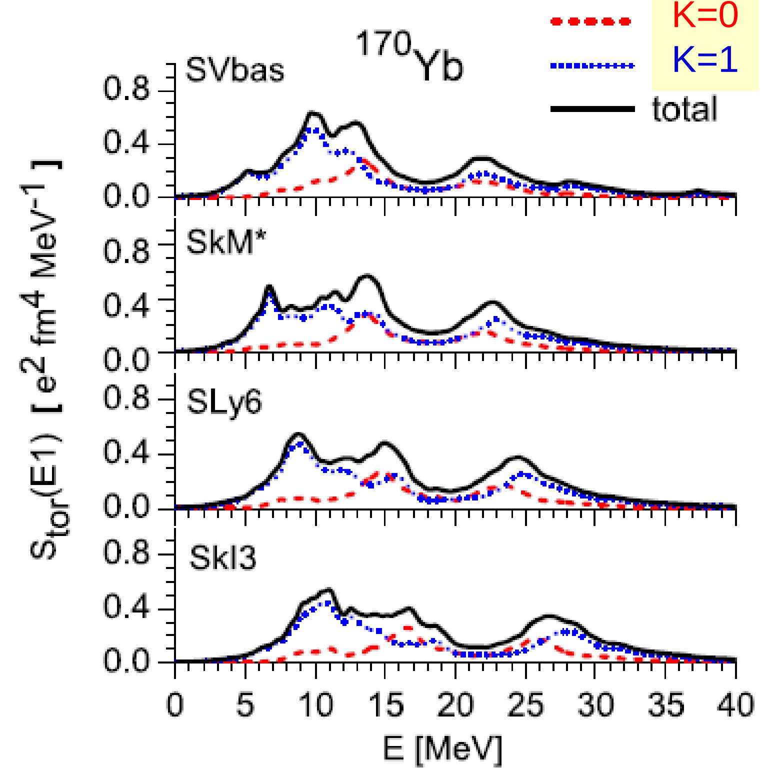
J. Kvasil, A. Repko,V.O.N, and P.-G. Reinhard,
“Pairing and deformation features of nuclear spectra”,
arXive: 1705.05436[nucl-th], subm. to EPJ(A).

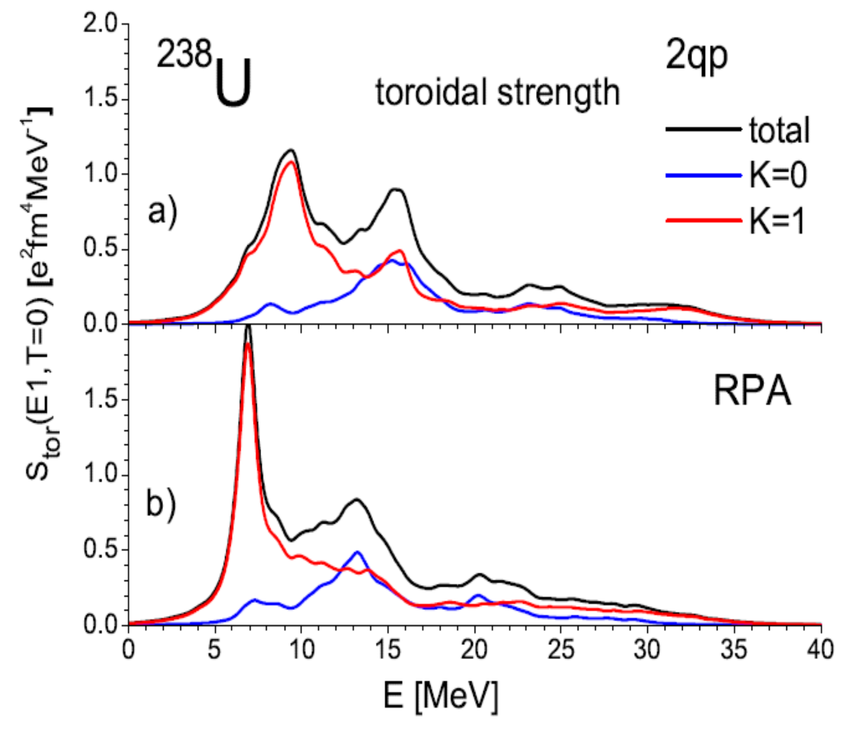
Deformation effects in the toroidal mode

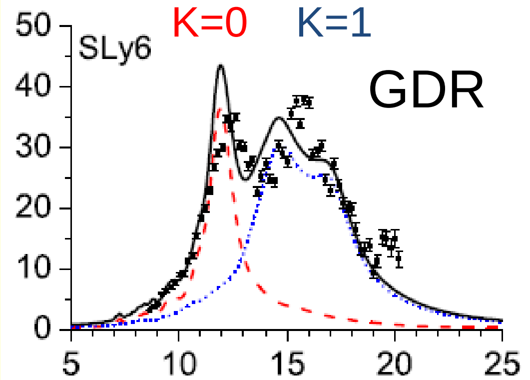
J. Kvasil, VON, W. Kleinig and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014)

V.O.N., J. Kvasil, A. Repko, W. Kleinig, P.-G.Reinhard,
Phys. Atom. Nucl. 79, 842 (2016).

toroidal strength





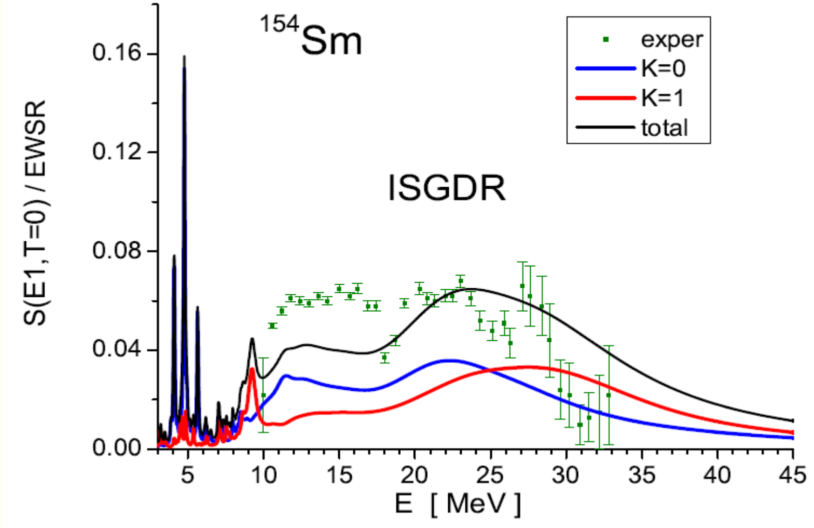


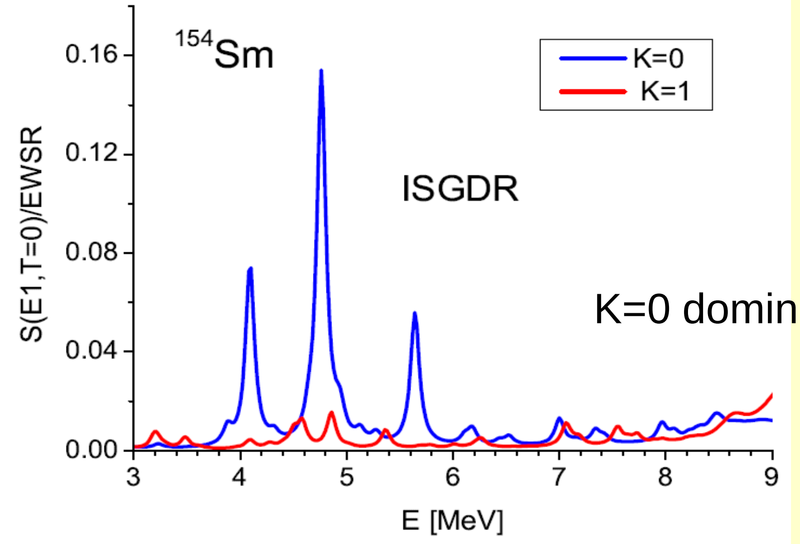
The low-energy TDR demonstrates the opposite, than in GDR, sequence of K=0 and K=1 branches.

154Sm, SLy6

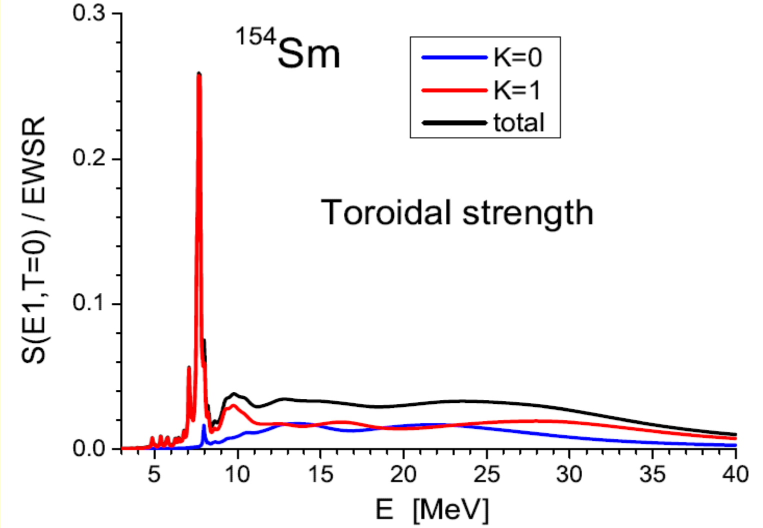
Energy-weighted strength functions

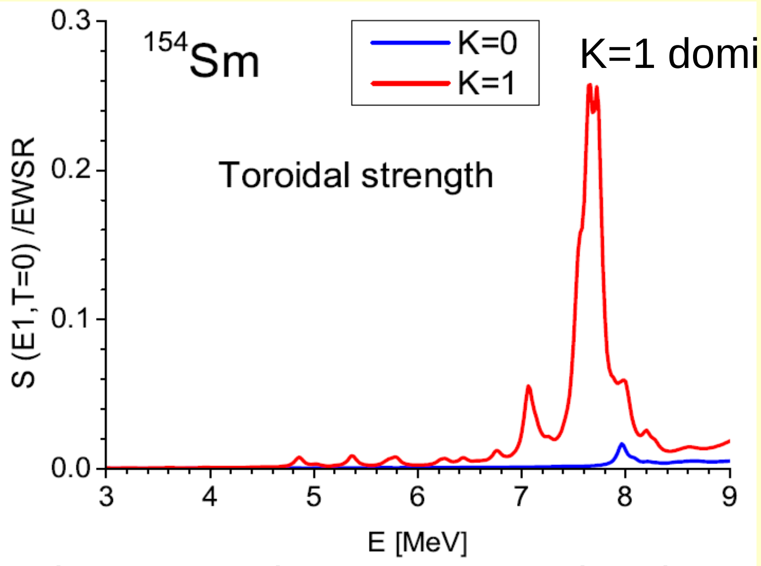
Exper:
D.H. Youngblood et al, PRC 69, 034315 (2004).





K=0 dominates



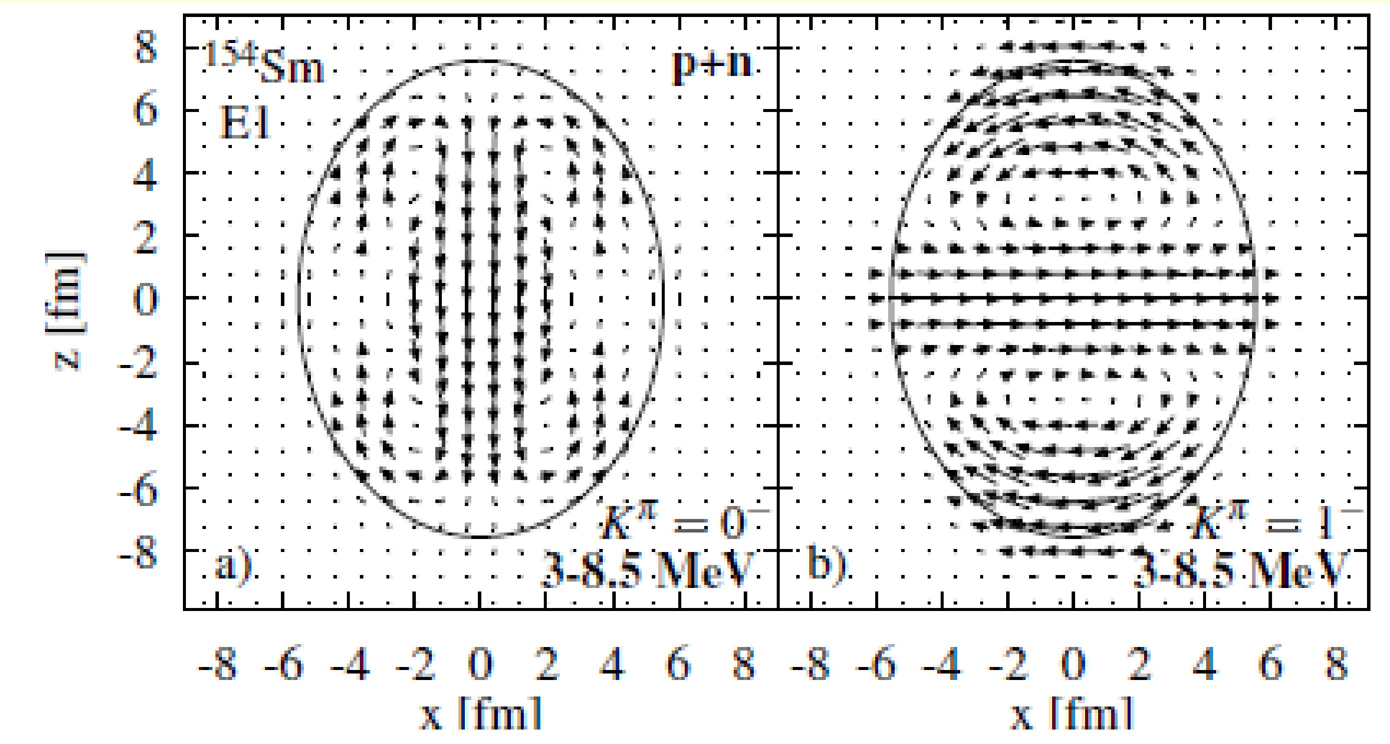


K=1 dominates !!!

-The available (α,α') experiments do not cover the TDR region 7-8 MeV and perhaps do not observe the vortical TDR. One has to probe the region 3-8 MeV !!!

Why K=1 branch dominates in peaked TDR?

Isoscalar current flows (transition densities) in z-x plane
for QRPA states at 3-8.5 MeV.



154Sm, SLy6

- K=1 flow dominates over K=0 one
- perhaps in prolate nuclei, the K=1 configuration is more suitable and thus more energetically favorable for the toroidal vortex-antivortex flow
- K=1 dominance can in principle be used as the TDR fingerprint in future experiments

Most suitable reactions to observe TDR

(α,α') and $(\alpha,\alpha'\gamma)$ are most suitable:

- isoscalar (T=0)
  - to eliminate dominant T=1 dipole excitations
  - the peaked TDR(T=0) is most clean and impressive
- surface (correspond to r^2 - dependence of toroidal operator)
- $(\alpha,\alpha'\gamma)$ for additional identification of TDR in deformed nuclei

  To use γ-decay for discrimination of the low-energy dominant K=1 branch (TDR fingerprint)

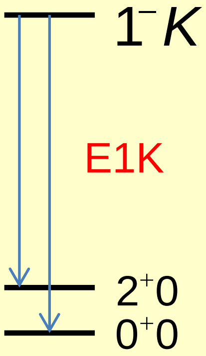
Alaga rules:

$$A_K = \frac{X(1^- K \rightarrow 0^+ 0)}{X(1^- K \rightarrow 2^+ 0)} = \left[\frac{\langle 1K1-K\,|\,00\rangle}{\langle 1K1-K\,|\,20\rangle}\right]^2$$

$$A_0 = 0.5$$

$$A_1 = 2$$

So K=1 branch can be discriminated



- E1(T=1) transitions because of the isospin admixtures
- TDR usually lies below the n,p-thresholds, then γ-decay is favorable.

  This is the case for 154Sm.

Conclusions

★ **Most probably PDR is a complex mixture of:**
- **IS/IV,**
- **collective/s-p,**
- **irrotational/vortical,**
- **TDR / CDR / GDR,**
- **complex configurations**

But the vortical toroidal flow dominates!

★ **In prolate deformed nuclei, TDR exhibits as a strong low-energy collective peak with dominant K=1 strength.
This feature can be used as TDR fingerprint.**

★ **IS reactions (α,α'), $(\alpha,\alpha'\gamma)$ look best to observe TDR.
Available experimental data inspect too high excitation energy where the peaked TDR is absent.
Experiments for lower energy intervals are necessary.**

★ **Familiar treatment of ISGDR experiments should be corrected: the data miss the main part of TDR .**

★ **Perspectives: individual lowest toroidal and compression states can exist in light deformed nuclei like 24Mg. Relation to cluster excitations?**

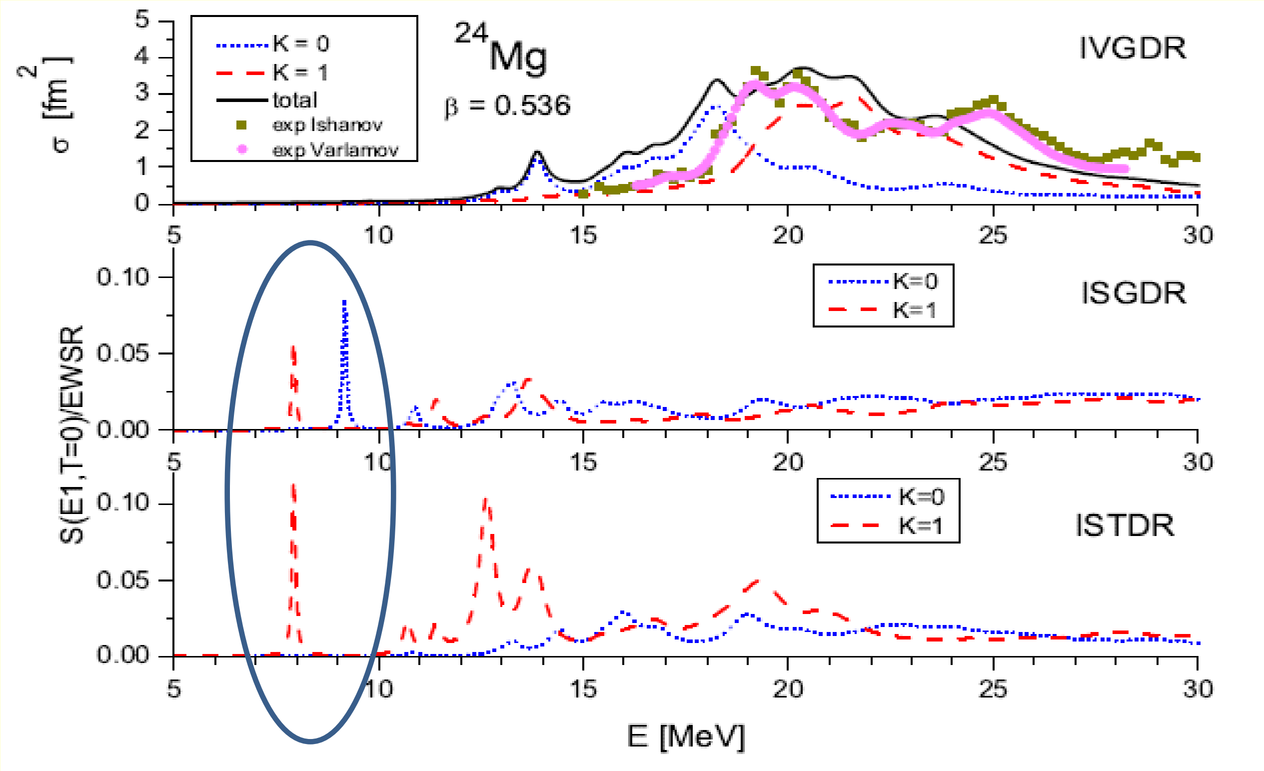
Relations: anapole dack matter, …

Thank you for attention!

^{24}Mg $\beta_2^{\exp} = 0.605$

- extremely large prolate deformation
- low-density low-energy spectrum
- cluster structure (6 α - particles)

→ Principle possibility for individual (!) toroidal and compression states



QRPA, SLy6

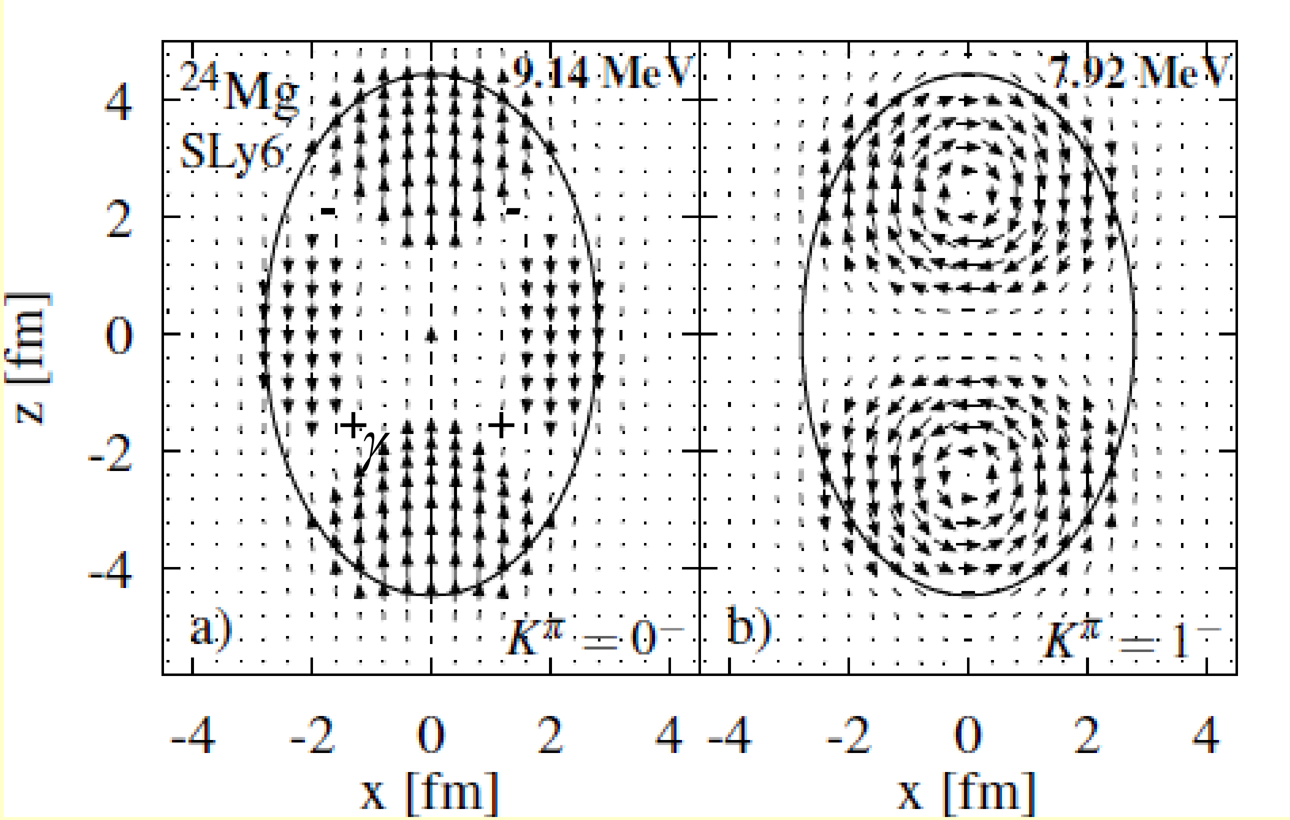
Excitation levels [MeV]

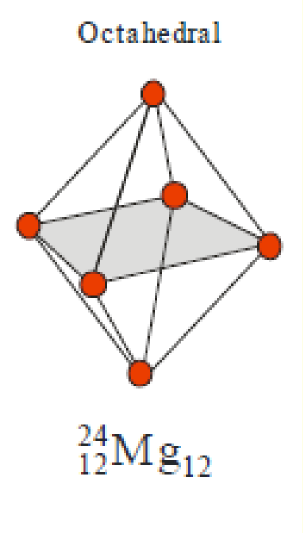
theor	exper
7.92	7.55
	8.44
9.14	9.15

Separate toroidal and compression dipole states!
Lowest by energy K=1 and K=0 dipole states
Possibility to test nuclear incompressibility using the compression state!

Nuclear isoscalar currents in 24Mg

C.-B. Moon,
arXiv:1605[nucl-th]



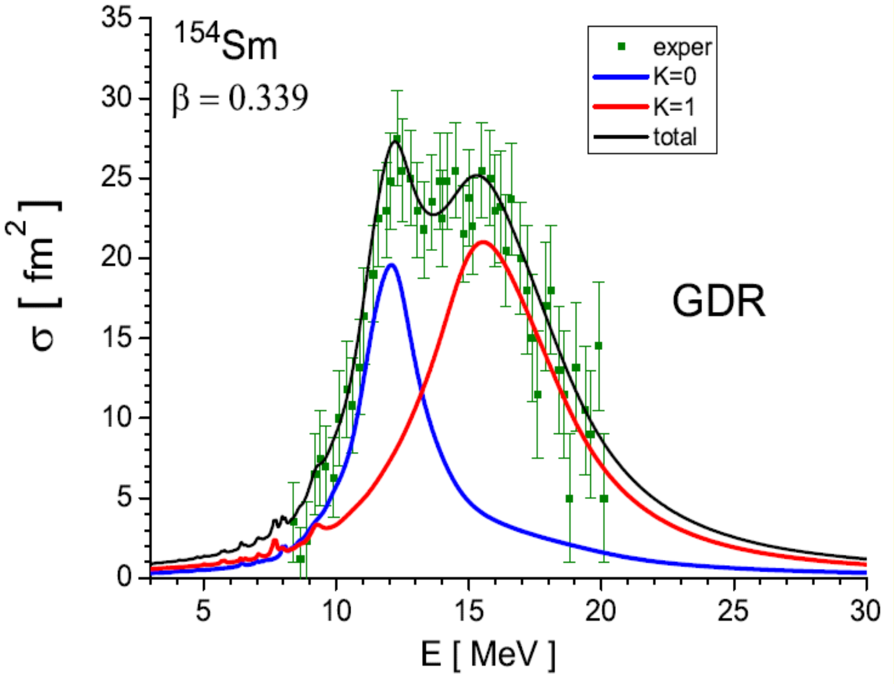


K=0: compression

K=1: toroidal

K=0:
-excitation energy E=9.14 MeV near α- particle threshold S_α=9.3 MeV
- reminds mutual oscillations and circulations of α -particles

154Sm,

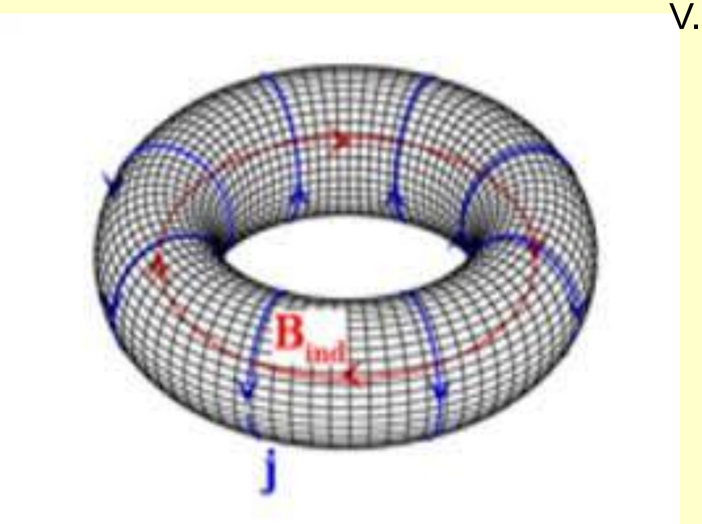


QRPA, SLy6

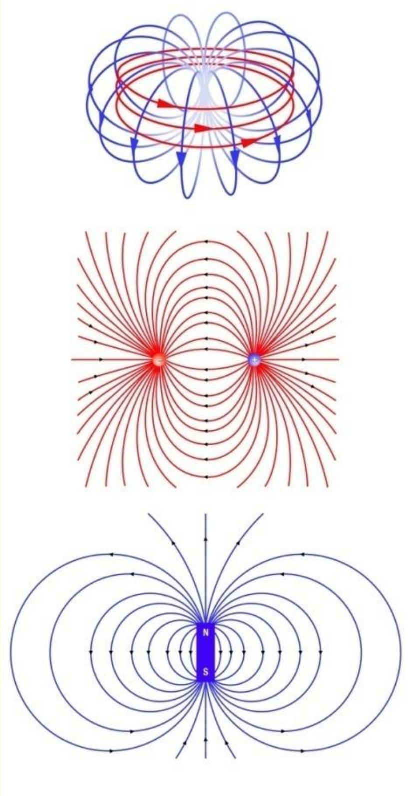
Good agreement for GDR: demonstration of accuracy of our QRPA method

Toroidal moment

Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 33, 1531 (1957)
V.M. Dubovik and L.A. Tosunyan, Part. Nucl., 14, 1193 (1983)



dipole moments



Zeldovich anapole: no electric and magnetic moments but the toroidal moment

$$\vec{T} = \frac{1}{10c}\int d\vec{r}\ [(\vec{j}\cdot\vec{r})\vec{r} - 2r^2\vec{j}]$$

$$T = \frac{\pi}{2c} jR_0^2 bn$$

Speculations with toroidal :

- Robert Scherrer and Chiu Man Ho (2013): attempt to explain dark matter by existence of Maiorana fermions with the anapole moment.

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

Toroidal moment:

- appears in multipole decomposition of nuclear current density

Following generalized Helmholtz theorem, vector functions can be decomposed as

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\chi(\vec{r})]$$

electric moments	magnetic moments	toroidal moments

Multipole electric operator

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1-\frac{(kr)^2}{2(2\lambda+3)}+\ldots]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

Toroidal operator appears as the second order term in long-wave expansion of the electric operator

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu} \longleftarrow$$

standard electric operator
In long wave approximation

Recent publications on TR/CR:

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely,
"General treatment of vortical, toroidal, and compression modes",
Phys. Rev. C84, n.3, 034303 (2011)

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
Phys. Rev. C87, 024305 (2013).

J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in {144-154}Sm: Skyrme-RPA exploration of deformation effect",
Eur. Phys. J. A, v.49, 119 (2013).

J. Kvasil, V.O. Nesterenko, A. Repko, W. Kleinig, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in 124Sn",
Phys. Scr., T154, 014019 (2013).

P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",
Phys. Rev. C89, 024321 (2014).

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",
Phys. Scri., v.89, n.5, 054023 (2014).

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
arXiv:1410.5634[nucl-th],

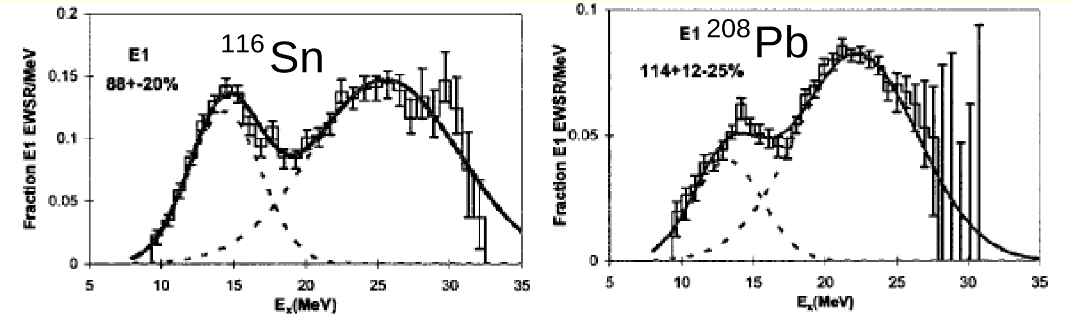
Available (α,α') experimental data

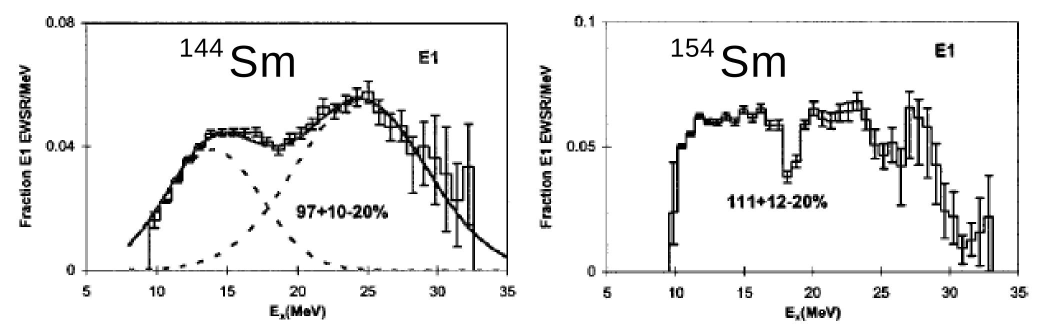
M.Uchida et al, PRC 69, 051301(R) (2004)

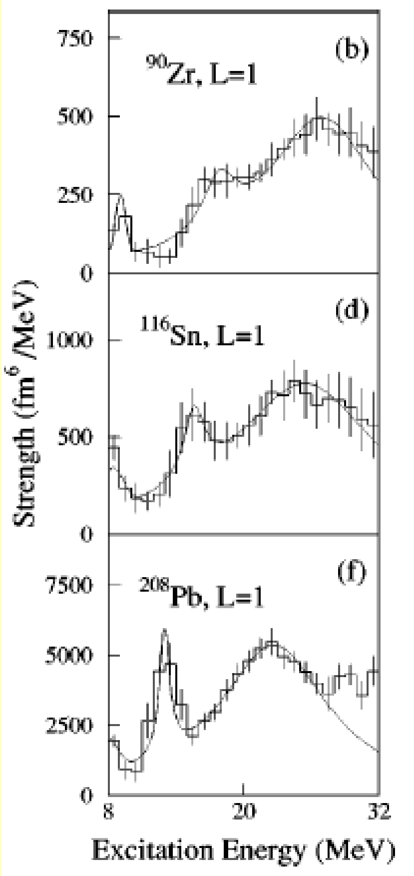
$^{90}Zr,^{116}Sn,^{144}Sm,^{208}Pb$

D.H. Youngblood et al, PRC 69, 034315 (2004).

$^{116}Sn,^{144}Sm,^{154}Sm,^{208}Pb$







-the only experiment for ISGDR in **deformed** nucleus

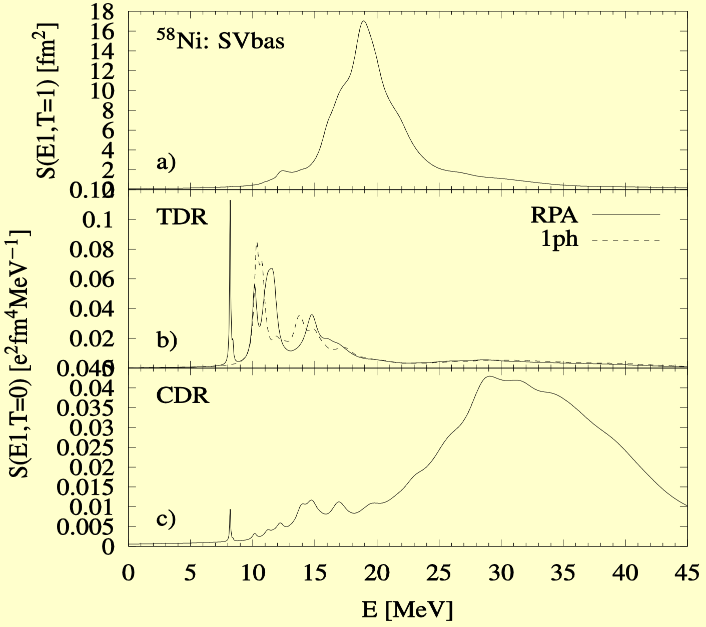
-**TDR is lost** since too high excitation energies (E>10 MeV) are considered

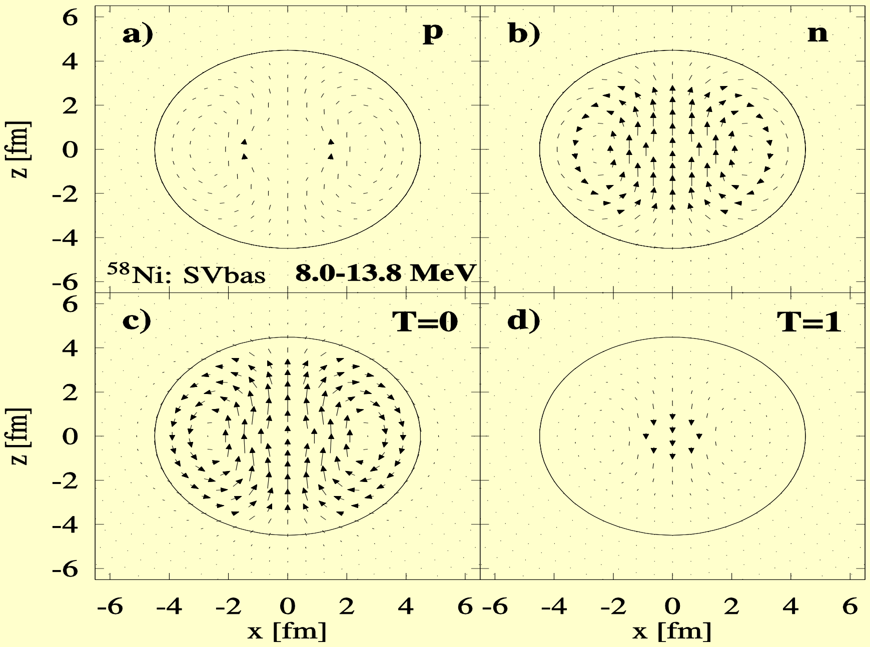
Perhaps TDR is observed at ~ 8 MeV

The familiar treatment of ISGDR experiments should be corrected:

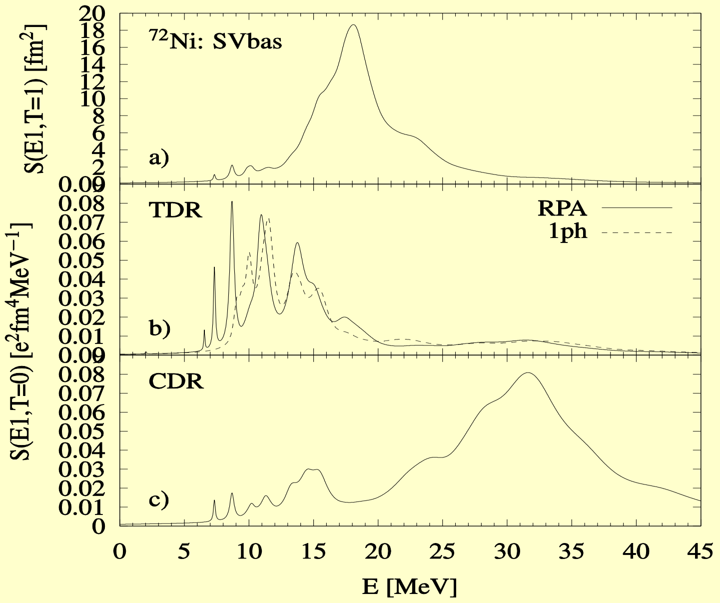
- the low energy broad bump is not TDR but some TDR/CDR mixture
- the main peaked TDR is at a lower energy

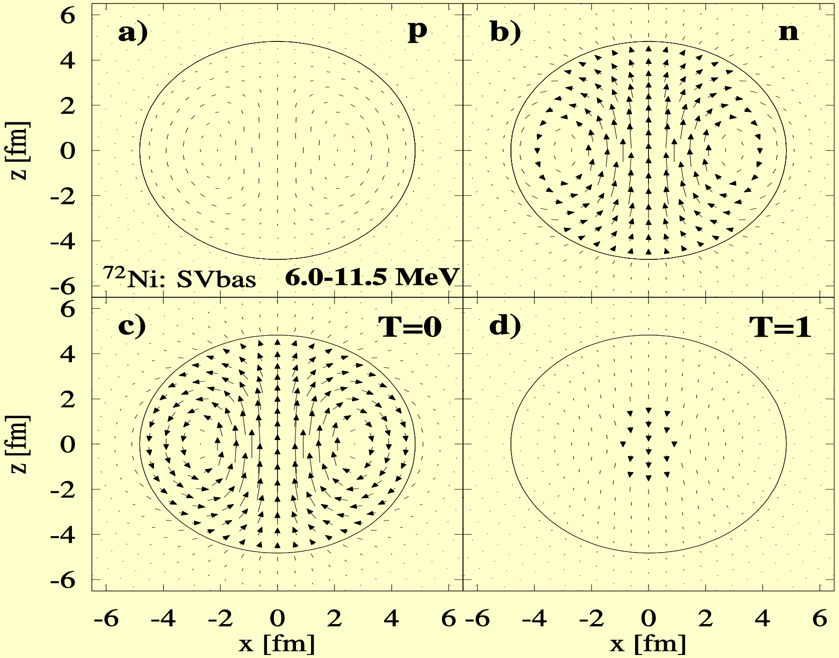
58Ni, SVbas





72Ni, SVbas





Previous studies

D. VRETENAR, N. PAAR, P. RING, AND T. NIKŠIĆ PHYSICAL REVIEW C **65** 021301(R)

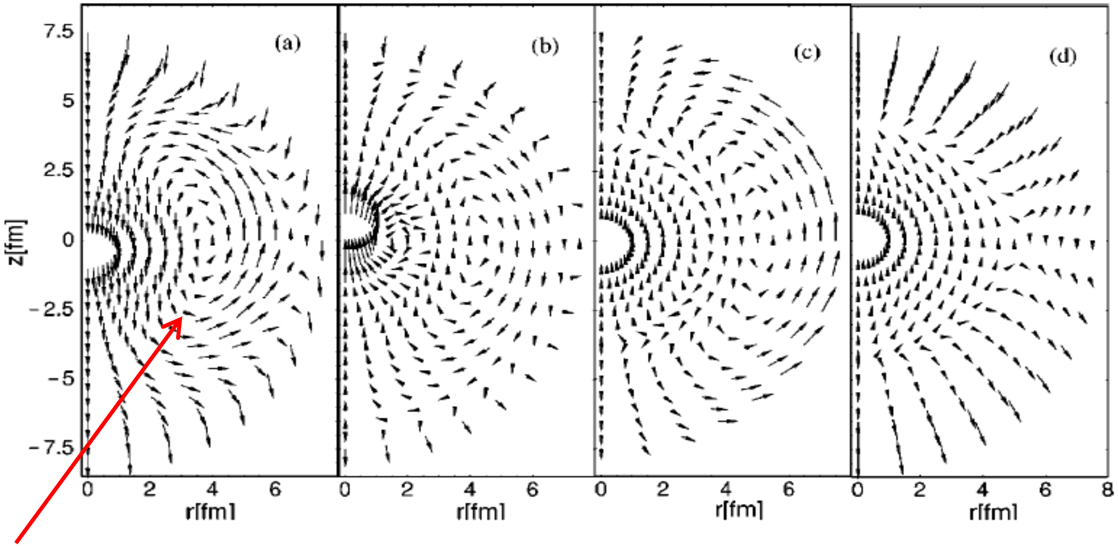


FIG. 3. Velocity distributions for the most pronounced dipole peaks in ^{116}Sn (see Fig. 2). The velocity fields correspond to the peaks at 8.82 MeV (a), 10.47 MeV (b), 17.11 MeV (c), and 30.97 MeV (d).

D. Vretenar et al,
relativistic mean field RPA

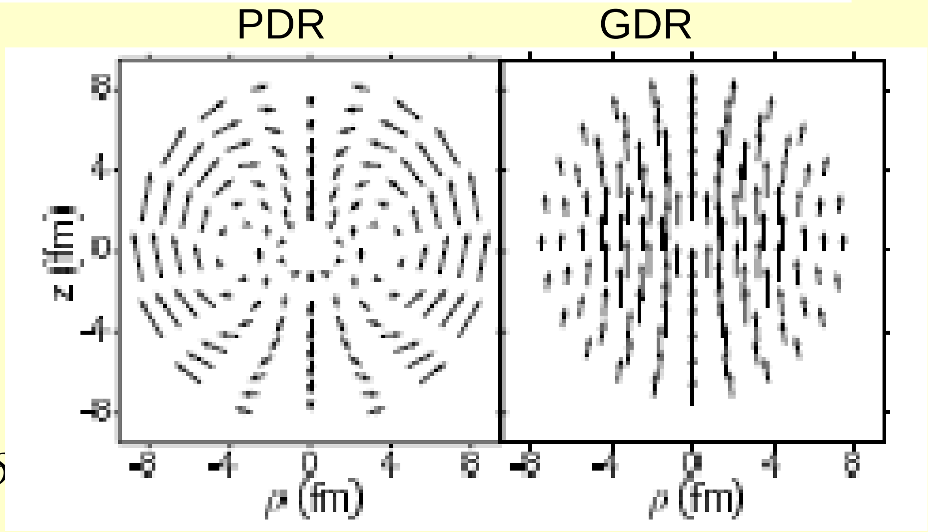
Toroidal-like flow in T=1 channel .

N.Ryezayeva et al, PRL 89, 272502 (2002).

QPM calculations taking into account complex configurations

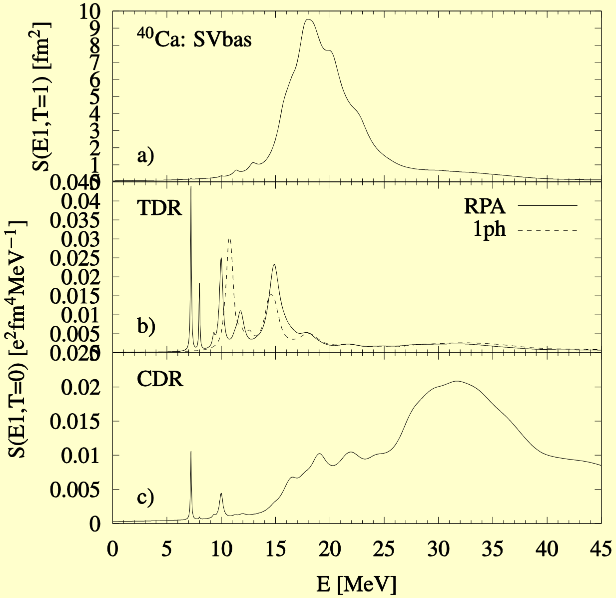
Summed QPM velocity fields in 6.5-10.5 MeV region

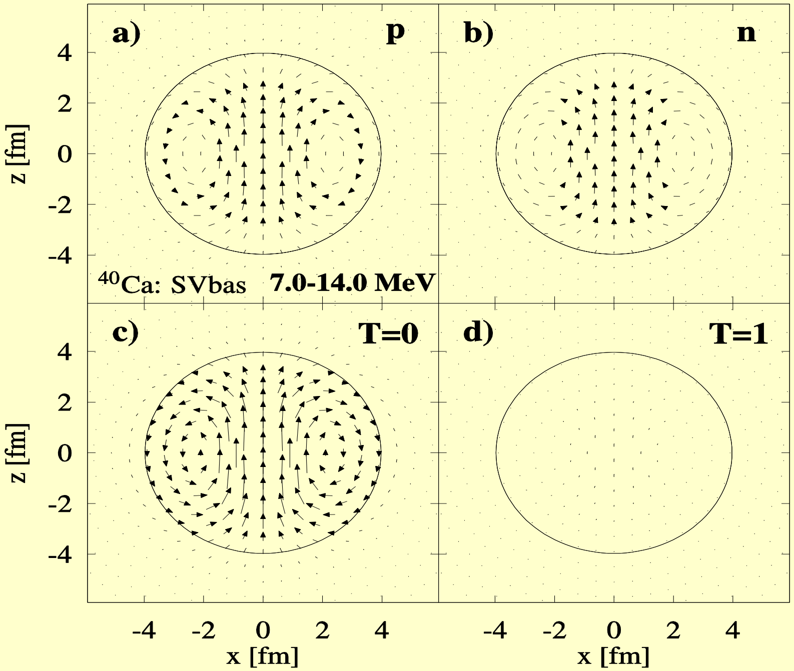
$$\delta\vec{v} = \frac{N}{A}\delta\vec{v}_p - \frac{Z}{A}\delta$$



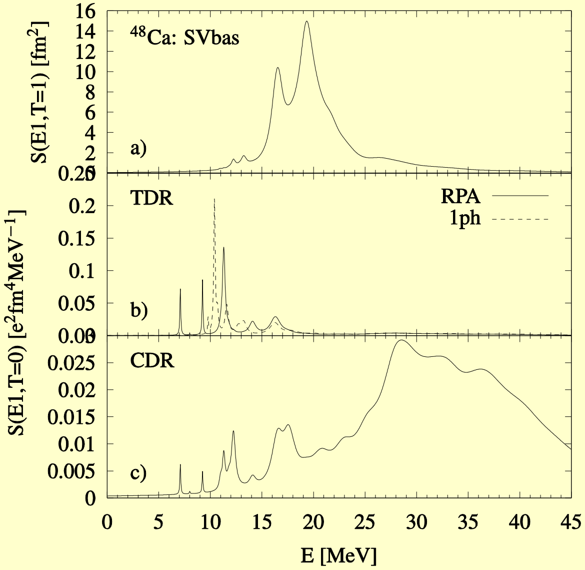
However none of these studies has clamed the toroidal origin of PDR

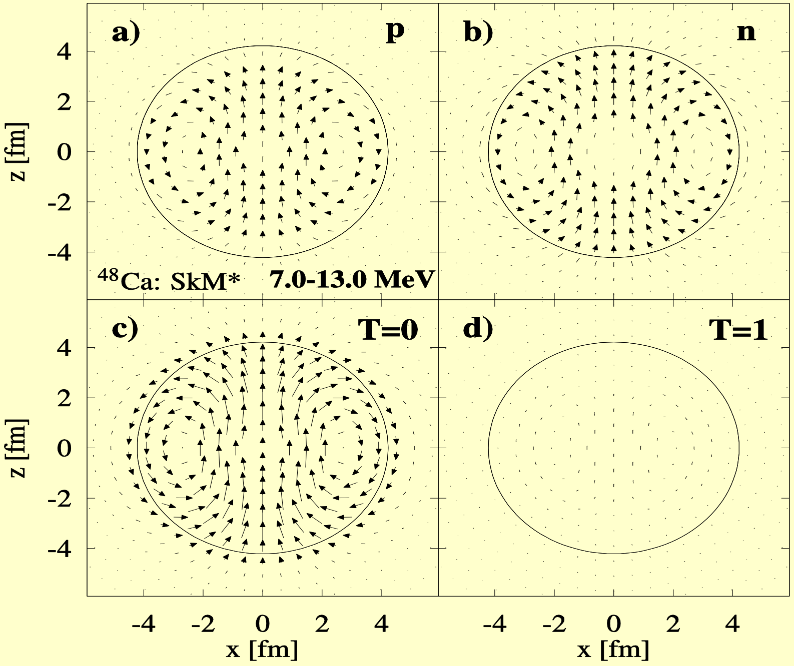
40Ca, SVbas





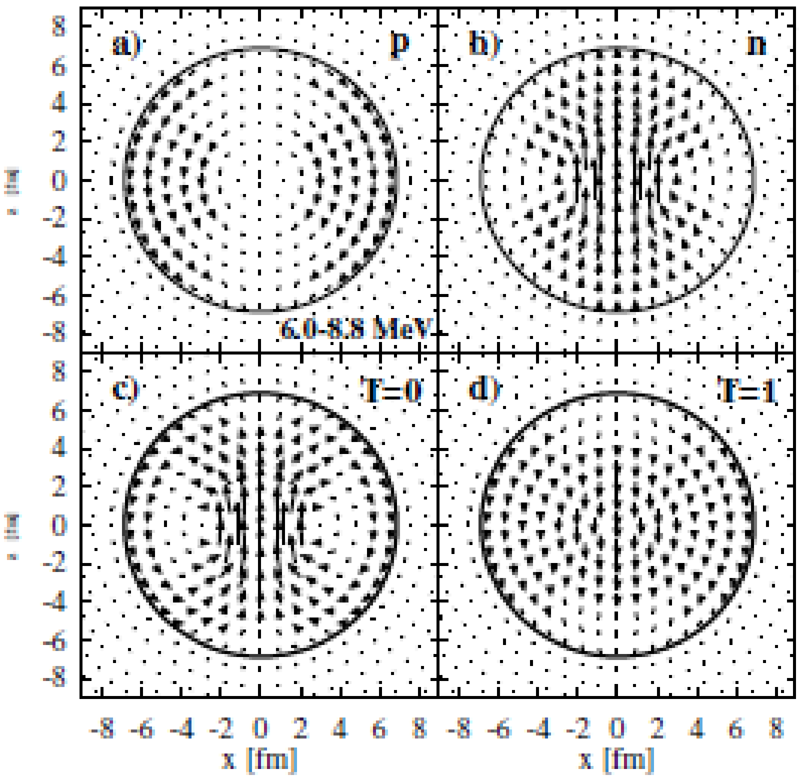
48Ca, SVbas



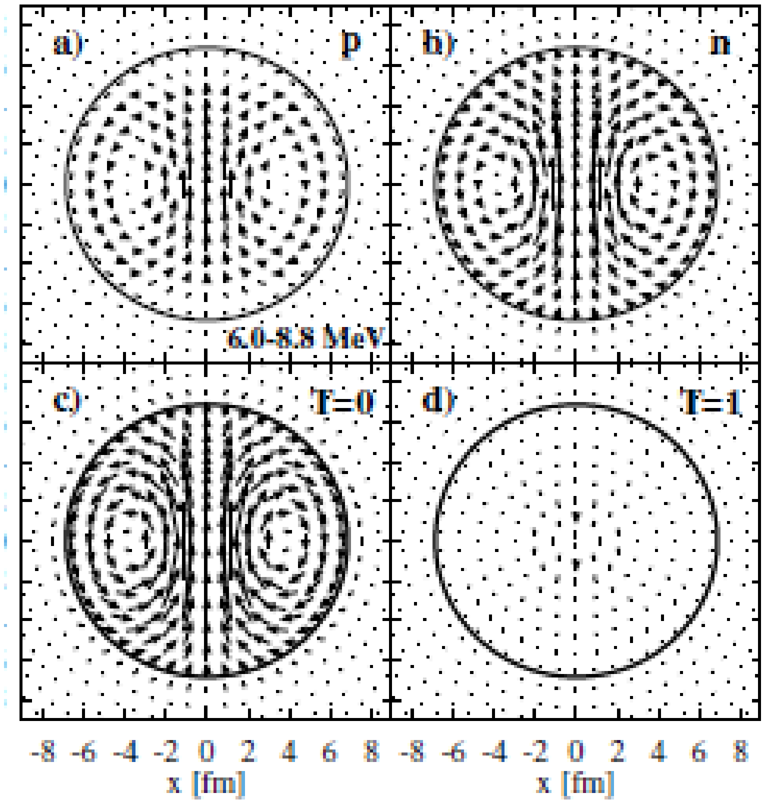


RPA vs 1ph

2qp



RPA



-both isoscalar and isovector
- toroidal flow mainly for neutrons

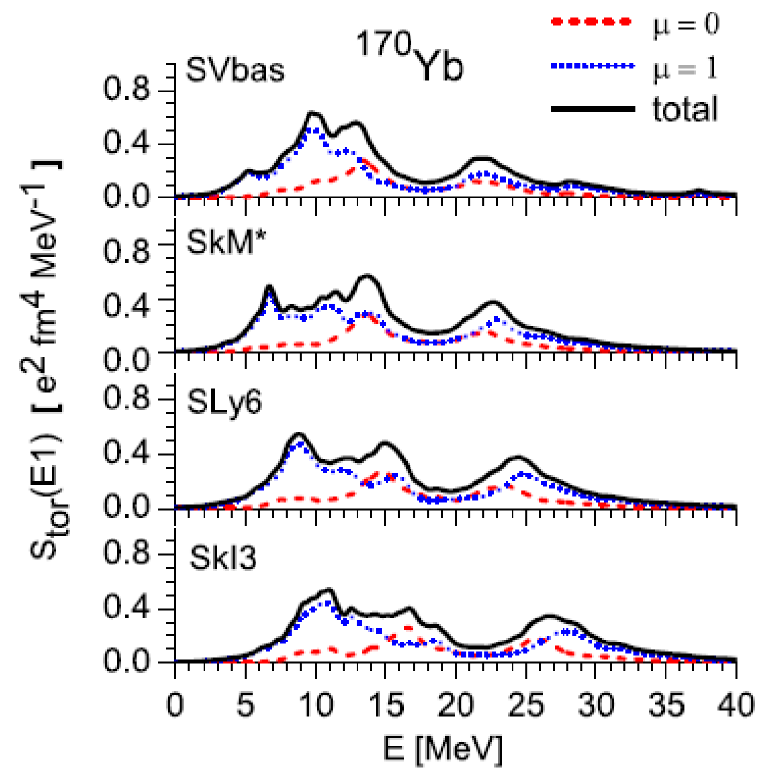
- mainly isoscalar
- toroidal flow for both n/p

So the toroidal flow is basically formed already by the mean-field.
But residual interaction makes it collective and more impressive.

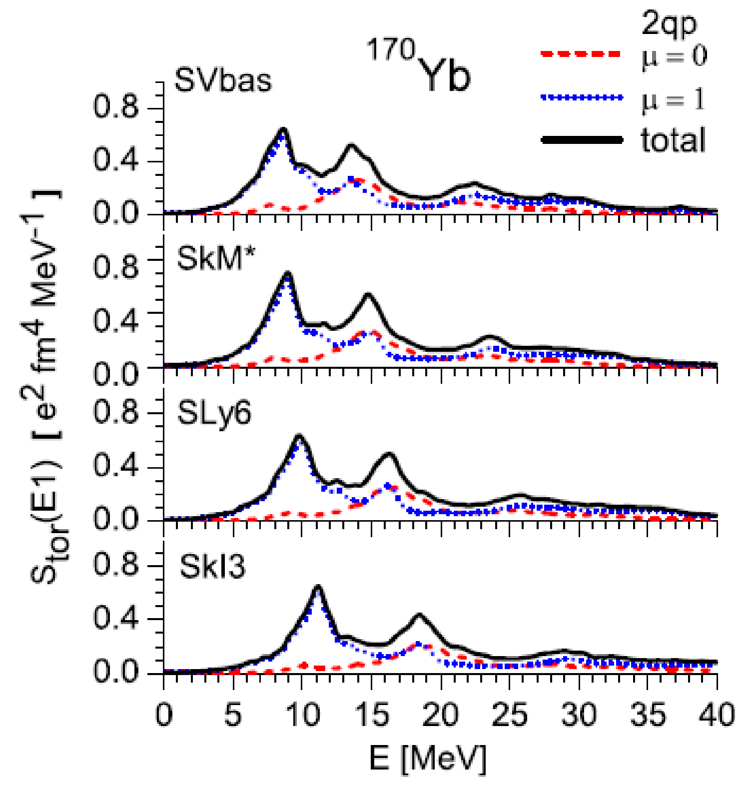
Deformation effects in the toroidal mode

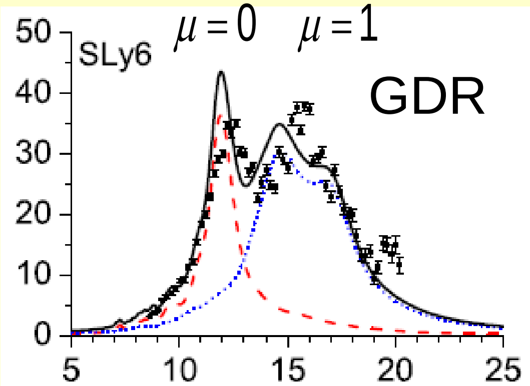
J. Kvasil, VON, W. Kleinig and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014)

RPA



2qp





GDR: $E(\mu = 0) < E(\mu = 1)$

TM: $E(\mu = 0) > E(\mu = 1)$

Unusual sequence of $\mu = 0$ and $\mu = 1$ branches

Deformation (not resid. Interaction) effect

Non-Tassie mode?

Should affect PDR properties

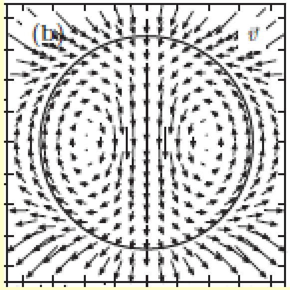
$$\nabla \times \vec{F} = 0, \nabla \cdot \vec{F} = 0$$

$$\vec{F} = \nabla \Phi, \Phi = r^{\lambda} Y_{\lambda\mu}$$

Two familiar conceptions of nuclear vorticity : HD, RW

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla}\times\vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$



$$(\vec{\nabla}\times\delta\vec{j}_{nuc}) \to \rho_0(\vec{r})(\vec{\nabla}\times\delta\vec{v}) \to \rho_0(\vec{r})\,\vec{w}(\vec{r})$$

2. RW vorticity

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

- **continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

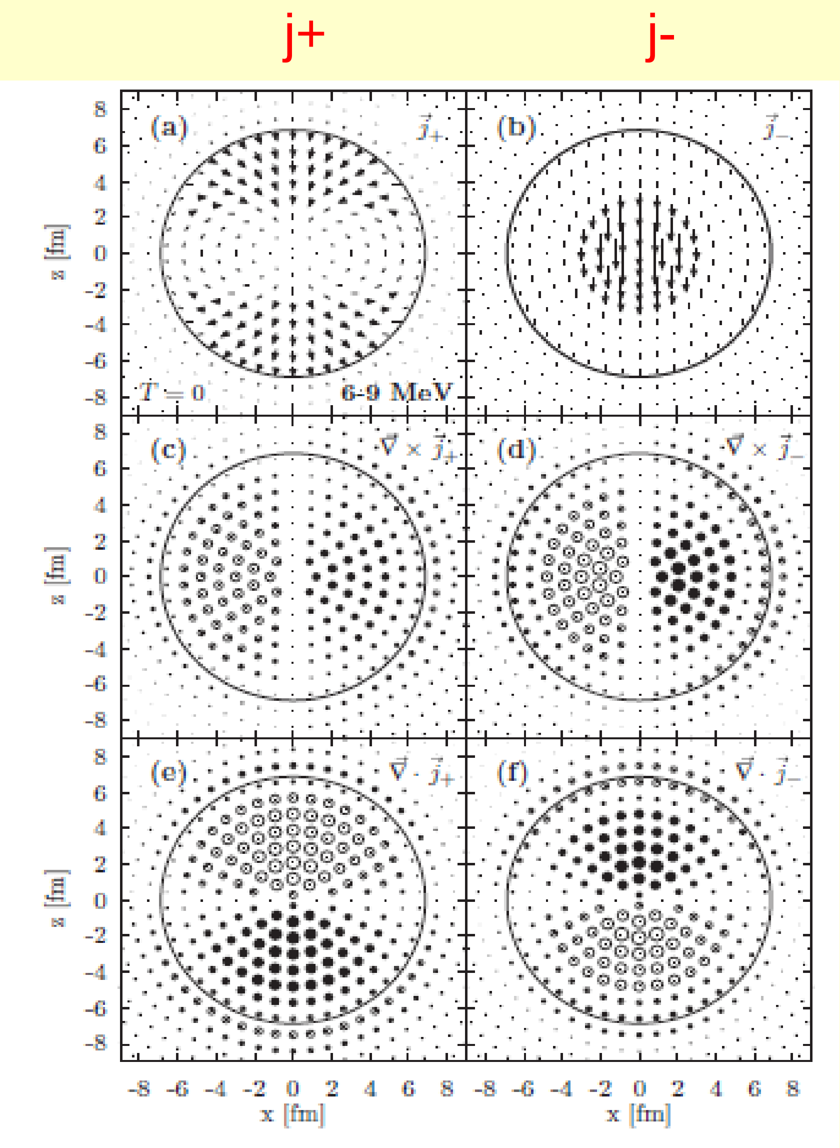
$$\delta\vec{j}^{\,\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}}[\underbrace{j^{\nu}_{10}(r)}_{j_-}\vec{Y}^{*}_{10\mu} + \underbrace{j^{\nu}_{12}(r)}_{j_+}\vec{Y}^{*}_{12\mu}]$$

- **current transition density**

$$\boxed{j^{\nu}_{+}(r)}$$

- independent part of charge-current distribution,
- decoupled from CE in the integral sense
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!



P.-G. Reinhard, V.O. N., A. Repko, and J. Kvasil,
Phys. Rev. C89, 024321 (2014).

208Pb:
all RPA states
at E=6-9 MeV

j+, j-:

- both have strong curl's and div's
- there is no any advantage of j+ over j- to represent the vorticity

The vortical or irrotational character of the flow is provided not by j+ or j- components separately but by their proper superposition.

So just the toroidal current but not j+ is the relevant measure of the nuclear vorticity .

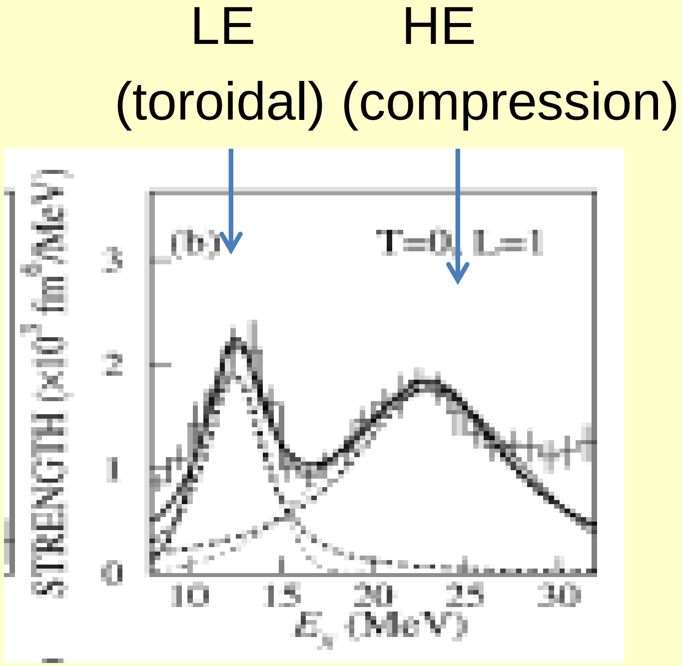
$$\langle \nu / \hat{M}_{tor}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{\sqrt{2}}{5} r^2 j_+^\nu(r) + (r^2 - \langle r^2\rangle_0) j_-^\nu(r)]$$

$$\langle \nu / \hat{M}_{com}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{2\sqrt{2}}{5} r^2 j_+^\nu(r) - (r^2 - \langle r^2\rangle_0) j_-^\nu(r)]$$

TR: experimental perspectives -1

Experiment: (α,α')

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)



(e,e'), - both IS/IV, strong magnetic form-factor
(p,p') - both IS/IV
photoabsorption, (γ,γ') - both IS/IV

} not good for IS-TR

Peripheral IS reactions (α,α') and $({}^{16,17}O,{}^{16,17}O')$ seem to be the best options:
To use $(\alpha,\alpha'\gamma)$ in deformed nuclei.
TR can be excited though its peripheral part (together with IS PDR and CR).

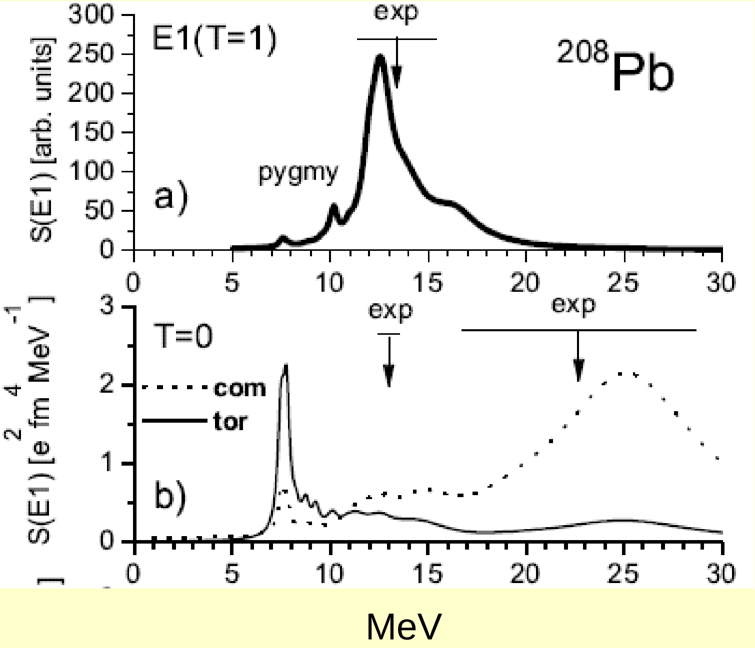
What we actually observe in (α,α')? Isoscalar PDR or TR?
This is yet unclear … .

J. Endres, et al, PRL, 105, 212503 (2010)
124Sn,$(\alpha,\alpha'\gamma)$
A. Bracco: $({}^{17}O,{}^{17}O'\gamma)$

?

So it is quite possible that PDR
is a peripheral part
of the dipole toroidal flow!

Benchmark examples



Current fields:

$$\hat{\vec{j}}(\vec{r}) = -i \sum_{q=n,p} e^q_{eff} \sum_{k \ni q} \left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k \delta(\vec{r}-\vec{r}_k)\right)$$

$$\delta\vec{j}_\nu(\vec{r}) = \left\langle \nu \,|\, \vec{j}\,(\vec{r}) \,|\, 0 \right\rangle$$

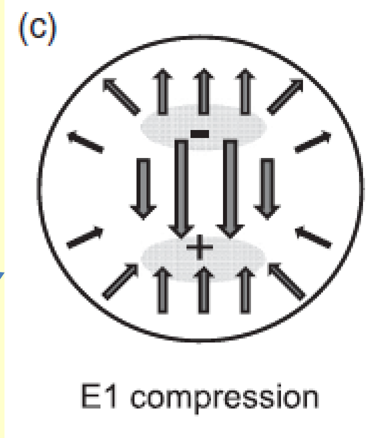
transition density
of the convection
current for the RPA state ν

T=0: $e^n_{eff} = e^p_{eff} = 1$

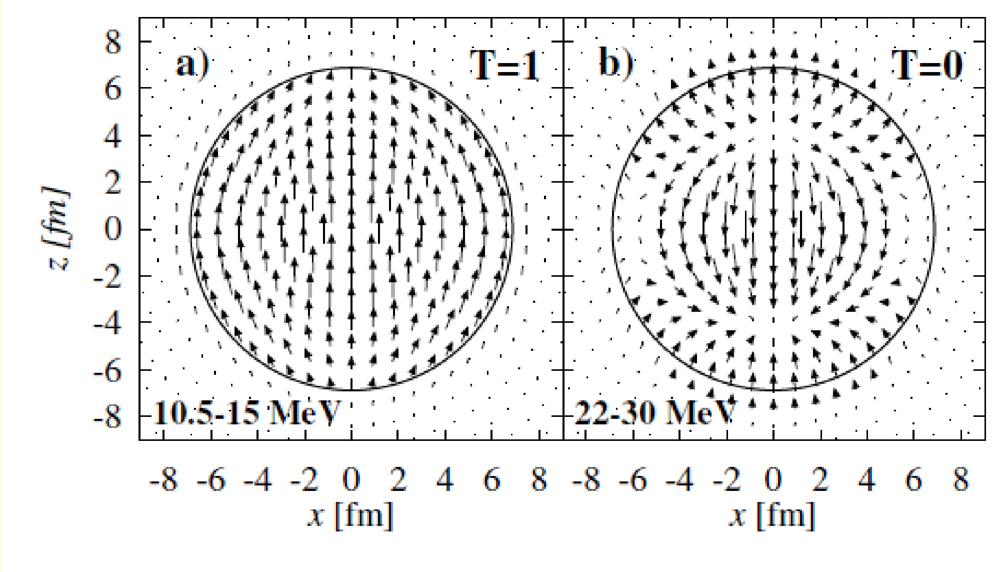
T=1: $e^p_{eff} = \frac{N}{A},\ e^p_{eff} = -\frac{Z}{A}$

p: $e^p_{eff} = 1, \quad e^n_{eff} = 0$

n: $e^p_{eff} = 0, \quad e^n_{eff} = 1$



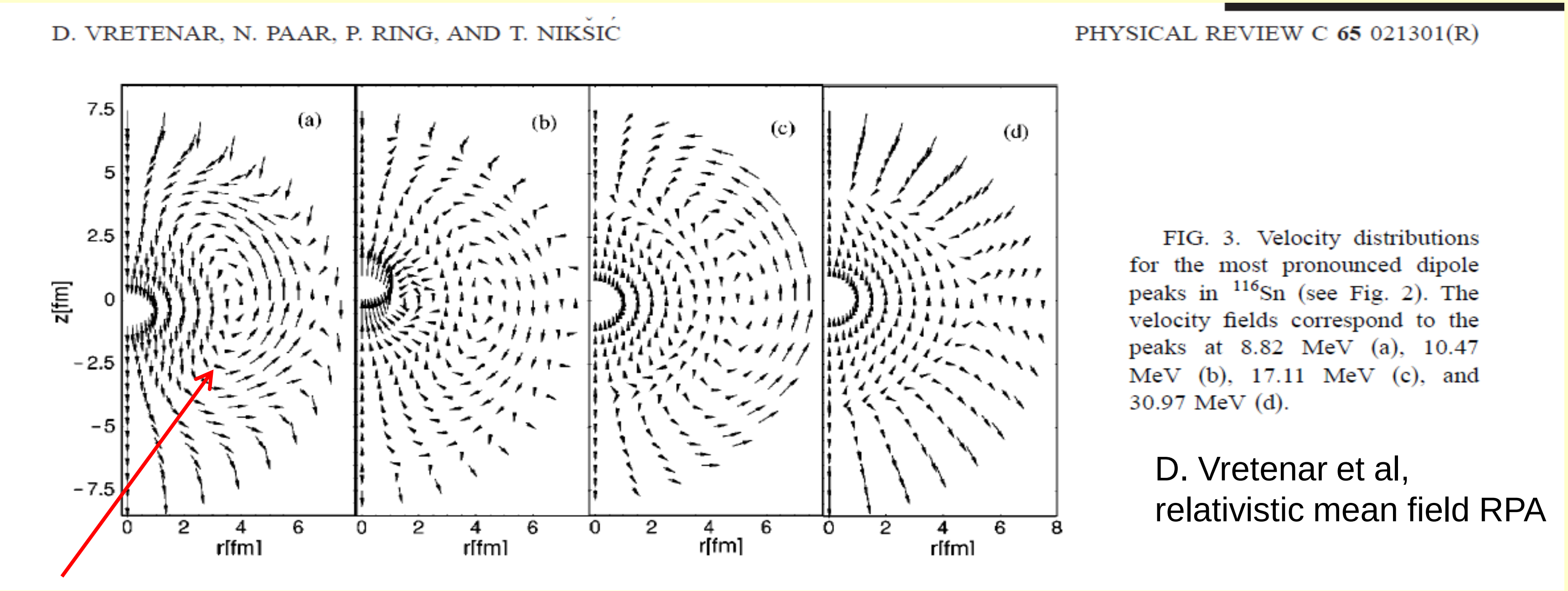
GDR compression



- good reproduction of known fields,

- justifies accuracy of our model

Previous studies



D. VRETENAR, N. PAAR, P. RING, AND T. NIKŠIĆ

PHYSICAL REVIEW C **65** 021301(R)

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D. Vretenar et al,
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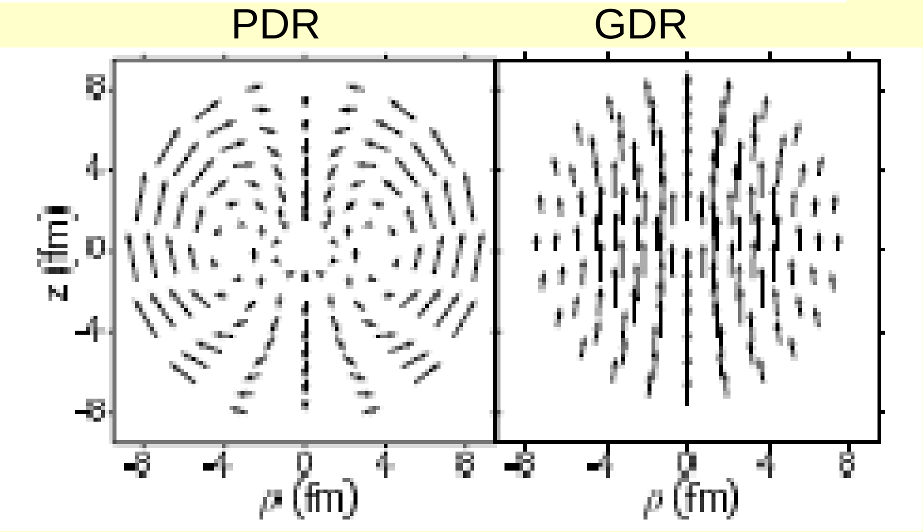

N.Ryezayeva et al, PRL 89, 272502 (2002).

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$$\delta\vec{v} = \frac{N}{A}\delta\vec{v}_p - \frac{Z}{A}\delta$$

However none of these studies has clamed the toroidal origin of PDR

The model

Ordinary dipole, toroidal, compression operators

$$\alpha = \{E1, com, tor\}$$

Strength function

$$S(E1;\omega) = \sum_{\nu\neq 0} \omega^{L} |< \Psi_{\nu} | \hat{M}_{\alpha} | 0 >|^{2} \, \varsigma(\omega-\omega_{\nu})$$

$$L = \begin{cases} 1 \text{ for E1} \\ 0 \text{ for com, tor} \end{cases}$$

$$\varsigma(\omega-\omega_{\nu}) = \frac{1}{2\pi}\frac{\Delta(\omega_{\nu})}{[(\omega-\omega_{\nu})^{2}+\frac{[\Delta(\omega_{\nu})]^{2}}{4}]} \longleftarrow$$

Lorentz weight with

$$\Delta(\omega_{\nu}) = \max\{0.4, (\omega_{\nu} - 8\,MeV)/3\}$$

Toroidal and compression operators

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[(r^{3}-\frac{5}{3}r<r^{2}>_{0})]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[(r^{3}-\frac{5}{3}r<r^{2}>_{0})Y_{1\mu}]\,[\vec{\nabla}\cdot\vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^{3}-\frac{5}{3}<r^{2}>_{0}\,r]Y_{1\mu} \qquad \hat{M}_{com}(E\lambda\mu) = -k\hat{M}'_{com}(E\lambda\mu)$$

Summed RPA transition densities and currents

$$\left.\begin{aligned}\delta\rho_\nu(\vec{r}) &= \left\langle \nu \,|\, \hat{\rho}\,(\vec{r}) \,|\, 0 \right\rangle \\ \delta\vec{j}_\nu(\vec{r}) &= \left\langle \nu \,|\, \vec{j}\,(\vec{r}) \,|\, 0 \right\rangle\end{aligned}\right\}$$

- are determined up to the general sign of RPA state ν ,
- being summed by ν may give ambiguous results

The problem may be cured by weighting TD and CTD by matrix elements

$D_{T\nu} = \left\langle \nu \,|\, \hat{D}(E1) \,|\, 0 \right\rangle$ of a probe operator $\hat{D}_{T\nu}$

$$\left.\begin{aligned}\delta\rho^{(D)}_\beta(\vec{r}) &= \left\langle \nu \,|\, \hat{\rho}\,(\vec{r}) \,|\, 0 \right\rangle = \sum_{\nu\in[\omega_1,\omega_2]} D^*_{T\nu} \sum_{q=n,p} e^q_\beta \, \delta\rho^q_\nu(\vec{r}) \\ \delta\vec{j}^{(D)}_\beta(\vec{r}) &= \left\langle \nu \,|\, \hat{\vec{j}}\,(\vec{r}) \,|\, 0 \right\rangle = \sum_{\nu\in[\omega_1,\omega_2]} D^*_{T\nu} \sum_{q=n,p} e^q_\beta \, \delta\vec{j}^q_\nu(\vec{r})\end{aligned}\right\}$$

- bilinear combinations of ν

-for the energy interval $[\omega_1,\omega_2]$

$$\hat{D}_1 = \frac{N}{A}\sum_i^Z (rY_1)_i - \frac{Z}{A}\sum_i^N (rY_1)_i$$

- relevant for photoabsorption and (e,e’)

e^q_β - effective charge

$$\hat{D}_0 = \sum_i^A (r^3 Y_1)_i$$

- relevant for isoscalar (α,α')

-The contributions of RPA states with a large D_T strength is enhanced

- There may be normalized weight

Nuclear current

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) \; + \; \hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{q}_{con}(\vec{r}) + \hat{\vec{j}}^{q}_{mag}(\vec{r}))$$

$$\hat{\vec{j}}^{q}_{con}(\vec{r}) = -ie^{q}_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right)$$

← used in the present calculations

$$\hat{\vec{j}}^{q}_{mag}(\vec{r}) = \frac{g_s}{2}\sum_{k\ni q}\vec{\nabla}_k \times \hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k)$$

T=0: $e^{n}_{eff} = e^{p}_{eff} = 1$

T=1: $e^{p}_{eff} = \frac{N}{A}, e^{p}_{eff} = -\frac{Z}{A}$

p: $e^{p}_{eff} = 1, \quad e^{n}_{eff} = 0$

n: $e^{p}_{eff} = 0, \quad e^{n}_{eff} = 1$

Center of mass corrections

$$\hat{O}=\sum_{k=1}^{A} o(\vec{r}_k) \quad\rightarrow\quad \hat{O}=\frac{1}{A}\sum_{k=1}^{A} z_k$$

$$\boxed{\begin{aligned}\delta\left\langle\hat{O}\right\rangle &= \int d\vec{r}\,\delta\rho(\vec{r})o(\vec{r})\\ &= \int d\vec{r}\,\delta\vec{j}(\vec{r})\cdot\vec{\nabla}o(\vec{r})=0\end{aligned}}$$

←

translation invariance:
perturbation $\delta\rho$ **does not change**
z-coordinate of the c.m.

$$\vec{\nabla}o(\vec{r})=\vec{\nabla}(rY_{10})=\sqrt{3}\,\vec{Y}_{100}$$

$$\sum_{\nu}\left\langle 0|\hat{\vec{j}}(\vec{r})|\nu\right\rangle\left\langle 0|\hat{F}|\nu\right\rangle=\frac{1}{2mi}\rho_0(\vec{r})\vec{\nabla}f(\vec{r})$$

$$\sum_{\nu}\omega_\nu\left\langle 0|\hat{\rho}(\vec{r})|\nu\right\rangle\left\langle 0|\hat{F}|\nu\right\rangle=-\frac{1}{2m}\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})]$$

$$\delta\vec{j}_\nu(\vec{r})=\left\langle 0|\hat{\vec{j}}(\vec{r})|\nu\right\rangle\propto\rho_0(\vec{r})\vec{\nabla}f(\vec{r})\propto\rho_0(\vec{r})\vec{v}(\vec{r})$$

$$\delta\rho_\nu(\vec{r})=\left\langle 0|\hat{\rho}(\vec{r})|\nu\right\rangle\propto\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})]\propto\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{v}(\vec{r})]$$

$$\boxed{\begin{aligned}\vec{v}_{vor}&=r^2\vec{Y}_{12\mu}+\eta\vec{Y}_{10\mu}\\ \vec{v}_{tor}&=\frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu}+(r^2-\eta)\vec{Y}_{10\mu}\\ \vec{v}_{com}&=\frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu}-(r^2-\eta)\vec{Y}_{10\mu}\end{aligned}}
\quad\Longrightarrow\quad
\boxed{\begin{aligned}\eta_{vor}&=0\\ \eta_{tor}&=\eta_{com}=\left\langle r^2\right\rangle_0\\ \eta'_{com}&=\frac{5}{3}\left\langle r^2\right\rangle_0\end{aligned}}$$