Phase diagram of hadron matter in effective theories of QCD

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The modern sketch of HIC

Experiment

The picture of the heavy ion collision's evolution

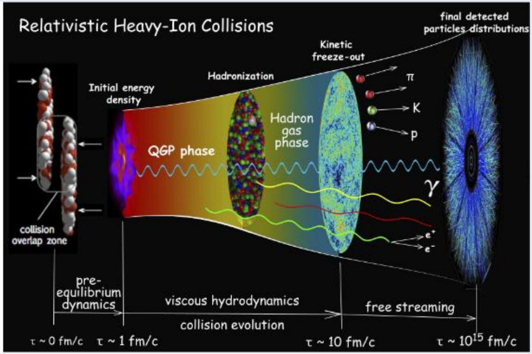
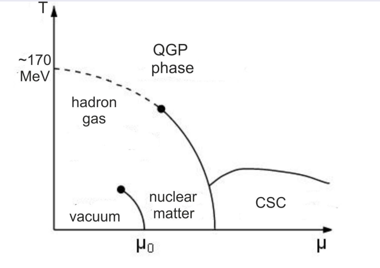


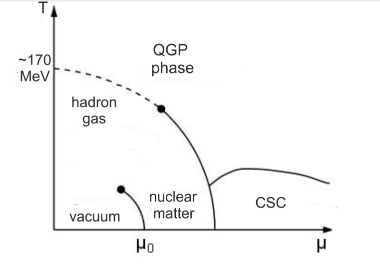
Figure 1: arXiv (nucl-th): 1304.3634

The QCD phase diagram



- chiral symmetry restoration (constituent quarks $\rightarrow$ current quarks);
- deconfinement;

The QCD phase diagram



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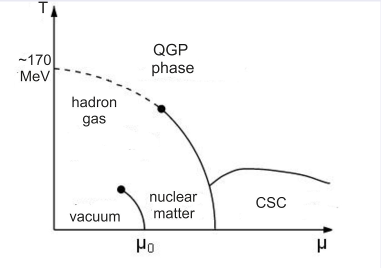
Do they coincide?

The QCD phase diagram



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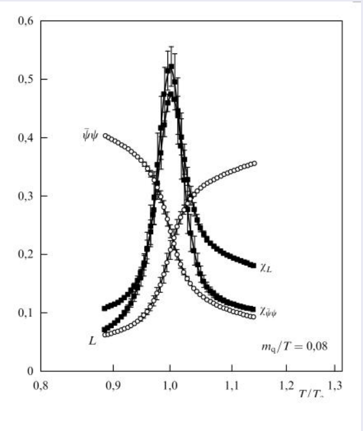
Lattice QCD

Hands S. Contemp. Phys. 42, 209 [2001], $T_c = 0.17$ GeV (SU(2))



The Nambu-Jona-Lasinio model

The Lagrangian

$$\mathcal{L}_{\rm NJL} = \bar{\rm q}\,({\rm i}\partial\!\!\!/ - \hat{\rm m}_0 - \gamma_0\mu)\,{\rm q} + {\rm G_s}\left[(\bar{\rm q}{\rm q})^2 + (\bar{\rm q}{\rm i}\gamma_5\vec{\tau}{\rm q})^2\right] \;,$$

G_s the effective coupling strength,
$\bar{q}$ и q - quark fields
$\hat{m}_0 = \mathrm{diag}\,(m_u^0, m_d^0)$, $m_u^0 = m_d^0$ - the current quark masses, $\vec{\tau}$ - Pauli matrices SU(2).

M. K. Volkov, Ann. Phys. 157,282 (1989); Sov. J. Part and Nuclei 17, 433 (1986) S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).

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We can:

- explain and describe spontaneous chiral symmetry broken as $m_q = m_0 + <\bar{q}q>$;
- describe light quarks and mesons properties,
- no deconfinement(local model) (!)

The Polyakov-loop extended Nambu-Jona-Lasinio model

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0 - \gamma_0\mu\right)q + G_s\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi, \bar{\Phi}; T\right)$$

C. Ratti, M. Thaler, W. Weise, PRD 73, 014019 (2006)

$q = (q_u, q_d)$ quark fields,

$\hat{m}_0 = \mathrm{diag}\,(m_u^0, m_d^0)$-current quark masses, $m_u^0 = m_d^0 = m_0$

$D^\mu = \partial^\mu - iA^\mu$ - covariant derivative,

$A^\mu(x) = g\mathcal{A}_a^\mu \frac{\lambda_a}{2}$, $\mathcal{A}_a^\mu$ the gauge field SU(3),

$A^\mu = \delta_0^\mu A^0 = -i\delta_4^\mu A_4$,

λ_a - Gell-Mann matrices,

G_s - scalar coupling strength.

The Polyakov field Φ is determined as: $\Phi[A] = \dfrac{1}{N_c}\mathrm{Tr}_c L(\vec{x})$,

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right],$$

$$\langle l(\vec{x})\rangle = e^{-\beta\Delta F_Q(\vec{x})}.$$

The effective potential

Polynomial fit:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

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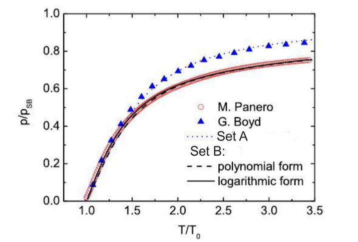
Logarithmic fit:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{1}{2}a\left(T\right)\bar{\Phi}\Phi + b\left(T\right)\ln\left[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\right],$$

$$a\left(T\right) = \tilde{a}_0 + \tilde{a}_1\left(\frac{T_0}{T}\right) + \tilde{a}_2\left(\frac{T_0}{T}\right)^2, b\left(T\right) = \tilde{b}_3\left(\frac{T_0}{T}\right)^3 .$$

The effective potential parametrization



M. Panero, PRL 103, 232001 (2009)
G. Boyd et. al, NPB 469, 419 (1996)

- $\Phi \to 1$, $p/T^4 \to 1.75$, where $T \to \infty$
- $\Rightarrow \tilde{a}_0 = 3.51$ for logarithmic fit
  $1.75 = a_0/2 + b_3/3 - b_4/4$ for polinomial fit
- $\frac{\partial \mathcal{U}(\Phi,\bar{\Phi},T)}{\partial \Phi}|\mu = 0$ ($\Phi = \bar{\Phi}$ at $\mu = 0$)
  $\Rightarrow$ the mean square method $\Rightarrow a_i$, b_i

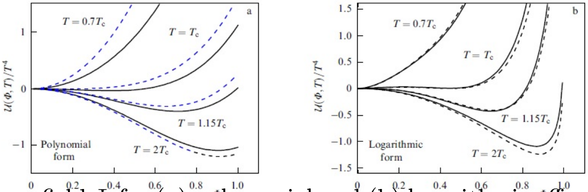
A. V. Friesen et al., IJMP A27, 1250013 (2012)

Parameters

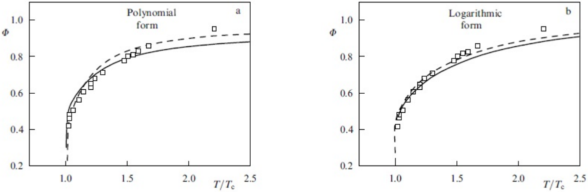
	$\tilde{a}_0$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{b}_3$	a_0	a_1	a_2	a_3	b_3	b_4
Set A	3.51	-2.47	15.2	-1.75	6.75	-1.95	2.625	-7.44	0.75	7.5
Set B	3.51	-5.121	20.99	-2.09	6.47	-4.62	7.95	-9.09	1.03	7.32

Symmetries restoration and breaking

Effective potential as function Φ for different temperatures.



Polyakov loop field Φ for (a) polynomial and (b) logarithmic effective potentials. The solid (dashed) curve corresponds to the sets B (A) parameters. (Lattice data from [Karsch F, Laermann E, Peikert A Phys. Lett. B 478 447 (2000)].)



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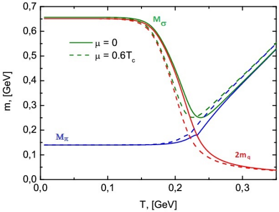
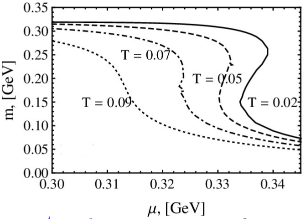


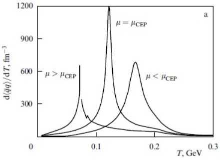


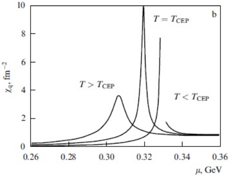
Crossover transition

$$\frac{\partial < \bar{\mathrm{q}}\mathrm{q} >}{\partial \mathrm{T}}\Big|_{\mu=\mathrm{const}}$$

1^{st} order transition: the quark susceptibility

$$\frac{\chi_{\mathrm{q}}(\mathrm{T},\mu)}{\mathrm{T}^2} = \frac{\partial^2(\mathrm{p}/\mathrm{T}^4)}{\partial(\mu/\mathrm{T})^2} = \frac{\partial}{\partial(\mu/\mathrm{T})}(\rho/\mathrm{T}^3).$$





The mean-field approximation

We can introduce the partition function

$$\mathcal{Z}[\bar{q}, q] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\int_0^\beta d\tau \int_v d^3x \left[\mathcal{L}_{NJL(PNJL)}\right]\right\}. \quad (1)$$

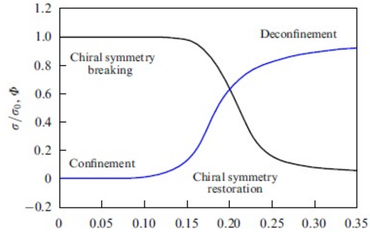
Then, using the mean-field approximation procedure, we get PNJL grand potential ($N_f = 2$):

$$\Omega(\Phi, \bar{\Phi}, m, T, \mu) = \mathcal{U}\left(\Phi, \bar{\Phi}; T\right) + G\langle\bar{q}q\rangle^2 -$$
$$-2N_cN_f \int \frac{d^3p}{(2\pi)^3} E_p - 2N_fT \int \frac{d^3p}{(2\pi)^3} \left[\ln N_\Phi^+(E_p) + \ln N_\Phi^-(E_p)\right],$$

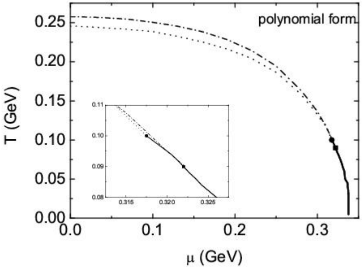
where $N_\Phi^\pm(E_p) = \left[1 + 3\left(\Phi + \bar{\Phi}e^{-\beta E_p^\pm}\right)e^{-\beta E_p^\pm} + e^{-3\beta E_p^\pm}\right]$
and $E_p = \sqrt{p^2 + m^2}$ - quark energy; $E_p^\pm = E_p \mp \mu$. The equations of motion

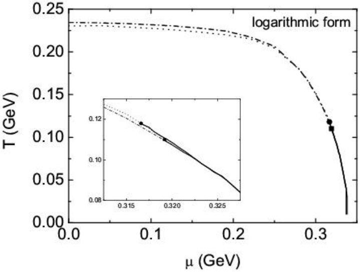
$$\frac{\partial \Omega_{MF}}{\partial \sigma_{MF}} = 0, \quad \frac{\partial \Omega_{MF}}{\partial \Phi} = 0, \frac{\partial \Omega_{MF}}{\partial \bar{\Phi}} = 0.$$

Phase diagram of PNJL model



Parameters: $m_0, \Lambda, G_s, a_i, b_i, T_0 = 0.27$ ГэВ





PNJL with vector interaction

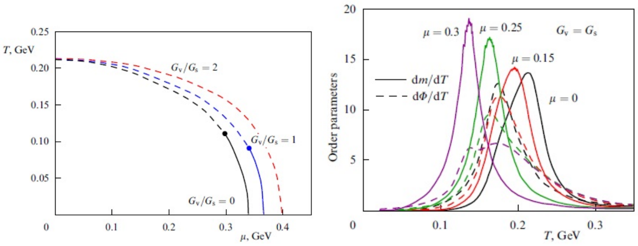
Introduction of vector interaction into model

$$\mathcal{L}_{\rm PNJL} \quad = \quad \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0 - \gamma_0\mu\right)q + G_s\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - G_v(\bar{q}\gamma_\nu q)^2 - \mathcal{U}\left(\Phi,\bar{\Phi};T\right)$$

leads to re-normalization of chemical potential:

$$\tilde{\mu} = \mu - 4G_vN_cN_f\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\left[f_\Phi^+ + f_\Phi^-\right].$$

$T_0 = 0.19$ GeV



A. V. Friesen et al. IJMP A 30, 1550089 (2015); Phys. Usp. 59 (4) 367 (2016)

Extended PNJL

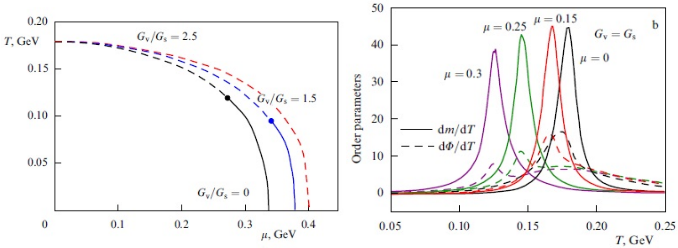
It is possible to introduce a phenomenological dependence of $G_s(\Phi)$ and $G_v(\Phi)$:

$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)],$$
$$\tilde{G}_v(\Phi) = G_v[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)],$$

with $\alpha_1 = \alpha_2 = 0.2$.

Y. Sakai et al PRD 82, 076003 (2010)

P. de Forcrand, O. Philipsen NPB 642, 290(2002)

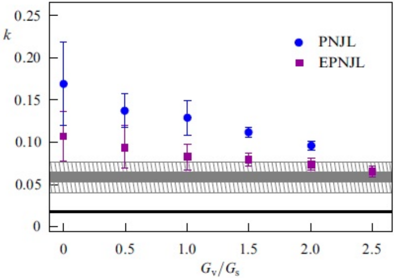


A. V. Friesen et al. IJMP A30 1550089 (2015)

Crossover curvature

It was suggested that critical curves for all physical quantities (chiral condensate, quark susceptibility, strange quark susceptibility, Polyakov loop) must meet at one point, which is the CEP (Kaczmarek O. et al. PRD 83, 014504 (2011)).

$$\frac{\mathrm{T_c}(\mu)}{\mathrm{T_c}(0)} = 1 - \mathrm{k}\left(\frac{\mu}{\mathrm{T_c}(\mu)}\right)^2 .$$

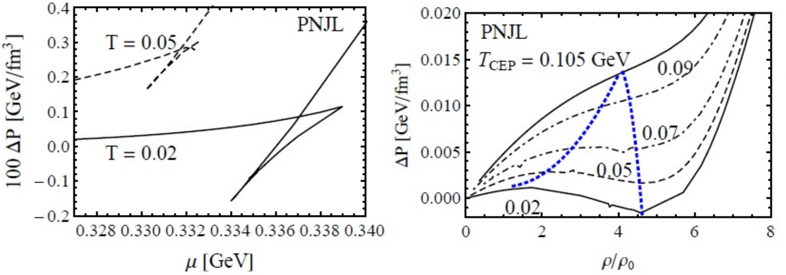


Enrödi G. JHEP (4) 1, 2011; Cea P. PRD89, 074512 (2014)

Equation of state

Using the grand canonical potential Ω we can build all thermodynamic quantities in theory:

$$\mathrm{p} = -\frac{\Omega}{\mathrm{V}}, \quad \mathrm{s} = -\left(\frac{\partial\Omega}{\partial \mathrm{T}}\right)_{\mu}, \quad \varepsilon = -\mathrm{p} + \mathrm{Ts} + \mu\rho, \quad \rho = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{\mathrm{T}}.$$



The mesons correlations

The partition function:

$$\mathcal{Z}[\bar{\mathrm{q}},\mathrm{q}] = \int \mathcal{D}\bar{\mathrm{q}}\mathcal{D}\mathrm{q}\exp\left\{\int_0^\beta \mathrm{d}\tau \int_\mathrm{v} \mathrm{d}^3\mathrm{x}\left[\mathcal{L}_{\mathrm{NJL(PNJL)}}\right]\right\}.$$

During the mean field approximation procedure we neglect the part of integral which is responsible for correlations:

$$\begin{aligned}\mathcal{Z}_{\mathrm{FL}}[\mathrm{T},\mathrm{V},\mu] \quad = \quad & \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\exp\left\{-\left[\int_0^\beta \mathrm{d}\tau\int_\mathrm{V}\mathrm{d}^3\mathrm{x}\,\frac{2\sigma\sigma_{\mathrm{MF}}+\sigma^2+\vec{\pi}^2}{4\mathrm{G_s}}\right]\right. \\ & \left. +\mathrm{Tr}\ln\left[1-\mathrm{S_{MF}}[\mathrm{m}]\left(\sigma+\mathrm{i}\gamma_5\vec{\tau}\vec{\pi}\right)\right]\right\},\end{aligned}$$

which eventually can be rewritten as

$$\mathcal{Z}_{\mathrm{FL}}^{(2)}[\mathrm{T},\mathrm{V},\mu] = \left[\det\left(\mathrm{D}_\sigma^{-1}\right)\right]^{-\frac{1}{2}}\left[\det\left(\mathrm{D}_\pi^{-1}\right)\right]^{-\frac{3}{2}}$$

with meson propagator

$$\mathrm{D}_\mathrm{M}^{-1} = \frac{1}{2\mathrm{G_s}} - \Pi_\mathrm{M}(\mathrm{q}_0,\vec{\mathrm{q}}).$$

The mesons correlations

The meson pressure:

$$\Omega_{\mathrm{M}}^{(2)}(\mathrm{T},\mu) = -\frac{\mathrm{N_M}}{2}\int \frac{\mathrm{d}^3\mathrm{q}}{(2\pi)^3}\int_0^{+\infty}\frac{\mathrm{d}\omega}{\pi}\frac{\mathrm{d}}{\mathrm{d}\omega}\left[-\omega + \mathrm{T}\ln[1-\mathrm{e}^{\beta(\omega-\mu)}] + \mathrm{T}\ln[1-\mathrm{e}^{\beta(\omega+\mu)}])\right]\frac{\mathrm{d}\Phi_{\mathrm{M}}(\omega,\vec{\mathrm{q}})}{\mathrm{d}\omega}$$

The phase shift

$$\Phi_{\mathrm{M}} = \frac{1}{2\mathrm{i}}\ln\mathcal{S}_{\mathrm{M}}(\omega,\vec{\mathrm{q}}) = \frac{1}{2\mathrm{i}}\ln\frac{1-2\mathrm{G_s}\Pi_{\mathrm{M}}(\omega-\mathrm{i}\eta,\overrightarrow{\mathrm{q}})}{1-2\mathrm{G_s}\Pi_{\mathrm{M}}(\omega+\mathrm{i}\eta,\overrightarrow{\mathrm{q}})}.$$

The phase shift is determined by polarization operator. In case pole approximation

$$1-2\mathrm{G_s}\Pi_{\mathrm{M}}(\omega,\vec{\mathrm{q}}) = (\omega^2-\mathrm{E_M^2})\times \mathrm{g^2_{Mqq}},$$

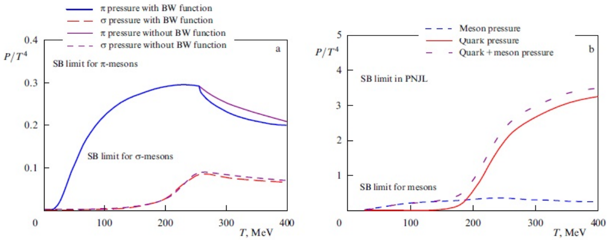
we have noninteracting mesons gas

$$\Omega_{\mathrm{M}} = \frac{\mathrm{N_M}}{2}\int\frac{\mathrm{d}^3\mathrm{q}}{\mathrm{q}}\left[\mathrm{E_M} + \mathrm{T}\ln[1-\mathrm{e}^{-\beta(\mathrm{E_M}-\mu)} + \mathrm{T}\ln[1-\mathrm{e}^{-\beta(\mathrm{E_M}+\mu)}]\right].$$

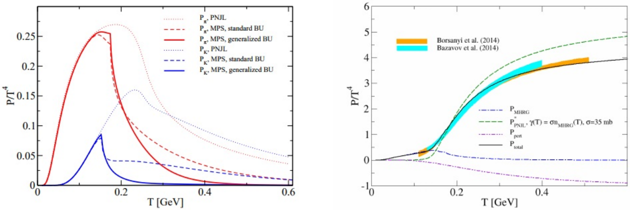
By introducing the width of mesons, we can take into account meson correlations

$$\frac{\mathrm{d}\Phi_{\mathrm{R}}(\mathrm{s},\mathrm{T})}{\mathrm{ds}} = \mathrm{A_R}(\mathrm{s},\mathrm{T}) = \mathrm{a_R}\frac{\mathrm{M_M}\Gamma_{\mathrm{M}}}{(\mathrm{s}-\mathrm{M_M})^2+(\mathrm{M_M}\Gamma_{\mathrm{M}})^2} = \frac{\pi}{\frac{\pi}{2}+\mathrm{arctg}\left(\frac{\vec{\mathrm{q}}^2+\mathrm{M_M^2}}{\mathrm{M_M}\Gamma_{\mathrm{M}}}\right)}\frac{\mathrm{M_M}\Gamma_{\mathrm{M}}}{(\mathrm{s}-\mathrm{M_M})^2+(\mathrm{M_M}\Gamma_{\mathrm{M}})^2}.,$$

A. Wergieluk et al. Phys.Part.Nucl.Lett. 10 (2013) 660-668;



D. Blaschke et al.: arXiv: 1612.09556; 1511.00338;



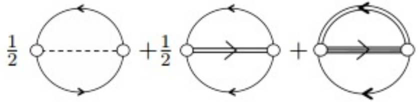
Conclusions and outlook

- NJL-like models let to describe the structure of phase diagram, light mesons and quarks properties, scattering and decay processes;
- describe the confinement properties & describe the chiral symmetry; check how additional interactions (vector interaction and extended couplings) effect on phase diagram;
- meson correlations can be taken into account

New ideas... Φ-derivable model

$$\Omega = \sum_{i=q,M,D,B\ldots} \frac{c_i}{2}\left\{\mathrm{Tr}\ln(G_i^{-1} - \mathrm{Tr}(\Sigma_i G_i))\right\} + \Phi(G_i)$$

$\Phi[\{G_i\}] =$



$\Sigma_q = \frac{\delta\Phi}{\delta G_q} =$

Thank you for attention