Cluster approach to the structure of heavy nuclei

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- *Clustering in medium and heavy mass nuclei*

Model

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- *Hamiltonian*
- *Multipole moments*

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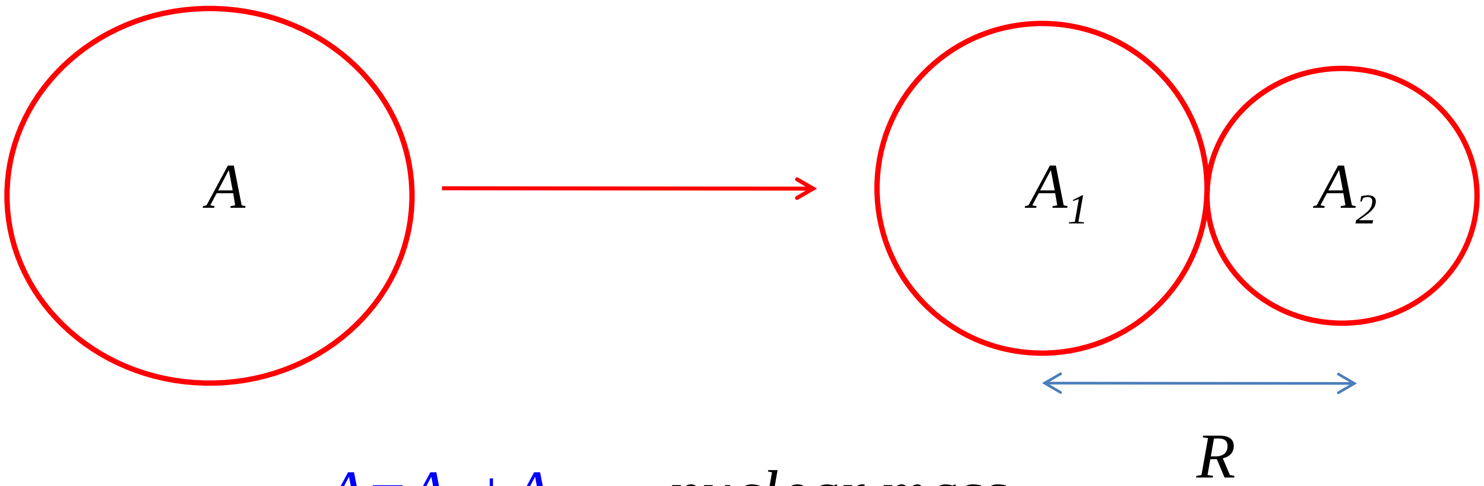
- *Applications to the actinides and rare-earth nuclei*
- *Multiple reflection-asymmetric bands in* ^{240}Pu

Summary

Clusters in nuclei



$A=A_1+A_2$ -- *nuclear mass*

R -- *relative distance*

$\xi=2A_2/A$ – *mass asymmetry*

Light nuclei : ξ is fixed, dynamics in R

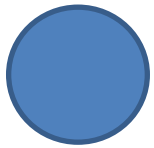
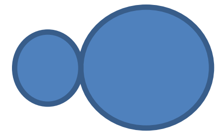
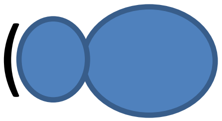
$$\psi_{ijk}\left(\vec{r}_1,\dots,\vec{r}_{A_0},\vec{R}\right)=\hat{A}\left[\phi_i(A_1)\phi_j(A_2)\chi_k\left(\vec{R}\right)\right]$$

Heavy nuclei: R is fixed in touching, dynamics in ξ

$$\Psi\left(\vec{r}_1,\dots,\vec{r}_{A_0}\right)=\sum_h\sum_{ijk}a^h_{ijk}\psi^h_{ijk}\left(\vec{r}_1,\dots,\vec{r}_{A_0},\vec{R}_{touch}\right)$$

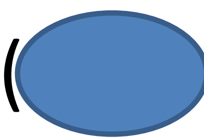
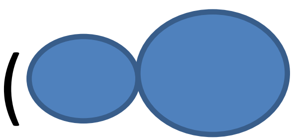
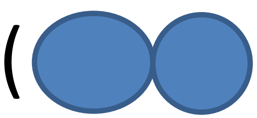
Dinuclear system (DNS) concept

The intrinsic nuclear wave function is a superposition of the mononucleus and different cluster configurations.

$\psi(^{A}Z)= \alpha\psi_{m}($  $)+\beta\psi_{\alpha}($  $)+\gamma\psi_{Li}($  $)+\ldots$

mononucleus $\quad$ $^{(A-4)}(Z-2)+{}^{4}He$ $\quad$ $^{(A-7)}(Z-3)+{}^{7}Li$

-Ground state spectra of actinides (T.M. Shneidman et al., EPJ A47, 34, 2011)
-Alpha-decay properties (S.N. Kuklin et al., EPJ A48, 112, 2012)

$\psi(^{A}U)=\delta\,\psi_{m}($  $)+\epsilon\psi_{Sr}($  $)+\chi\psi_{Zr}($  $)+\ldots$

mononucleus $\quad$ $^{A1}Sr+{}^{A2}Xe$ $\quad$ $^{A1}Zr+{}^{A2}Te$

The weight of dinuclear systems are determined by the potential energy in mass asymmetry coordinate.

Cluster effects in the structure of nuclei

There are many theoretical and experimental indications that clustering is a common feature of nuclei in different mass regions and for different deformations.

- SD and predicted HD bands in medium-light nuclei with $N \approx Z$

W.D.M. Rae, IJMP A3, 1343 (1988), M.Freer et al., NPA 587, 36 (1995)&JPG 23, 261 (1997), H. Horiuchi, NPA552, 257 (1991), F. Michel et al., Prog. Theor. Phys. Suppl. 132, 7 (1998), G.G. Adamian et al., PRC67, 054303 (2003), J. Darai et al., PRC 84, 024302 (2011).

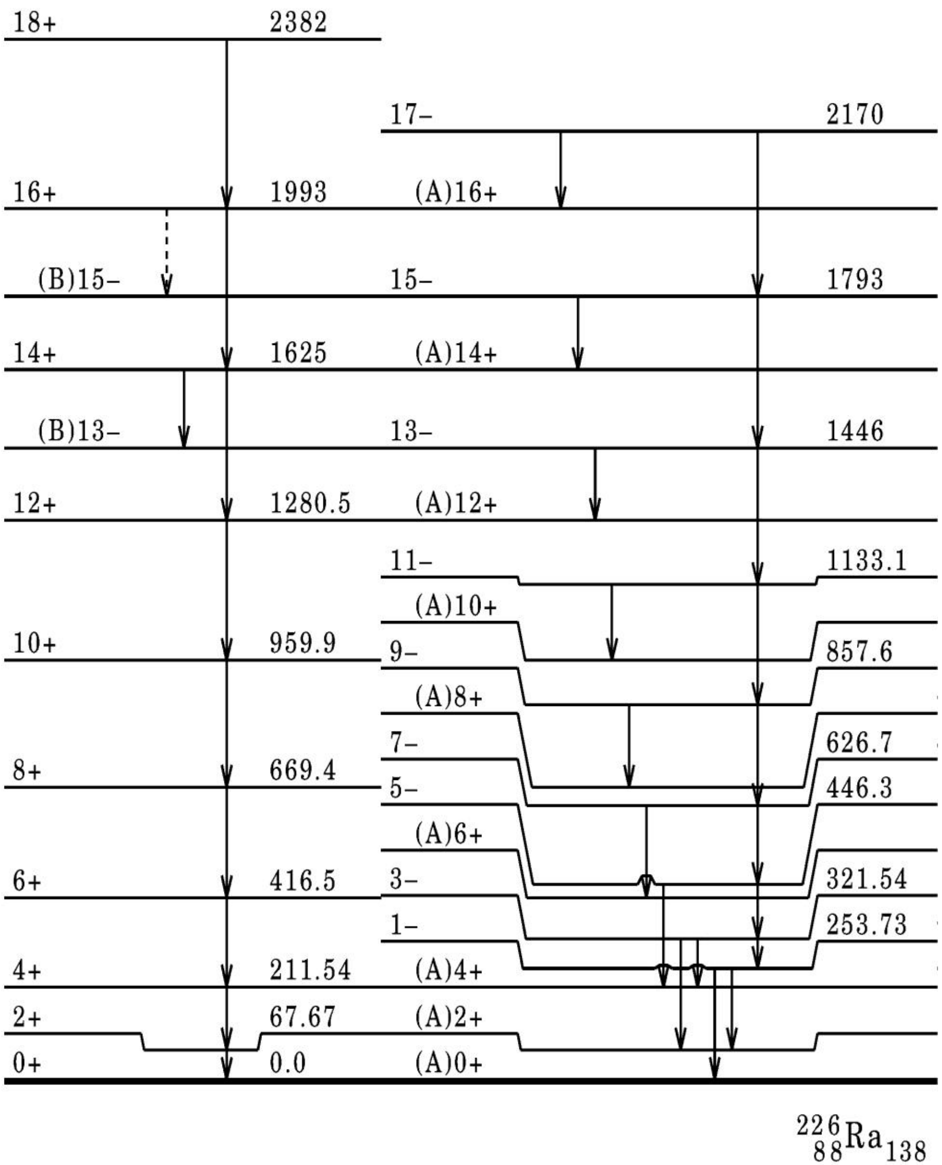
- Reflection-Asymmetric Deformations in actinides and rare-earth nuclei

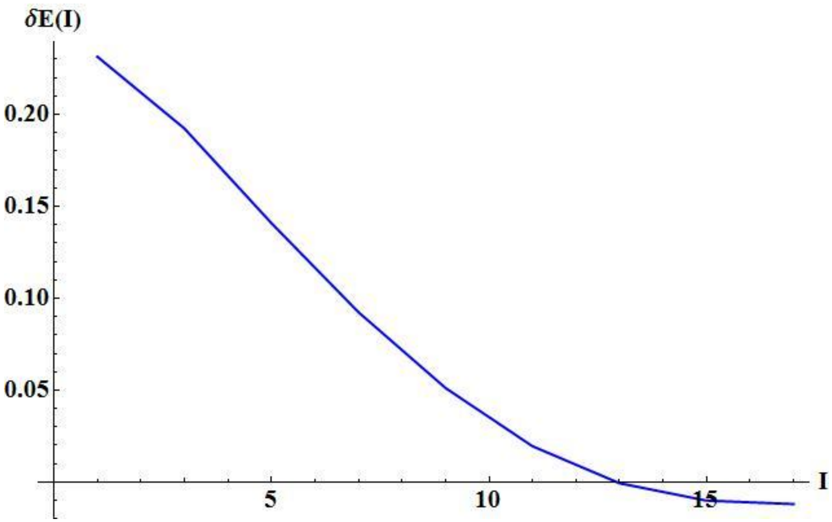
Yu. S. Zamiatin et al., Phys. Part. Nucl. 21, 537 (1990),
B. Buck et al, PRC58, 2049 (1998)&PRC61, 024314 (2000),
T.M. Shneidman et al., PLB526, 322 (2002)&EPJA 47, 34 (2011)

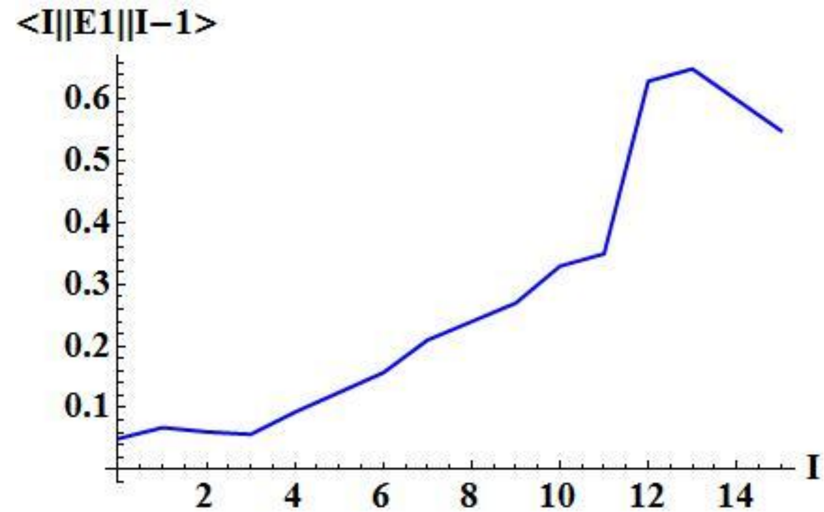
- Strongly elongated fission isomers in heavy nuclei.

V.V. Pashkevich et al., NPA 624, 140 (1997), S. Aberg et al., Z. Phys. A349, 205 (1994), S. Cwiok et al., PLB 322, 304 (1994)

Excitation spectrum of nucleus with R.-A. deformation







Reflection Asymmetric Deformation in Heavy Nuclei

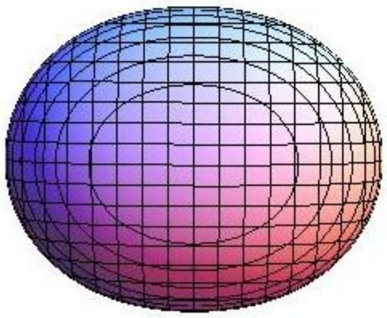
- Low negative parity rotational bands
- Strong dipole (*E1)* and octupole (*E3*) transitions connect negative parity states with members of the ground state band.

Conclusion:

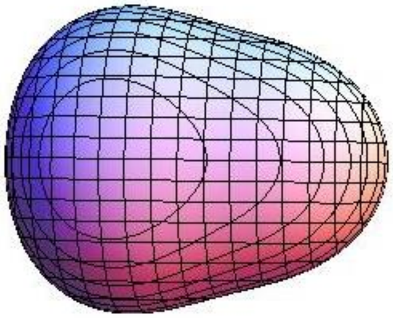
Some nuclei (actinides and rare-earth nuclei) might have **reflection asymmetric shapes**

Example (octupole deformation)

$$R(\Omega) = c(\beta)R_0 \left[1 + \sum_{\mu=-2}^{2} \beta_{2\mu} Y^*_{2\mu}(\Omega) + \sum_{\mu=-3}^{3} \beta_{3\mu} Y^*_{3\mu}(\Omega)\right]$$



β_{20}=0.6, β_{30}=0.0

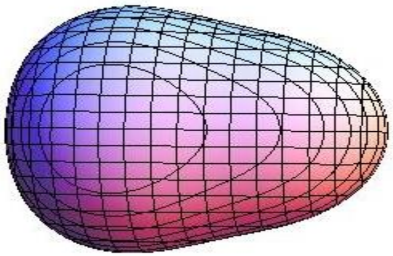


β_{20}=0.6, β_{30}=0.5

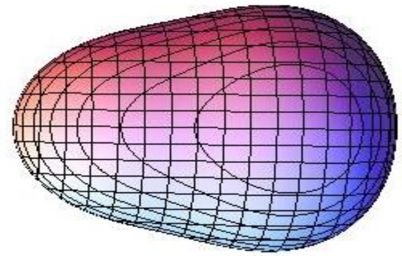
Reflection Asymmetric Deformation

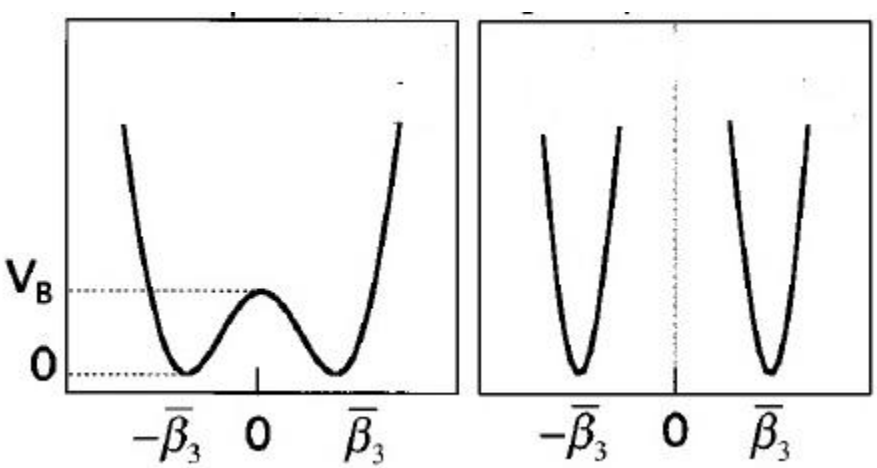
Intrinsic states $\Psi(\beta_{30})$ and $\Psi(-\beta_{30})$ are physically equivalent.

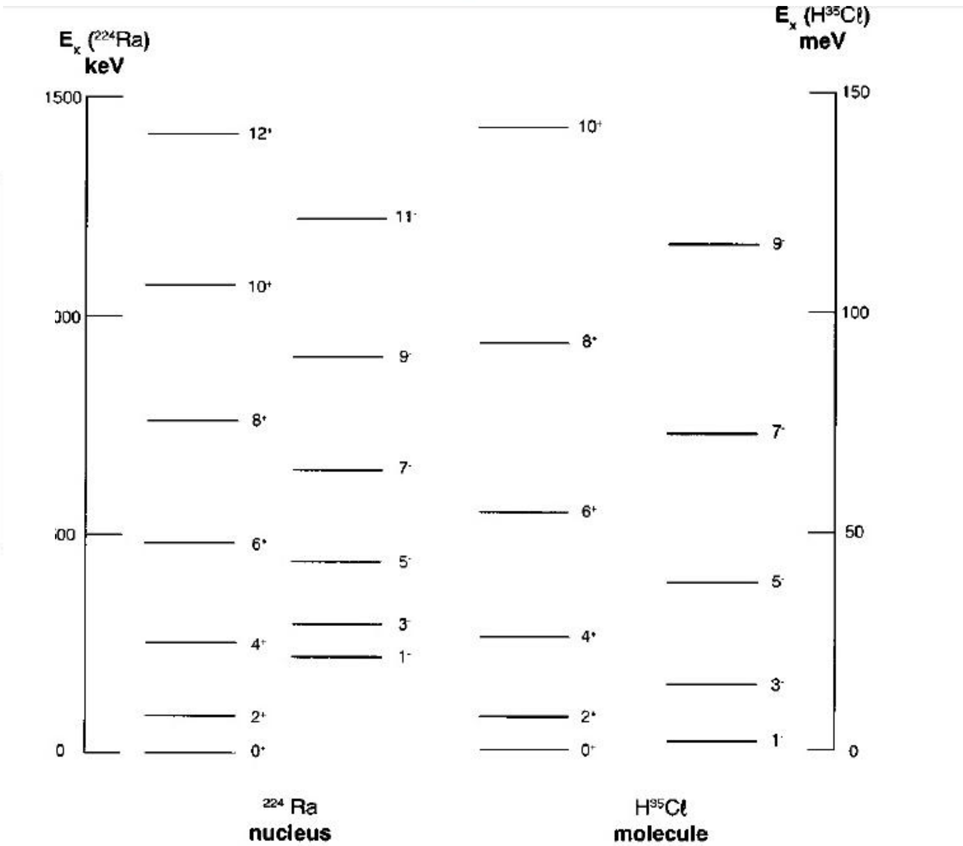
β_{20}=0.6, β_{30}=-0.5



β_{20}=0.6, β_{30}=0.5







Motion in mass asymmetry

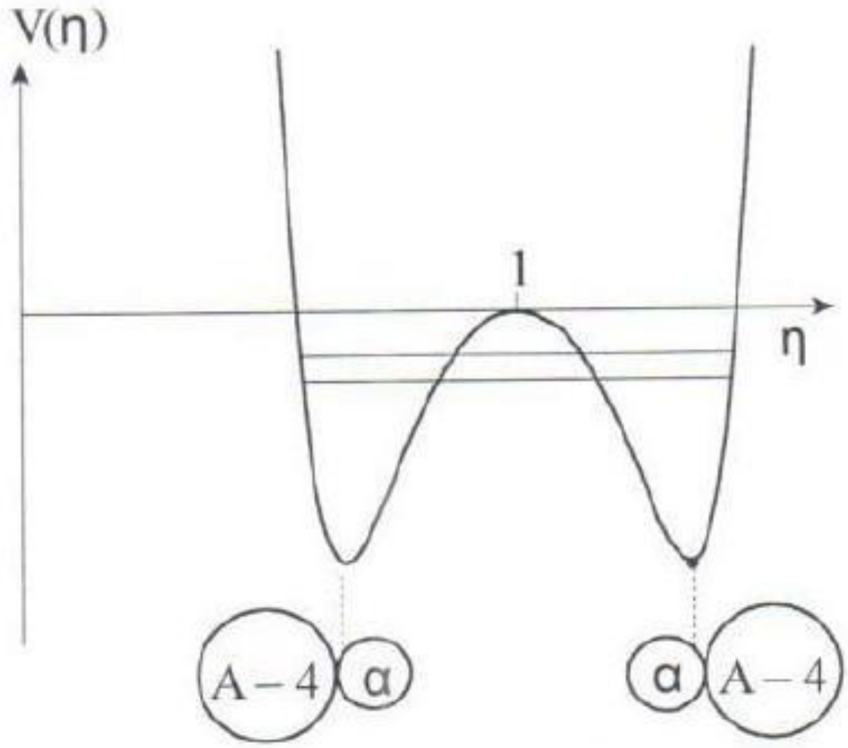
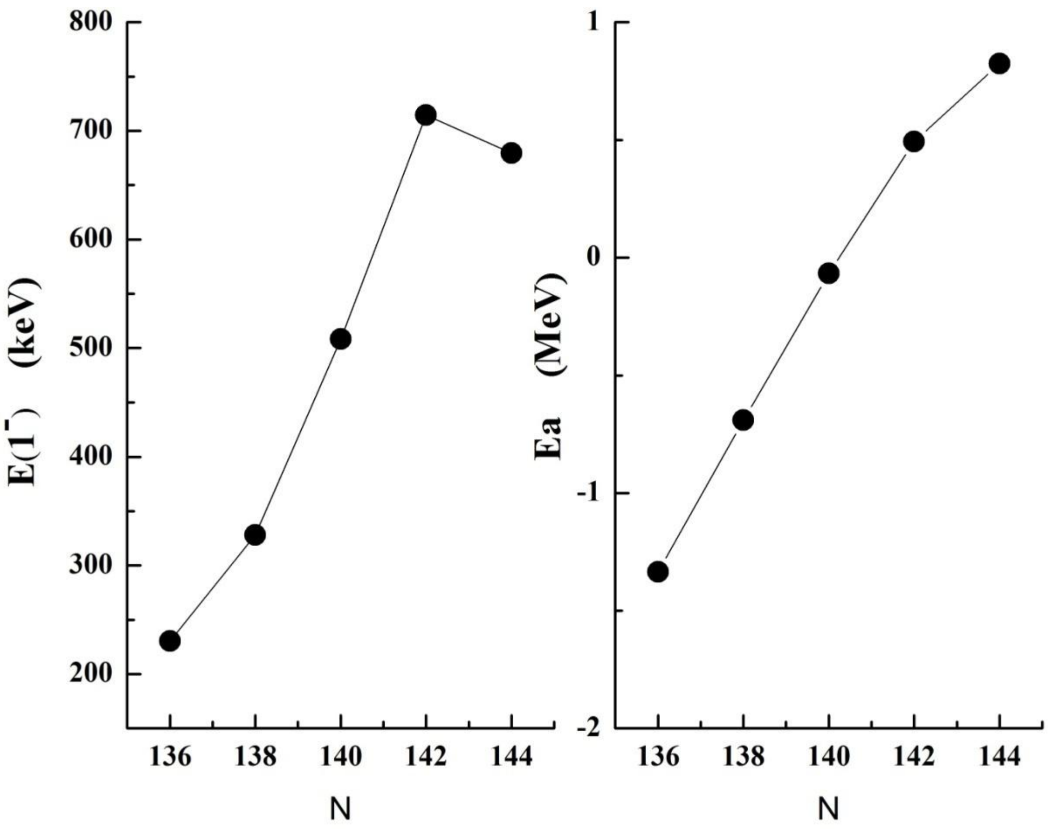


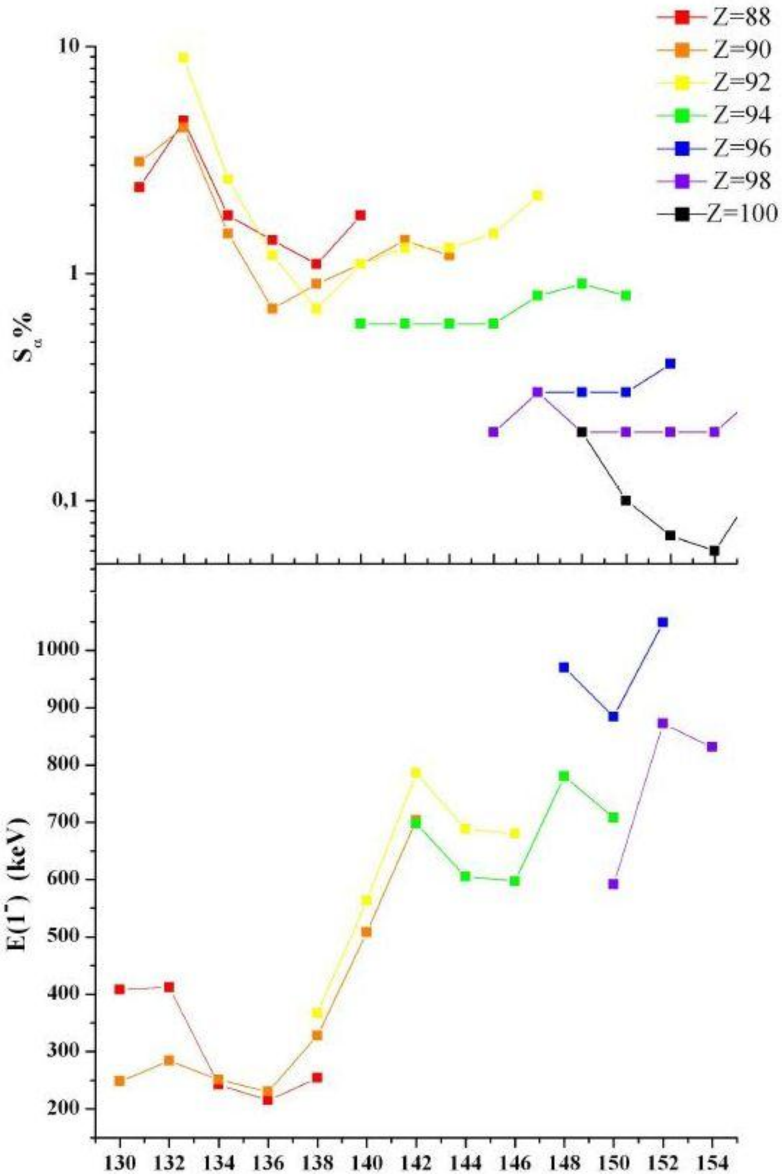
Fig. 2. Schematic picture of the potential in the mass asymmetry and of the two states with different parities (parallel lines, lower state is with positive parity, higher state is with negative parity).

Energy of α–DNS for Th isotopes



The dynamics of the reflection asymmetric collective motion can be treated as the motion in mass-asymmetry degrees of freedom.

Role of α–DNS



The value of α-particle preformation factor obtained from the experiment as:

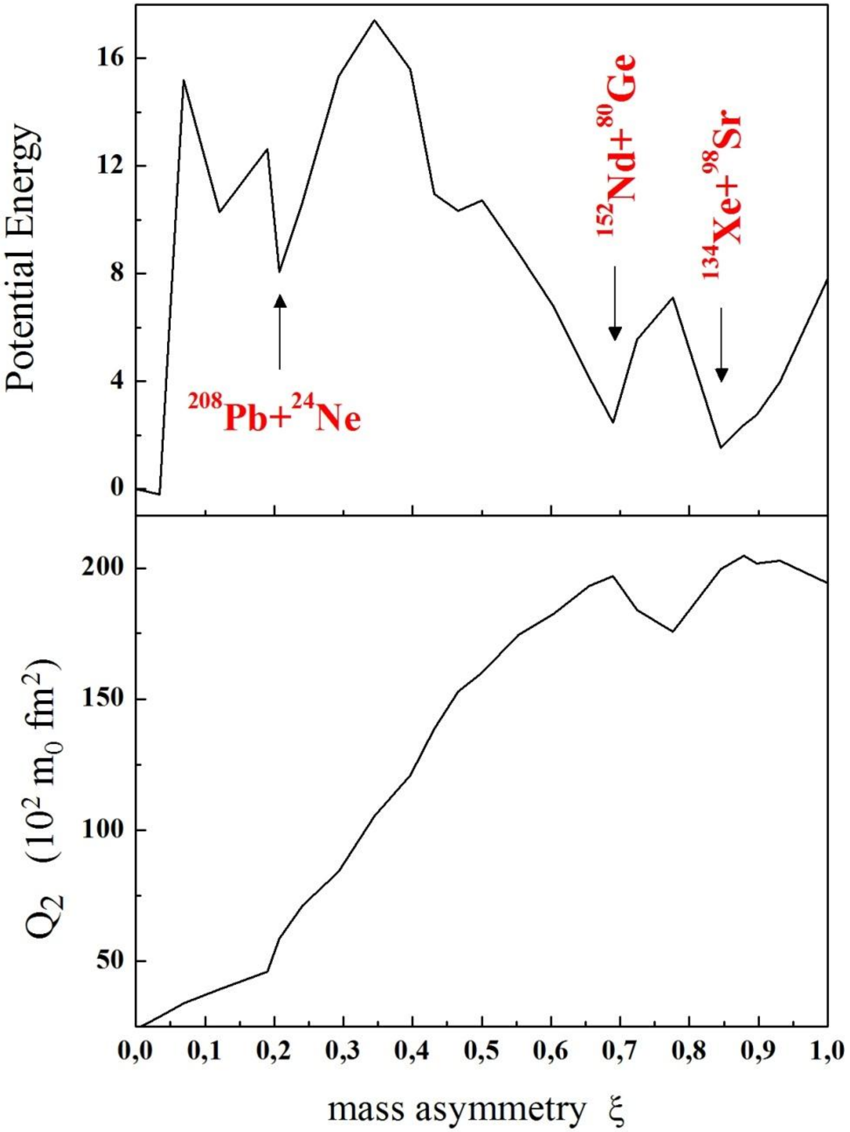
$$S_\alpha^{exp} = T_{1/2}^{\alpha}/T_{1/2}^{exp}$$

$T_{1/2}^{\alpha}$ – *half-life of α-particle dinuclear system.*

Energies of $E(1^-)$ – *states as a function of neutron number.*

Dinuclear System (DNS) model

Driving Potential for ^{232}U



The potential energy of the DNS

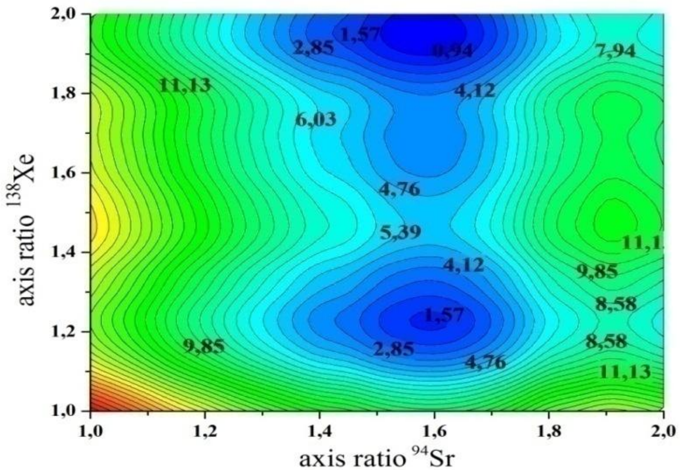
$$V(\xi) = E_1(\xi) + E_2(\xi) + V_N(R,\xi) + V_C(R,\xi)$$

Mass quadrupole moments of the DNS

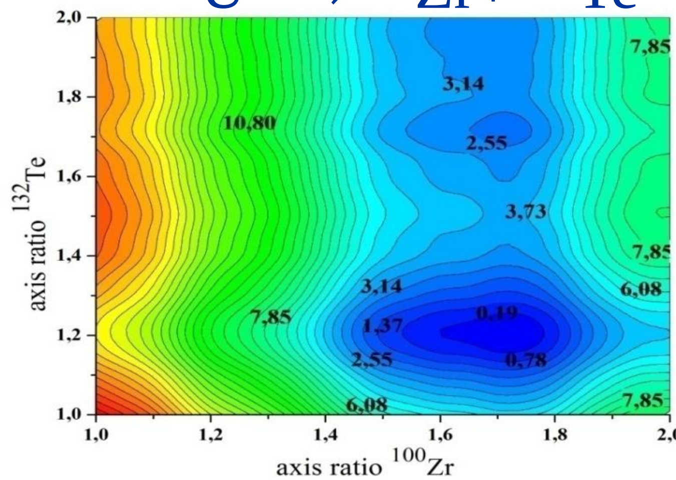
$$Q_2(\xi, R) = 2m_0 \frac{A_1 A_2}{A_1 + A_2} R^2 + Q_2(A_1) + Q_2(A_2)$$

Hyperdeformation as a cluster state

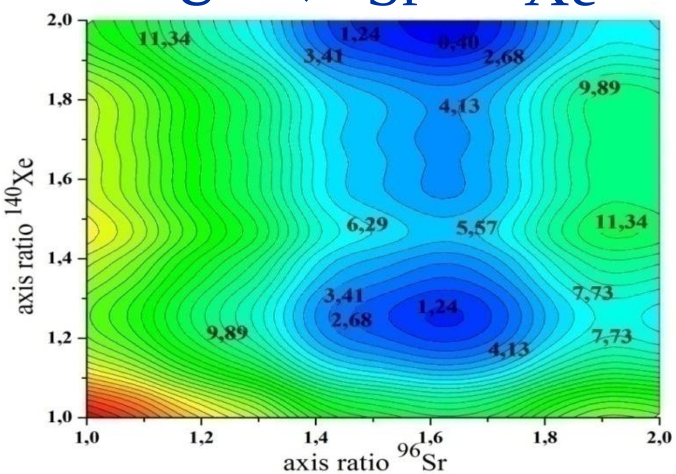
$^{232}U \longrightarrow {}^{94}Sr + {}^{138}Xe$



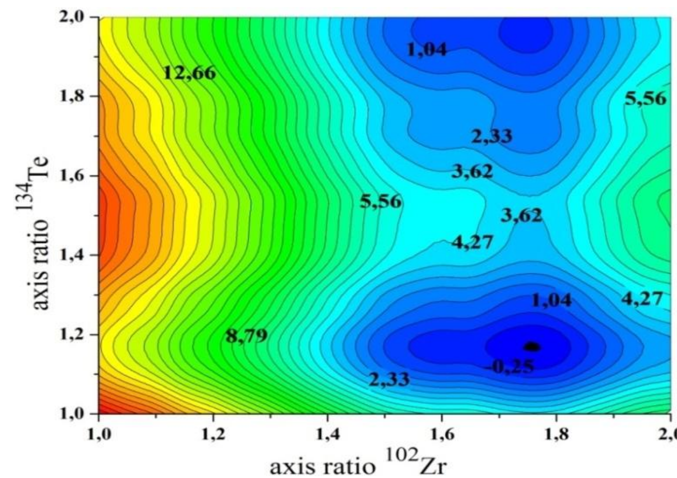
$^{232}U \longrightarrow {}^{100}Zr + {}^{132}Te$



$^{236}U \longrightarrow {}^{96}Sr + {}^{140}Xe$



$^{236}U \longrightarrow {}^{102}Zr + {}^{134}Te$



Characteristics of HD minima in U isotopes

Nucleus	^{232}U	^{234}U	^{236}U	^{238}U
DNS	^{94}Sr+^{138}Xe	^{96}Sr+^{138}Xe	^{96}Sr+^{140}Xe	^{98}Sr+^{140}Xe
Energy (MeV)	3.06	2.6 (3.1±0.4)	2.81 (2.7±0.4)	3.49
Rot. Const. (keV)	1.825 (1.96±0.11)	1.772 (2.1±0.2)	1.751 (2.4±0.4)	1.697
Q_2 (10^2 e fm^2)	92.37	93.021	93.466	96.772
Q_3 (10^3 e fm^3)	29.96	28.48	29.92	27.84

Exp: L. Csige et al., Journal of Physics: Conference Series **312** (2011) 092022;
A. Krasznahorkay et. al, AIP Conf. Proc. 819, 439 (2006) .

Dinuclear system (DNS) concept

The intrinsic nuclear wave function is a superposition of the mononucleus and different cluster configurations. The weight of different cluster components are determined by the Schrodinger equation in mass-asymmetry coordinate.

$$\Psi_{p,IMK} = \sqrt{\frac{2I+1}{16\pi^2}}\left(\Phi_{n,K}(\xi)D^I_{MK} + p(-1)^{I+K}\Phi_{n,\overline{K}}(\xi)D^I_{M,-K}\right)$$

Wave function in ξ defined by the equation:

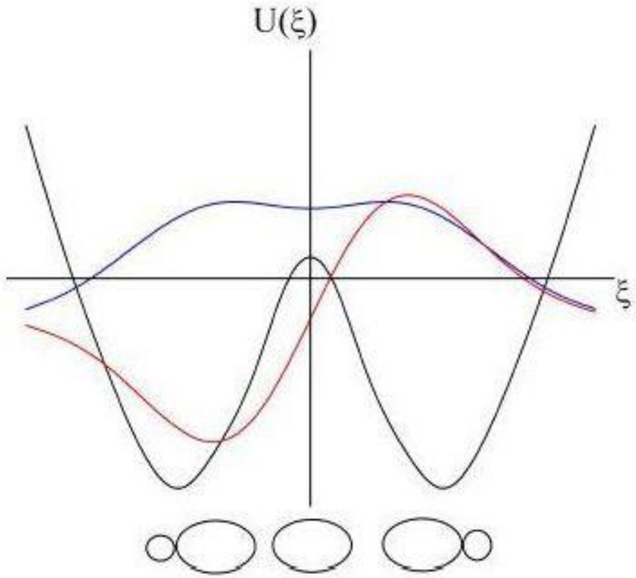
$$\left(-\frac{\hbar^2}{2B_\xi}\frac{d^2}{d\xi^2} + U(\xi) + \frac{\hbar^2}{2\Im(\xi)}I(I+1)\right)\Psi_{n,K}(\xi) = E_{n,K}\Psi_{n,K}(\xi),$$

where

$$\Im(\xi) = 0.85(\Im_1^r + \Im_2^r + m_0\frac{A_1A_2}{A}R^2)$$

Exitation spectra:

$$I^p(\text{ for } K=0) = 0^+, 1^-, 2^+ \ldots$$
$$I^p(\text{ for } K\neq 0) = K^\pm, (K+1)^\pm \ldots$$



Potential Energy of the Dinuclear System

$$U(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = B_1(\beta^{(1)}) + B_2(\beta^{(2)}) - B_{12} + V(R,\xi,\beta_{2\mu})$$

where, B_1, B_2 and B_{12} are the binding energies of the fragments and the compound nucleus, respectively.

The nucleus-nucleus potential

$$V(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = V_{Coul}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) + V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$$

is the sum of the nuclear interaction potential $V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$

and of the Coulomb potential

$$V_{Coul}(R,\xi,\beta_{2\mu}) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3} R_{01}^2 \sum_{i,\mu} \beta_{2\mu}^{(i)*} Y_{2\mu}(\theta_i,\phi_i) + \ldots$$

Nuclear Interaction in Dinuclear System

$$V_{nucl}(R, \xi, \beta_{2\mu}) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{R} - \mathbf{r}_2)F(\mathbf{r}_1 - \mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2$$

$$\rho_i(\mathbf{r}) = \frac{\rho_{00}}{1 + \exp\left(\frac{s(\mathbf{r})}{a_{0i}}\right)}, \quad \rho_{00} = 0.17 \text{ fm}^{-3}$$

$$F(\mathbf{r}_1 - \mathbf{r}_2) = C_0\left(F_{in}\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}} + F_{ex}\left(1 - \frac{\rho_0(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r})$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1 - Z_1}{A_1}\,\frac{N_2 - Z_2}{A_2}$$

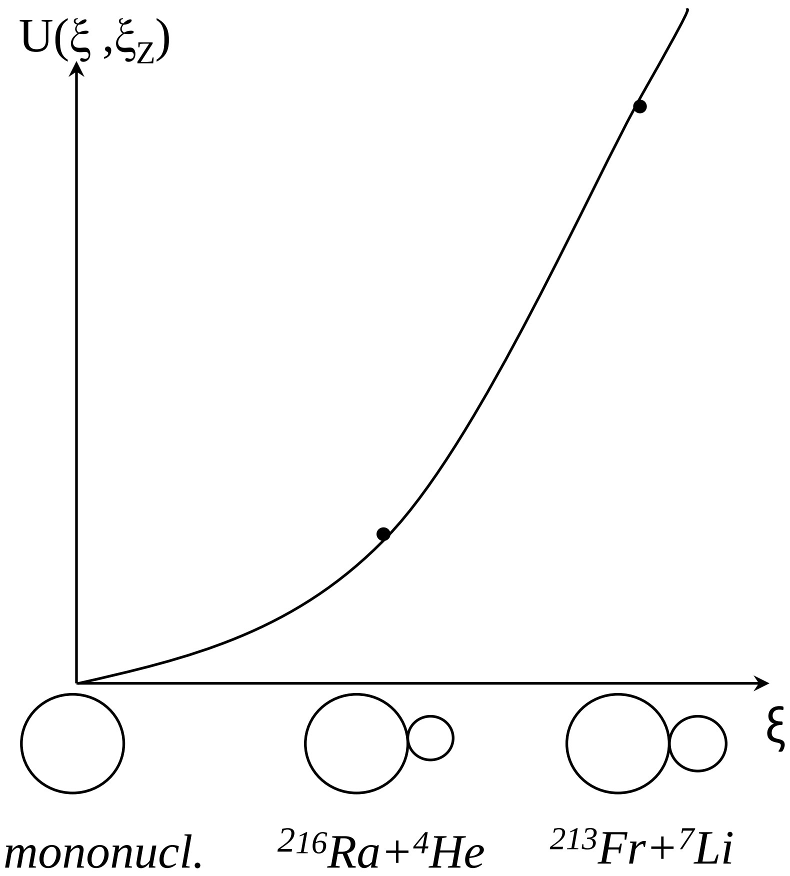
$$C_0 = 300 \text{ MeV fm}^3, \; f_{in} = 0.09, \; f_{ex} = -2.59, \; f'_{in} = 0.42, \; f'_{ex} = 0.54$$

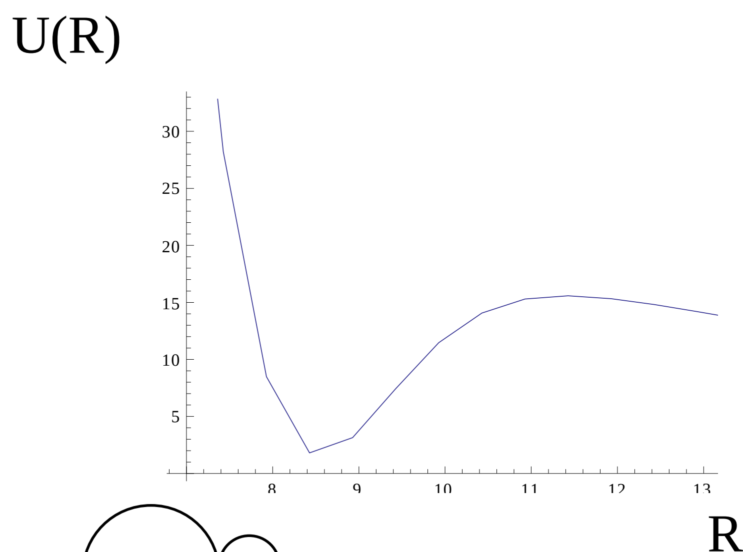
Potential Energy of the Dinuclear System

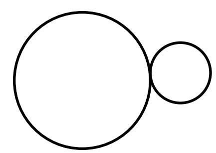
$$U(\xi, \xi_Z) = B_1(\xi, \xi_Z) + B_2(\xi, \xi_Z) - B + V(\mathbf{R} = \mathbf{R_m}, \xi, \xi_\mathbf{Z})$$

$$V(\mathbf{R}, \xi, \xi_\mathbf{Z}) = \mathbf{V}_\mathbf{coul}(\mathbf{R}, \xi, \xi_\mathbf{Z}) + \mathbf{V_N}(\mathbf{R}, \xi, \xi_\mathbf{Z})$$

$$V_N(\mathbf{R}, \xi, \xi_Z) = \int \rho_1(\mathbf{r_1})\rho_2(\mathbf{r_2})F(\mathbf{r_1} - \mathbf{r_2} + \mathbf{R})d\mathbf{r_1}d\mathbf{r_2}$$

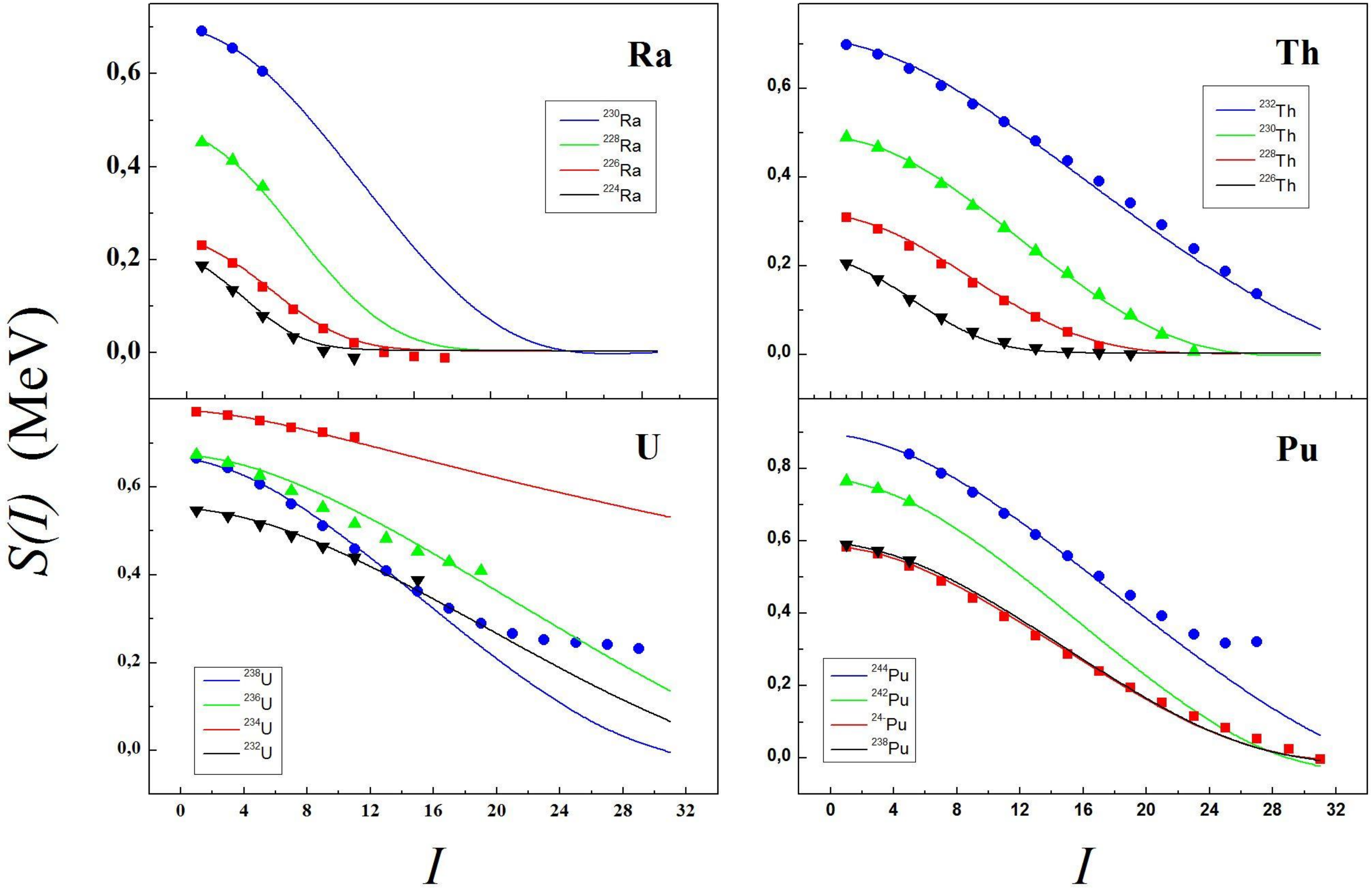






Touching configuration
$R=R_m$

Parity splitting in alternating parity bands



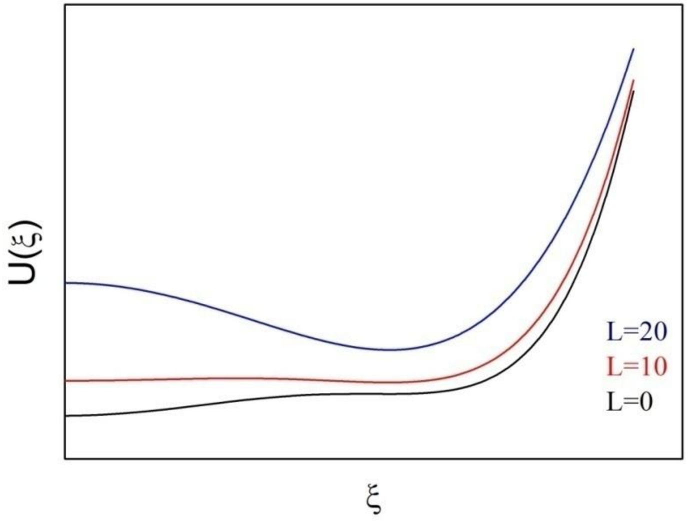
$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}$$

EPJ WC 107, 03009, (2016)

Angular momentum dependence of the parity splitting

Hamiltonian in mass asymmetry

$$H(\xi, L) = -\frac{\hbar^2}{2B}\frac{1}{\xi^{3/2}}\frac{\partial}{\partial \xi}\xi^{3/2}\frac{\partial}{\partial \xi} + U_0(\xi) + \frac{\hbar^2 L(L+1)}{2J(\xi)}$$



$$\xi = 0;$$

$$U(\xi, L) = U(\xi, L=0) + \frac{\hbar^2}{2}\frac{L(L+1)}{J_h}$$

$$\xi = 1;$$

$$U(\xi, L) = U(\xi, L=0) + \frac{\hbar^2}{2}\frac{L(L+1)}{J_{tot}}$$

$$J_{tot} > J_h$$

As a result the parity splitting decreases with angular momentum.

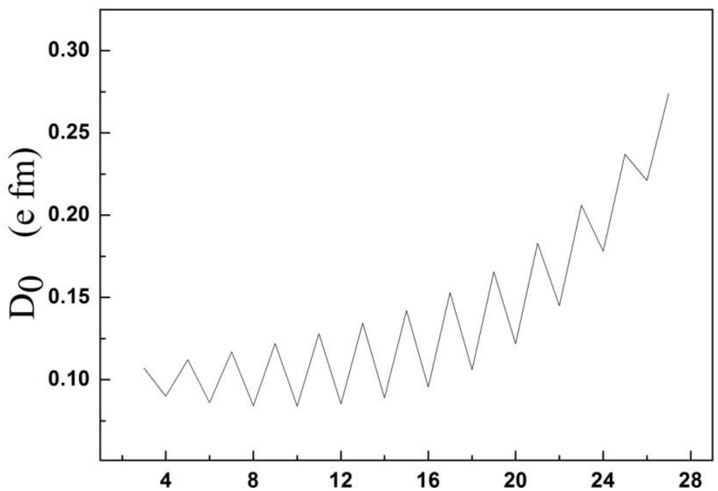
Electromagnetic transition probabilities

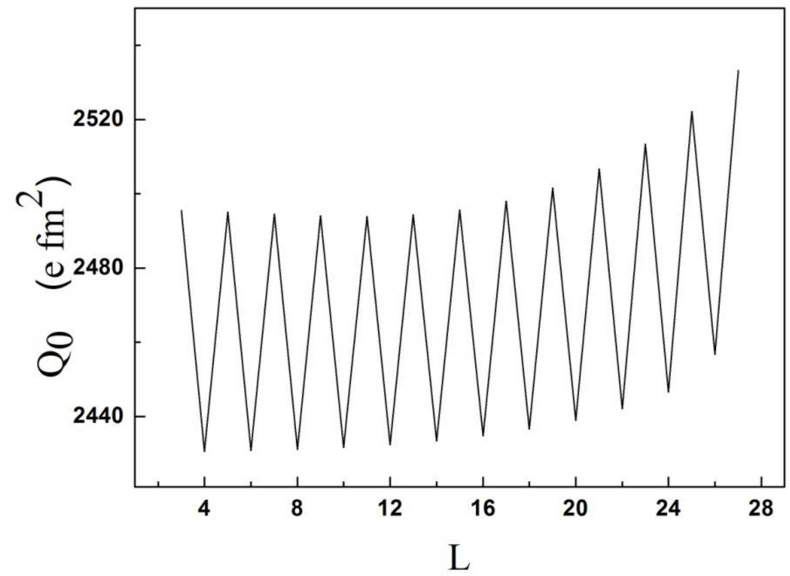
*(Exp.data are taken from: H.J. Wollersheim et al., Nucl. Phys. A**556 (1993) 261***

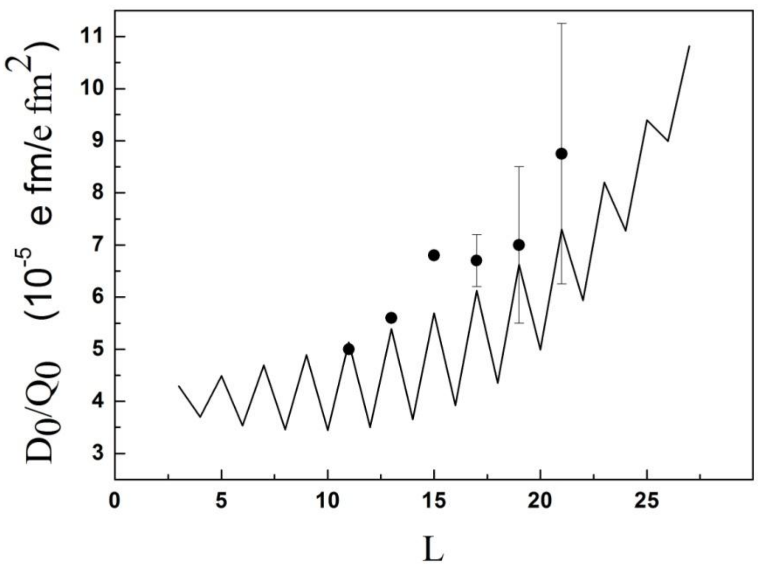
PRC 67, 014313 (2003)

Electromagnetic transition in ^{240}Pu

(*I. Wiedenhöver et al., Phys. Rev. Lett. 83, Number 11, (1999)*)



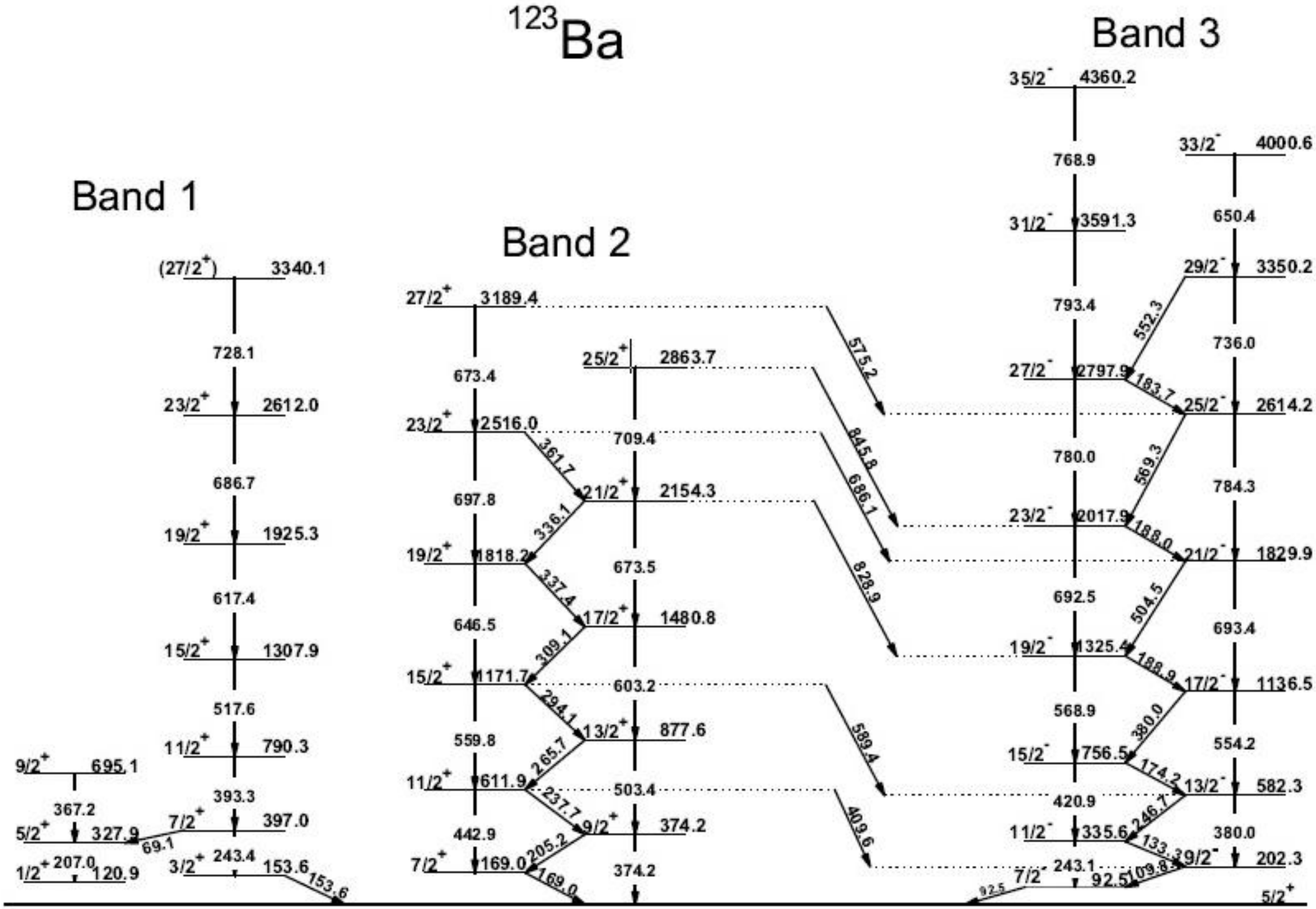




Ratio of transition dipole and quadrupole moments extracted from the *E1* and *E2* branchings $E1(I^- \longrightarrow (I-1)^+)/E2(I^- \longrightarrow (I-2)^-)$ as a function of the initial spin *I*.

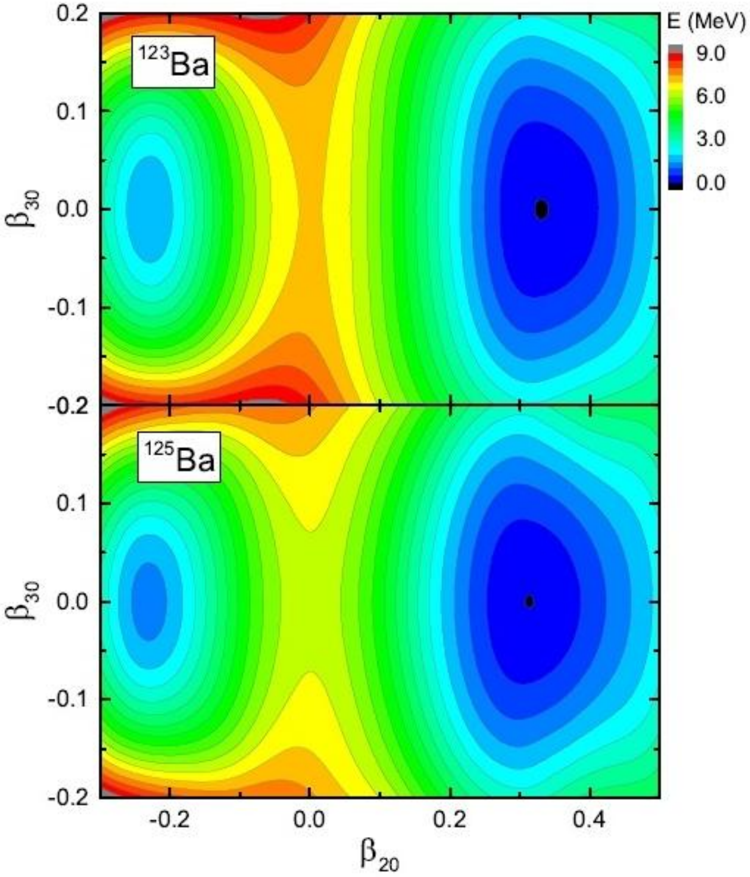
PRC92, 034302 (2015)

Reflection-asymmetric correlations in ^{123}Ba



PRC94, 021301(R) (2016)

PES for ^{123,125}Ba



Calculations have been performed in the frame of MDC-RMF model.

Although the minimum of the nuclear potential energy corresponds to the reflection-symmetric shape, PES for ^{123,135}Ba are very soft with respect to the reflection-asymmetric deformation.

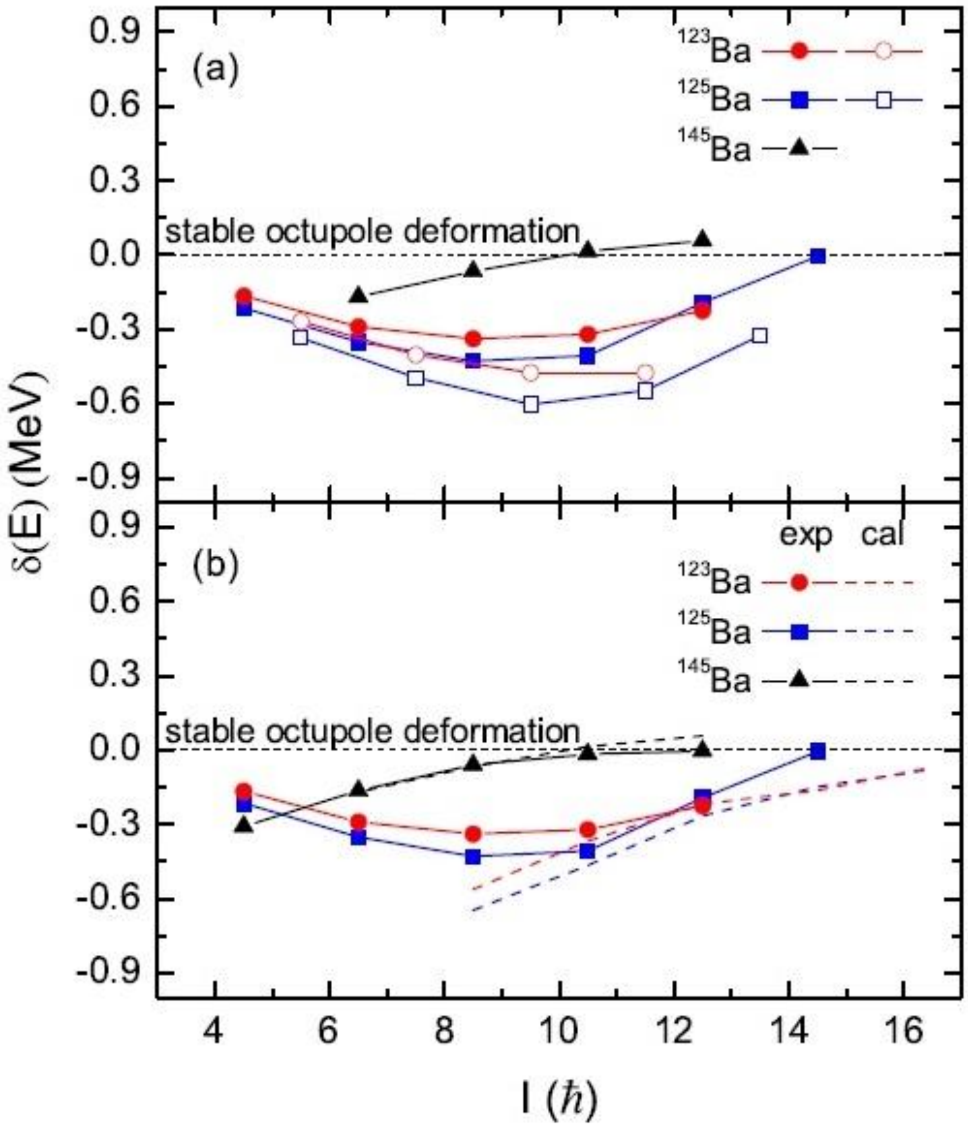
Using the DNS model one can estimate the critical value of angular momentum at which the stable reflection-asymmetric is developed.

$$I_{crit} \approx 13\hbar \quad \text{- for } ^{123}\text{Ba},$$

$$I_{crit} \approx 12\hbar \quad \text{- for } ^{125}\text{Ba}.$$

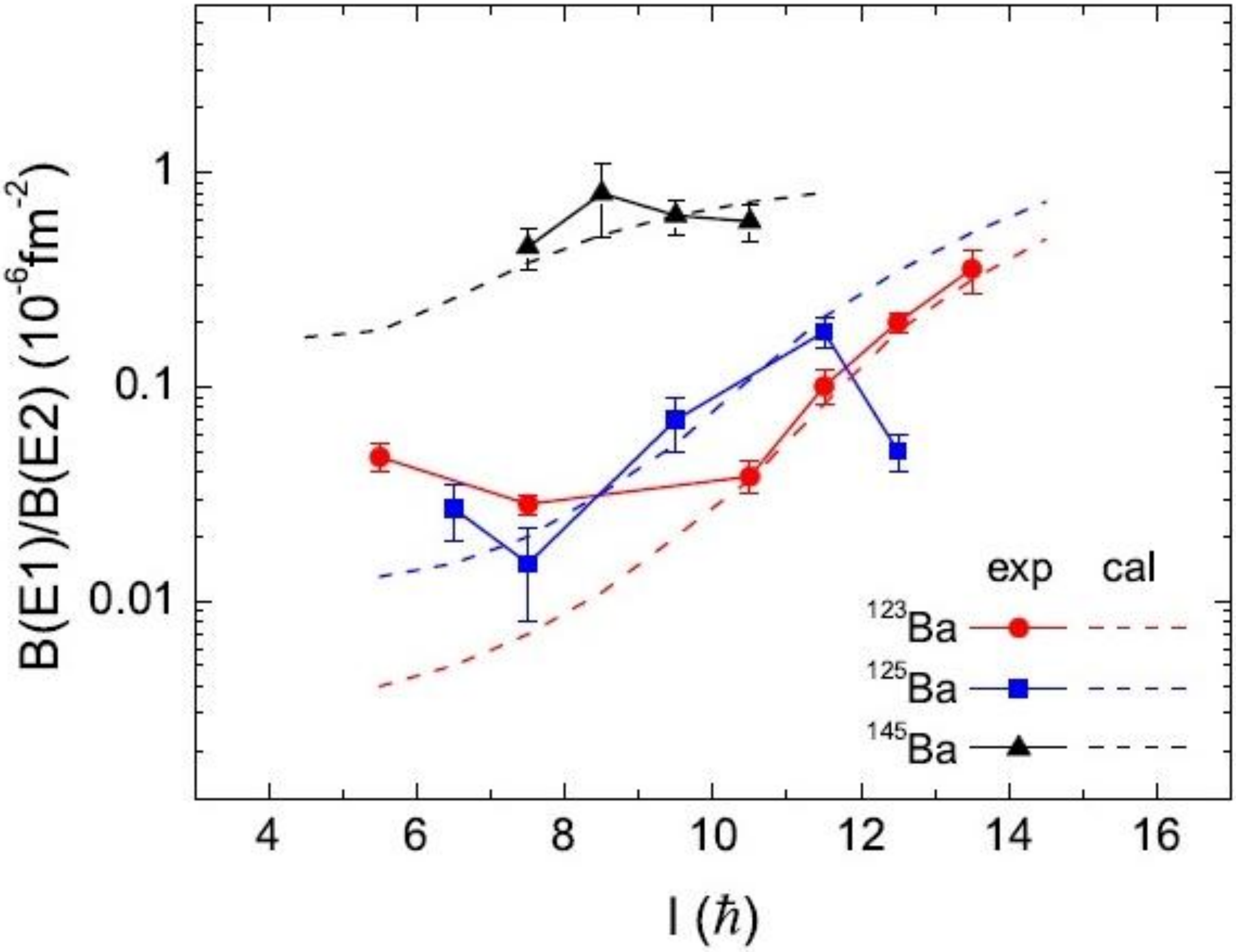
PRC94, 021301(R) (2016)

Parity splitting of ^{123,125,145}Ba



PRC94, 021301(R) (2016)

B(E1)/B(E2)-values for ^{123,125,145}Ba



PRC94, 021301(R) (2016)

Degrees of Freedom of Dinuclear System

The dinuclear system (A,Z) consists of a configuration of two touching nuclei (clusters) (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$, which keep their individuality.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$

Hamiltonian of the DNS model

The kinetic energy operator of the DNS then becomes

$$\hat{T} = -\frac{\hbar^2}{2B(\xi_0)}\frac{1}{\mu^{3/2}(\xi)}\frac{\partial}{\partial\xi}\mu^{3/2}(\xi)\frac{\partial}{\partial\xi} - \frac{\hbar^2}{2\mu(\xi)}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R}$$

$$+\frac{\hbar^2}{2\mu(\xi)R^2}\hat{l}_0^2 + \frac{\hbar^2}{2}\sum_{n=1}^{2}\sum_{k=1}^{3}\frac{\hat{l}_{(n)k}^2}{I_k^{(n)}(\beta_n,\gamma_n)} \qquad \left(\equiv \hat{T}_{rot}\right)$$

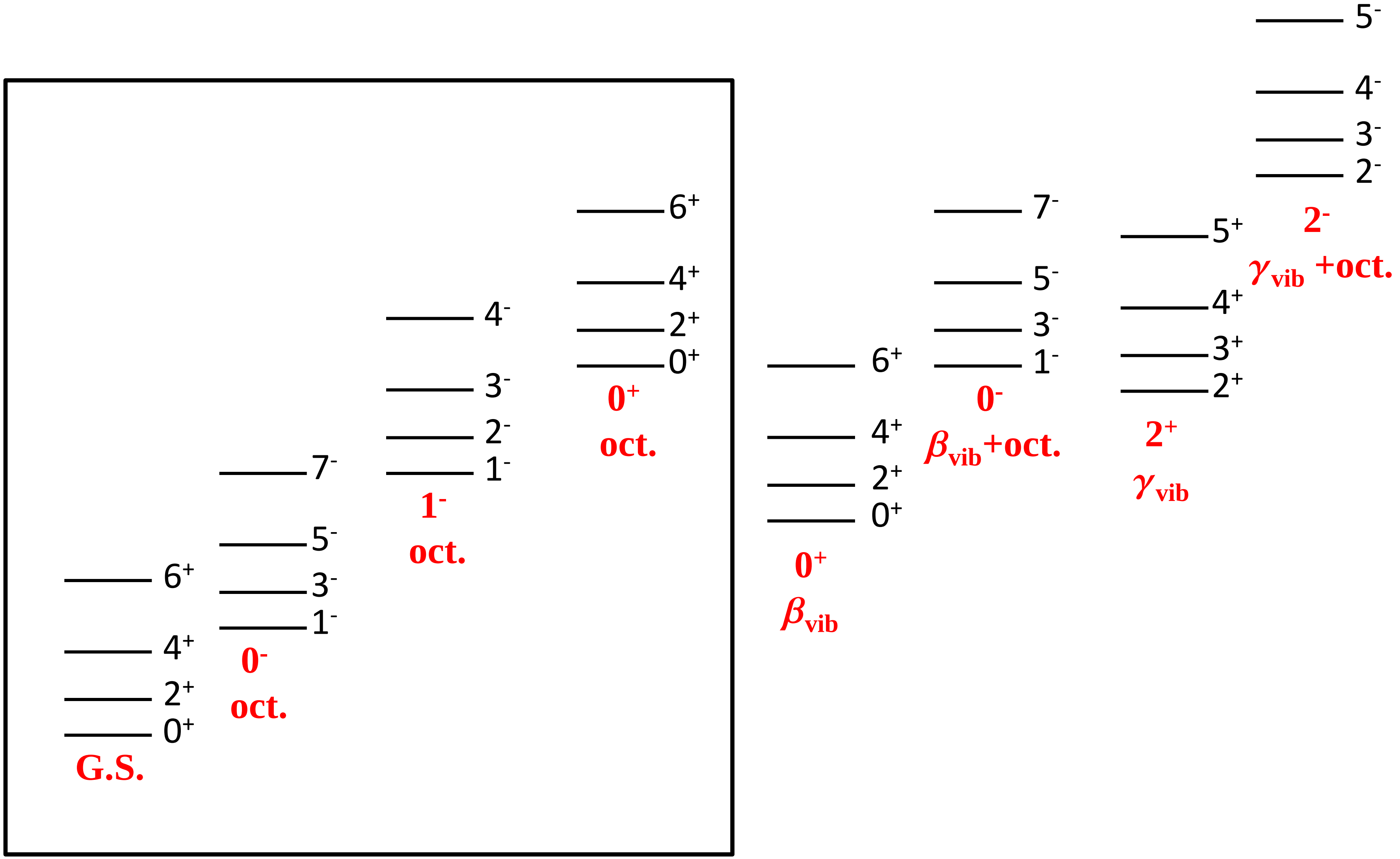
$$-\frac{\hbar^2}{2}\sum_{n=1}^{2}\frac{1}{D_n(\xi_0)}\left(\frac{1}{\beta_n^4}\frac{\partial}{\partial\beta_n}\beta_n^4\frac{\partial}{\partial\beta_n} + \frac{1}{\beta_n^2}\frac{1}{\sin 3\gamma_n}\frac{\partial}{\partial\gamma_n}\sin 3\gamma_n\frac{\partial}{\partial\gamma_n}\right)$$

$$\left(\equiv \hat{T}_{\mathrm{int}r}\right)$$

The potential energy of the DNS is

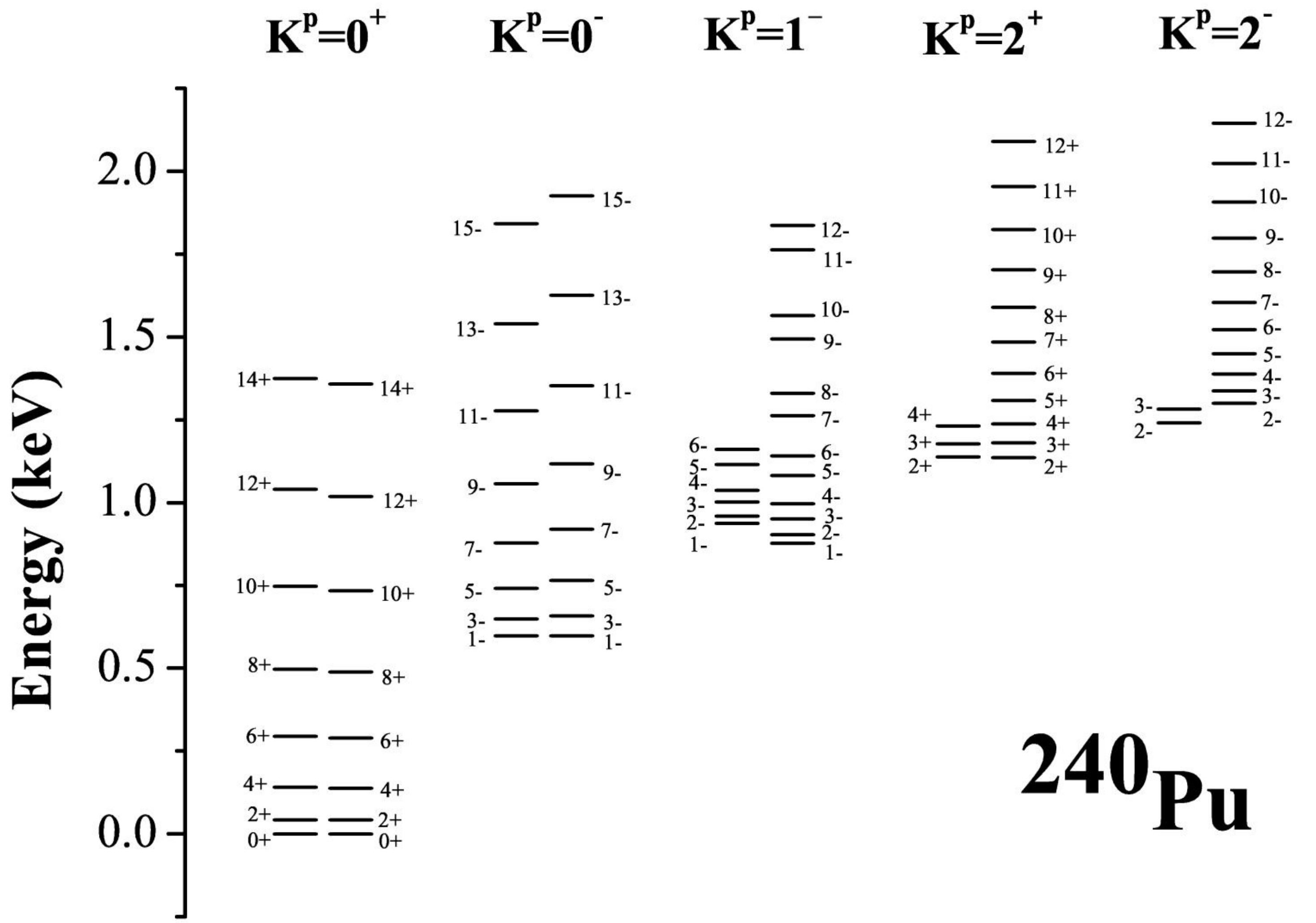
$$V(\xi) = E_1(\xi,\beta_1,\gamma_1) + E_2(\xi,\beta_2,\gamma_2) + V_N(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}}) + V_C(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})$$

Schematic Spectrum Produced by DNS Hamiltonian



Ground-State Well (^{240}Pu)

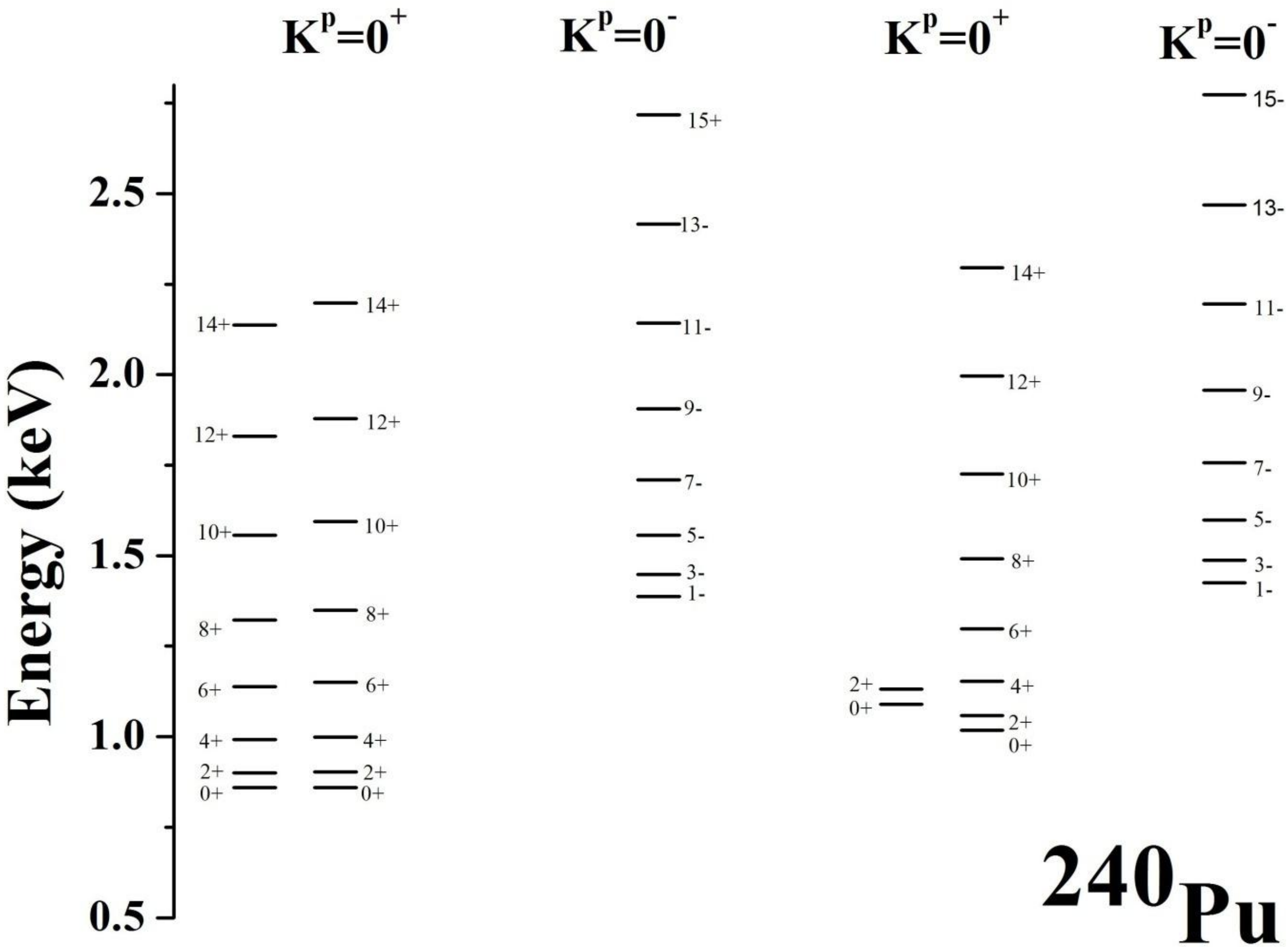
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

Ground-State Well (^{240}Pu) –continued

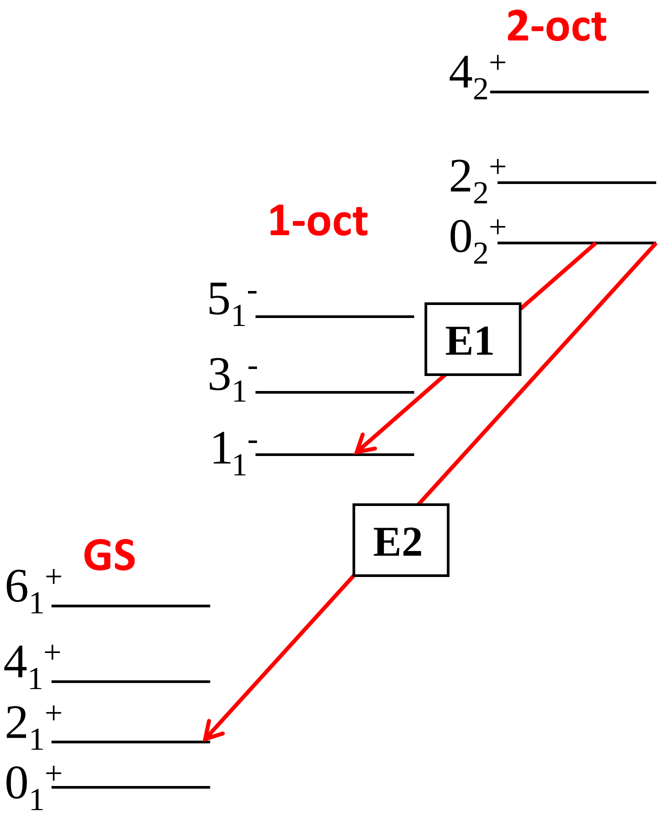
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

Electromagnetic Transition in ^{240}Pu

(Exp. Data are from *M. Spieker et al., Phys. Rev. C88, 041303(R), (2013)*)



Experimental B(E1)/B(E2) ratios (R_{exp}) are compared to the calculation of our model for the low-spin members of the $K\pi = 0^+{}_2$ rotational band in ^{240}Pu.

I_i^π	$I_{f,E1}^\pi$	$I_{f,E2}^\pi$	R_{exp} (10^{-6} fm^{-2})	R_{DNS} (10^{-6} fm^{-2})
0_2^+	1_1^-	2_1^+	13.7(3)	19.17
2_2^+	1_1^-	0_1^+	99(15)	99.95
2_2^+	1_1^-	2_1^+	26(2)	39.15
2_2^+	1_1^-	4_1^+	5.9(3)	8.57
2_2^+	3_1^-	0_1^+	149(22)	165.60
2_2^+	3_1^-	2_1^+	39(2)	64.9
2_2^+	3_1^-	4_1^+	8.9(5)	14.2
4_2^+	3_1^-	6_1^+	4.4(11)	6.9
4_2^+	5_1^-	6_1^+	4.7(13)	10.59

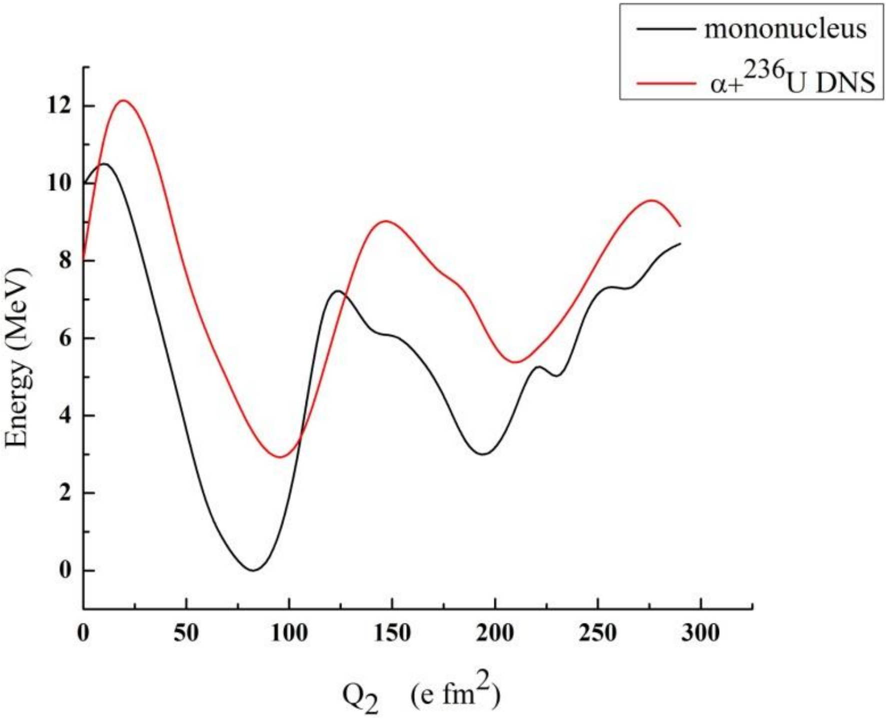
PRC92, 034302 (2015)

Fission Isomers of ^{240}Pu

The DNS model is applied to the study of ^{240}Pu as a compound fissioning nucleus, owing to the detailed experimental information on the spectrum in the second well of the barrier (as reviewed by ***P. G. Thirolf and D. Habs, Prog. Part. Nucl. Phys. 49 (2002) 325***).

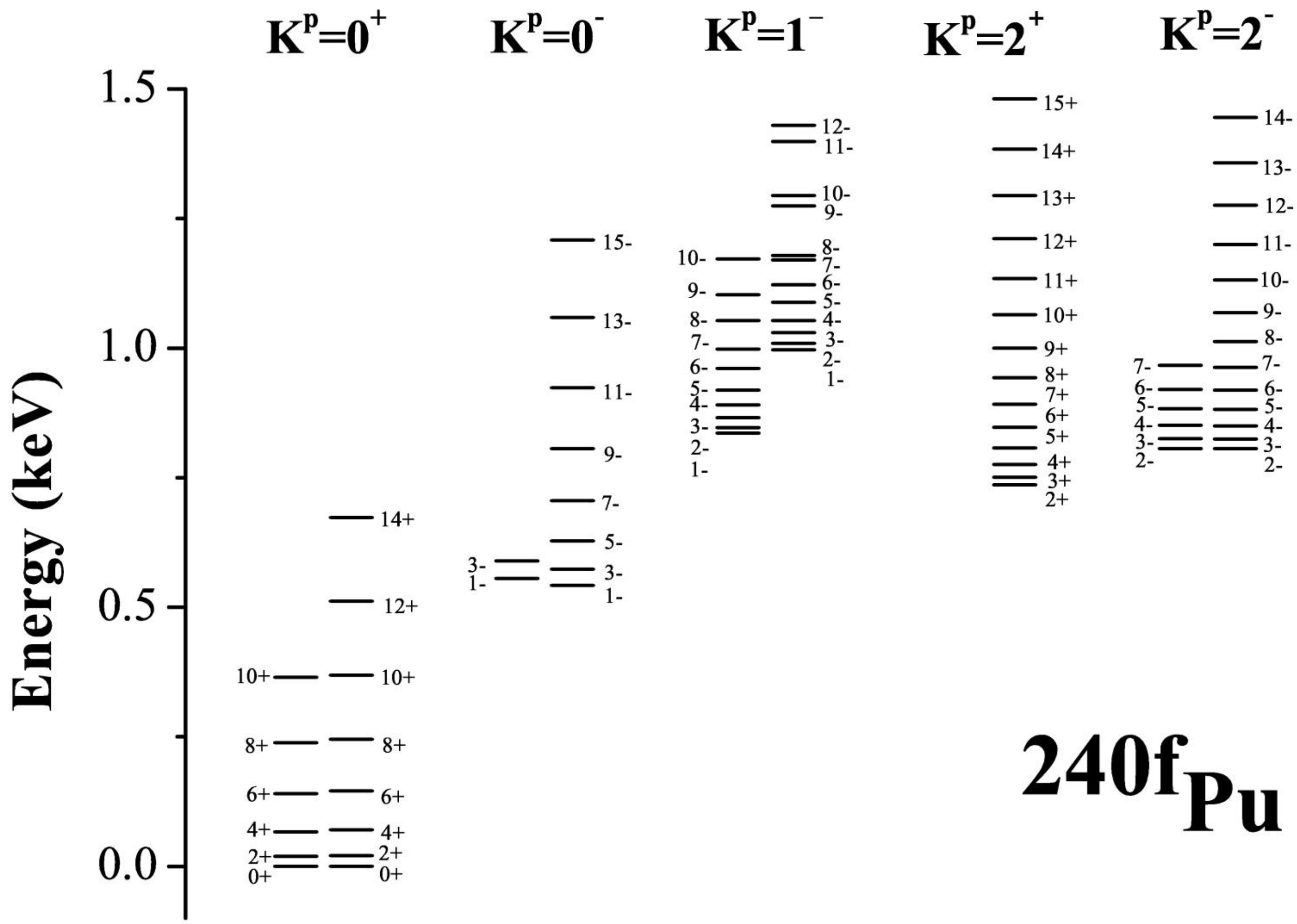
At ground state deformation and in the **superdeformed well** the favored combination is the mononucleus. The weight of $^{236}U+\alpha$ configuration is small (reflection-asymmetric vibrations).

At saddle-point deformations, the configuration $^{236}U+\alpha$ comes down in energy yielding a reflection-asymmetric shape.



Isomeric (Superdeformed) Well (^{240}Pu)

(Exp. data are from: P. G. Thirolf and D. Habs, Prog. Part. Nucl. Phys. **49** (2002) 325



EPJ WC, 38, 07001 (2012)

Conclusion:

- We suggested a cluster interpretation of the multiple negative parity bands in actinides and rare-earth nuclei assuming collective oscillations of nucleus in mass-asymmetry degree of freedom.
- The angular momentum dependence of the parity splitting and electromagnetic transition probabilities *B(E1)* and *B(E2)* are described. The results of calculations are in good agreement with experimental data.
- To take care of non-axially symmetric reflection asymmetric modes, the rotational and vibrational degrees of freedom of the heavy DNS fragment are considered.
- The excited 0^+ bands of reflection-asymmetric nature are explained as a bands built on the first exited state in mass asymmetry degrees of freedom.